Author:
Helliwell, Tracy

Title:
Developing a narrative-enactivist methodology for becoming a mathematics teacher educator

Developing a narrative-enactivist methodology for becoming a mathematics teacher educator

A dissertation submitted to the University of Bristol in accordance with the requirements for award of the degree of Doctor of Philosophy in the Faculty of Social Sciences and Law.

(Word count: 79,764)

Tracy Helliwell
School of Education
University of Bristol
May 2021

Abstract

This is a methodological thesis. The motivation for this study is a personal and professional one, which arose out of my need to learn a new set of practices and a new way of being with mathematics teachers as a new mathematics teacher educator. Over the course of this study, I explore a range of methodological issues that are pertinent to a self-based narrative inquiry, researching how I am becoming a mathematics teacher educator. In considering these methodological issues, I uncover a number of methodological principles that inform my approach to analysing one conversation between myself and an experienced mathematics teacher by using creative analytical practices. Through analysing that conversation, a central issue is revealed that provokes the need for an explanatory theory (the enactivist theory of cognition) and a conceptualisation of the process of developing expertise as a mathematics teacher educator.

Having uncovered a range of methodological principles, in relation to narrative inquiry and the enactivist theory of cognition, I bring these principles together, to formulate a narrative-enactivist methodology for *researching* how I am becoming a mathematics teacher educator which I express as a set of eight key methodological principles. This research methodology informs my systematic approach to analysing a set of audio-recorded conversations with a collaborative group of in-service mathematics teachers. Through enacting this research methodology, which combines categorical analysis with the process of telling stories, a narrative-enactivist methodology for becoming a mathematics teacher educator emerges. This methodology for becoming a mathematics teacher educator is ultimately expressed as five methodological dimensions, specifically, *using dissonance; staying with the detail; finding conviction; making it real;* and *going meta*. Each of the five methodological dimensions are described in relation to three distinct methodological levels: learning to teach mathematics and mathematics teachers; researching how I am becoming a mathematics teacher educator; and a way of working with mathematics teachers.

Acknowledgements

I thank the wonderful mathematics teachers with whom I have had the privilege to work. You have allowed me to learn from you, and alongside you, witnessing the care and attention you continually give to your students.

To Laurinda and Alf, for constantly challenging me over the last 22 years. You continue to make mathematics teaching and teacher education a wonderful world to be a part of. Somehow, without ever really telling me what to do, you have supported me to grow as a teacher and as a researcher. You both embody the kind of ethical expertise that I have done my best to articulate in my writing. I consider myself extremely lucky to have been able to work so closely with you both, for such a long time, and I look forward to that continuing.

Thank you to my family for having so gracefully put up with me and given me constant encouragement. To Chris for your continuing reassurance and support (and especially for feeding me and the children). To Oscar, Harriet and Sebastian for giving me plenty of distractions.

Author's declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

Signed: Tracy Helliwell

Date: 10th May 2021

For my family:

Pat and Alec, who are dearly missed,

Chris,

Oscar, Harriet and Sebastian.

Contents

Appendices

Tables and figures

List of tables

List of figures

Notes on how to read this dissertation

This is a set of explanatory notes on the following aspects: Progression of theoretical ideas; use of names; transcript and quotation conventions; referencing conventions; and use of footnotes, abbreviations/acronyms, and glossary terms.

Progression of theoretical ideas

This dissertation is best read start to finish. However, individual chapters can be read in isolation. As you move through the chapters, you will see ideas unfolding. Unlike some other dissertations, this dissertation is deliberately designed to show the development of ideas rather than having them all set them up in the beginning chapters. For instance, the theory of enactivism is not introduced until chapter five, at the point where it became necessary to use an explanatory theory.

Use of names

In terms of dealing with the individuals that appear in this study, I have, in almost all cases, used pseudonyms for anonymity (e.g., for the participating teachers). There are two exceptions to this: Laurinda and Alf, my doctoral supervisors. The reason for this is that they have been so influential in terms of my practice, both as a mathematics teacher and as a mathematics teacher educator. They appear explicitly in the stories I tell, and implicitly (Laurinda is sometimes referred to as the "little bird sitting on my shoulder"). It is worth now briefly outlining my history of connections with them both.

Laurinda has been part of my life since 1999. She was a tutor on an undergraduate education module I was taking, during my mathematical studies. In 2002, I began my PGCE with Laurinda as my university tutor. In 2004, I began my master's degree in mathematics education, again, with Laurinda as my unit tutor and dissertation supervisor. From 2012-2015, Laurinda and I have been in research collaboration, with Laurinda observing me teaching mathematics. In 2013, I began part-time doctoral study with Laurinda as my first-supervisor. In 2015, I was appointed to the role at the University of Bristol that Laurinda was flexibly retiring from, continuing to be my doctoral supervisor to this point (in 2021). Thus, Laurinda has been my mentor, tutor, supervisor and colleague for 22 years.

I first met Alf when I was on my second school placement during my initial teacher education course. I was placed, by Laurinda, in a school where Alf was both head of the

mathematics department and my school-based mentor (see glossary, page 291). I taught mathematics in that school, with Alf as my head of department, for five years until I became head of the mathematics department myself; Alf becoming an assistant head teacher in the same school. In 2010, Alf and I went our separate ways when I moved to be head of mathematics in a new school, and Alf went to work with Laurinda at the university. In 2015, I moved to the university as a university tutor alongside Alf. Alf has been my second doctoral supervisor since 2013.

Transcript and quotation conventions

Transcriptions (full and partial) can be found throughout appendix two and appendix three. I use simple conventions in the appendix versions, which includes the name of the speaker, a time stamp and the transcribed dialogue. A new row appears for each new turn.

Where I use transcribed dialogue in the main text, I have included punctuation to help express my interpretation of what is being said.

[…] is inserted in transcriptions and quotations to indicate a portion of text that is not used (i.e., a gap in the original transcript or quotation).

[ ] a letter inserted between square brackets indicates a change in letter case, from the originating text.

[ ] a word or phrase inserted between square brackets indicates additional information/context in order to make sense of the quotation or point to an observed action in a transcript.

… in a transcription suggests an interruption.

American spellings within quotes have not been edited with UK spellings (e.g., organizing).

A time-stamp is used before the beginning of any new section of transcript in the main body, using notation [session 2, 00:22:04] to indicate the feedback session (e.g., feedback session 2 denotes the second feedback session) and the time as marked in the full transcript in appendix two or three.

Referencing conventions

For referencing, I have used an adapted version of APA7, with a slightly modified use of capitalising in the references list.

Use of footnotes, abbreviations/acronyms, and glossary terms

There are occasional footnotes throughout this dissertation. I use footnotes when there is some contextual information needed. A footnote is used (as opposed to inserting in parentheses) either when there is too much text to add in parentheses or adding in parentheses would break the flow (e.g., in a diary extract, or in transcript).

There is a list of abbreviations and acronyms on page xv. Acronyms are spelled out fully in every first instance of use within a chapter.

There is a glossary of terms in appendix one. I use the glossary to explain any technical/culturally-specific terminology, signposting each first instance within each chapter.

List of abbreviations

The following is a list of the abbreviations and acronyms used in this dissertation. A number of these terms also appear in the glossary in appendix one.

BSRLM	British Society for Research in the Learning of Mathematics
CERME	Congress of European Research in Mathematics Education
DfE	Department for Education
HEI	Higher Education Institution
ITE	Initial Teacher Education
NCETM	National Centre for Excellence in the Teaching of Mathematics
NQT	Newly Qualified Teacher
Ofsted	Office for Standards in Education
PGCE	Postgraduate Certificate in Education
PISA	Program for International Student Assessment
QTS	Qualified Teacher Status
PME	Psychology of Mathematics Education
TIMSS	Trends in International Mathematics and Science Study

Chapter one

Beginnings: On not starting from scratch

I taught children mathematics for the majority of my adult life. Being a teacher was a huge part of who I was. When I left the classroom (summer 2015), where did that part of me go? I am not sure it went anywhere. Perhaps that is part of the problem.

Though I am still very much that teacher, I have come to recognise changes in myself since moving to university as a mathematics teacher educator. It is difficult to articulate such changes, but I might describe an element of this as me being more engaged with the world around me, or perhaps it is that I engage with the world in different ways. I have certainly felt awake in a way that I am not sure I have done in the past, or perhaps I am experiencing what it feels like to be out of my depth.

The move away from teaching children mathematics in secondary schools (students aged 11-19 years) to teaching teachers of mathematics at university, has proven to be more disruptive than I had allowed myself to imagine it might be. I thought, naively, that I would have what I needed, that I was equipped with my years of experience (13 years of teaching) and my convictions about mathematics teaching. Evidently, it was not that simple. There have been times where my convictions about teaching mathematics have

felt more like obstacles to becoming a mathematics teacher educator than any necessary foundation. As a new mathematics teacher educator, I found myself wondering if it might be easier if I could somehow start from scratch, to let go of my convictions about mathematics teaching and start over again. Of course, it was impossible to disregard what I had come to value so strongly, my experiences were part of who I had become, they shaped the way that I interacted with the world. Instead of trying to start afresh, I needed to work at seeing things differently, at broadening my perspective, at becoming a mathematics teacher educator. This meant developing a different set of convictions to that which I had developed as a mathematics teacher.

Having revealed a sense of the problem I was facing as a novice mathematics teacher educator, I now offer some further detail, concerning some of my significant encounters as a new mathematics teacher educator, encounters that motivated this study. I reflect on these encounters in relation to my experiences as a mathematics teacher towards an initial formulation of the research problem.

1.1 Recognising myself as a novice classroom observer

I officially started teaching at the University of Bristol in February 2016, having spent a period on maternity leave following the birth of my third child. Beginning in February meant that I joined the initial teacher education programme (a one year Post Graduate Certificate in Education course (PGCE), see glossary, page 291) as a university tutor, mid-way through the course. Within a week, I had gone along to a secondary school to observe my colleague Alf (see notes on how to read this dissertation, page xii), an experienced university tutor, during a visit to one of the prospective mathematics teachers from his tutor group[1]. As a university tutor, a school visit usually consists of observing a mathematics lesson (taught by the prospective teacher) alongside the school-based mentor (see glossary, page 291). The lesson is then followed by a three-way "lesson de-

[1] At the university, the PGCE mathematics group consists of around 30 prospective teachers who are usually taught as a whole group. The whole group is also divided into tutor groups. The university tutor is responsible for their own tutor group which involves visiting them in schools to observe them teach.

brief conversation" (Brown, Brown, Coles & Helliwell, 2019) between the university tutor, mentor, and prospective mathematics teacher.

As an observer during this initial school visit, I was interested in learning about the process involved in the de-brief conversation (see glossary, page 291). One feature that I observed was a set of three prompts, used to provoke the three-way discussion and directed at the prospective teacher: "What went well? What didn't go well? What would you do differently?" These were prompts I was already familiar with, having used them to begin conversations both as a school-based mentor myself working with prospective mathematics teachers, and as head of faculty working with more experienced mathematics teachers. Before moving to the university, my experience of mentoring teachers had always been in the context of a familiar setting (in a school where I was teaching). As a new mathematics teacher educator, I was apprehensive about what I should be offering during a lesson de-brief conversation, now that the setting would be less familiar. I was not accustomed to the schools, nor did I have a grasp of the way mathematics was being taught across the range of departments. I was not familiar with the different school cultures, the children, the school-based mentors, and I had only met the prospective teachers on one occasion[2] prior to commencing my school visits.

When I was a school-based mentor, not only was I operating within a familiar setting, but my role as a mentor was influenced by my personal commitment to developing a mathematics curriculum for *all* students at my school. This personal commitment meant that, as a school-based mentor, and as a head of department, I worked with mathematics teachers on developing their teaching in a way that did not contradict my image of a mathematics curriculum, in fact, I actively encouraged certain classroom practices. It may sound as though I was trying to create identical classrooms, but this was never my intention. I worked hard to develop a culture within my department, where teachers were curious about their teaching and where students in their classrooms were curious about mathematics. My hope was to enable the mathematics teachers that I worked alongside, to develop their own convictions about teaching mathematics but within a context that

[2] This was during a university *recall day* where the group of prospective teachers return to university during a period on school placement. There are two recall days during the longest school placement that takes place from early January for approximately 12 weeks.

prioritised mathematical thinking as a process of inquiry (that involves asking questions, finding patterns, making conjectures, and formulating arguments and proofs).

Following that initial school visit with Alf, I immediately began visiting those prospective mathematics teachers that made up my own tutor group. For the majority of these prospective mathematics teachers, my visit to them was their third and final school visit from a university tutor. I was ready with the set of three prompts ("What went well? What didn't go well? What would you do differently?") to begin the de-brief conversation and tried to imagine myself in a supporting role, supporting each prospective teacher in coming to their own set of development targets based on the lesson taught, which would be firmly situated within their own school contexts. I told myself not to be too concerned about operating in these unfamiliar environments, I could teach mathematics after all, I just needed to resist any urge to control the outcome of the conversations.

It turned out that those early school visits were less straightforward than I had anticipated, and I was right to be a little apprehensive. At times, during the de-brief conversations, I experienced moments of self-consciousness, accompanied by an awareness that I did not know what to do. I was not sure when to speak and when not to, when to offer my own views and when not to, or whether it was my place to be making suggestions at all. I wanted to be able to respond appropriately in the moment of the conversation, but I did not know how to do that. I felt uncomfortable, not constantly, but frequently. I found it was helpful having a set of prompts to begin the de-brief conversation, but what then?

In trying to rehearse in my mind what I had observed Alf doing during that initial school visit, I found I was unable to make any useful distinctions, beyond the three prompts. Maybe I had not paid enough attention. More likely was that my attention was on something else, on my own emerging thoughts and ideas about the mathematics lesson being discussed. I had not prepared myself to observe that lesson de-brief conversation, nor did I realise I might need to. Caught up in the moment, I suspect I involved myself too much in the conversation rather than attend to the process of observing the de-brief and what Alf was doing as the university tutor. I did not observe in a way that would help me learn.

I was a novice observer in that de-brief situation, in the same way that, as a prospective teacher, I was a novice observer in the mathematics classroom. In fact, I can still bring to mind a clear image of one particular lesson that I observed as a prospective teacher, around twenty years ago now. I am sat at the back of the classroom, observing a mathematics lesson, and thinking to myself, "wow, these kids are doing such amazing things, yet the teacher doesn't seem to be doing very much at all to make that happen!" Of course, the teacher was doing a great deal of things, I just could not see them yet. I have listened to a number of prospective teachers recounting similar experiences to my own. Seemingly, it is a common experience for classrooms to feel like mysterious places, as Bishop and Goffree (1986) explain:

> despite the student teachers' strong motivations for learning […] when they see a skilled teacher […] in action, it is difficult for them to know how anything happens. The pupils seem to know what they are supposed to do in their groups, the classroom discussion seems to happen almost naturally and without much obvious effort by the teacher, and abstract mathematical ideas are being debated sensibly by quite young children. (Bishop & Goffree, 1986, p. 310)

According to Bishop and Goffree, the task of the teacher educator is that of "*demystifying* the expert performance" which involves "removing the mystery and revealing the skills, techniques and knowledge of classroom teaching in order that they can become accessible to student teachers" (p. 310). I clearly needed to demystify for myself before I could think about doing that for the prospective teachers.

In schools, I learned to observe the teachers that I worked closely with, firstly as a new teacher observing my expert colleagues but then eventually as a school-based mentor, and a head of department, as an expert teacher myself. These were familiar classrooms and with that familiarity I developed a lens for observing mathematics lessons in parallel with my developing image of the mathematics curriculum, and my image of what mathematics classrooms *should* look like. I suppose, in that familiar context, I might have been described as "an expert in classroom observation of mathematics lessons" (Brown, 2015, p. 192), yet beyond that familiar context, I was no longer an expert. I was a novice observer, both in observing Alf during that de-brief conversation and as an observer of

mathematics lessons in unfamiliar classrooms as a new mathematics teacher educator. I needed to develop new ways of observing that were no longer inhibited by my own ideologies and biases. In a time of need, I found myself, as I have done in the past, turning to my diary. Before I share two diary entries that I wrote as a new mathematics teacher educator, I briefly explore some of the pertinent issues in relation to the process of diary writing.

1.2 On writing a diary

Lejeune (2009, p. 151) suspects that we turn to our diaries with passion, either during our adolescence, or when we are going through a crisis. Perhaps I was experiencing a crisis of sorts having moved to the university as a mathematics teacher educator. Greene (1995, p. 75) suggests that we shape the materials of lived experience into a narrative as a way of meaning making. I think a diary offers a space in which to do this shaping and to make meaning. I have found, on a number of occasions, that turning to my diary helps me to work things out, on occasions I have preferred just to forget.

I have been writing in a professional diary of sorts since 2002, not consistently, but occasionally. I tend to use a diary when I encounter a problem within my practice that I want to consider more carefully; I have done this since I started teaching. As a newly qualified teacher it became fairly routine for me to write in my diary. Sometimes I would produce an account of a lesson, carefully reconstructing the detail of who said what, before adding my retrospective reflections to these accounts. On other occasions, I would simply record a particular moment from a lesson, or an enduring thought. Occasionally I would make a pact with myself or devise a course of action. Whatever form my diary writing has taken, the *process* of writing in my diary has become a mechanism for working on my practice, a way of reflecting on a recent experience that I have wanted to dwell upon further, a way of resolving an issue that I have felt the need to resolve. Although I have worked at changing my practice as a result of writing in my diary, these intentions have not always been realised (as you will get to hear more about later on). Writing in my diary gives me a way of finding out, a way of uncovering meaning that may

have otherwise remained concealed, a way of questioning certain practices as well as confirming others. Writing in my diary helps me to develop new insights where my practice is concerned, and, I suppose, new insights where I am concerned.

I am also aware, from personal experience, that I have to be careful when it comes to writing in my diary. The issue I am pointing to here is articulated by Roquentin, the protagonist in Sartre's (1965) philosophical novel *Nausea* (also known as *La Nausée*), who tells the story of his daily observations through the medium of his diary entries. Roquentin begins the novel with a cautionary note about using his diary, when he warns, "I mustn't put strangeness when there's nothing" (Sartre, 1965, p. 9). He is concerned about the dangers of keeping a diary where the writer, who is "on the look-out", is inclined to "exaggerate everything", and to "continually stretch the truth" (p. 9). In essence, I am looking for something when I write in my diary, so it is important to keep in mind the dangers that Roquentin warns against. Unlike Sartre's protagonist, I am not searching for the meaning of reality itself, but I do write in my diary to find meaning in relation to my practice, both as a mathematics teacher and now as a mathematics teacher educator.

I can imagine, for example, how keeping a diary could become problematic if it were to become a completely isolated activity, that is, if you only ever considered your practice from your own perspective. After all, as Mason (2002) counsels, "[s]tudying oneself can become solipsistic and even narcissistic, if gaze is always inward" (p. 174), and it is quite conceivable, in this scenario, that certain aspects of my practice could well get exaggerated, become unhelpful fixations, or develop based on misinterpretations of situations. I may well be able to convince myself of all sorts of things, if I repeat them to myself regularly enough, if I "continually force the truth" so to speak. I might not have access, on my own, to different ways of interpreting a situation or to resolving a problem, I may be prone then to casting judgements, judgements that come from my existing ways of viewing the world of mathematics teaching.

Perhaps the problem I am exploring here stems from the traditional view of a diary as something that is private, something for the writer's eyes only. I have been lucky enough to work with and be supported by people who I feel safe to share my diary writing with.

The act of sharing the content of a diary, however, brings with it a whole new set of issues worth exploring. For instance, as Bruner (2003) suggests, "[t]elling others about oneself is [...] no simple matter. It depends on what *we* think *they* think we ought to be like" (p. 66). The fundamental issue that Bruner is calling attention to goes beyond diary writing to address any self-based writing that is made public, an issue that I will return to at different points throughout this thesis.

In terms of sharing my diary, I console myself that I have always aimed to remain open regardless of whom I have shared my diary with, and, after all, it is a diary about my teaching practice and not my inner most secrets. Why would I try to paint a different picture? Bruner's argument follows that in telling about ourselves we are *always* writing for an audience since there exists a "tacit *pacte autobiographique* governing what constitutes appropriate public self-telling" (Bruner, 2003, p. 66), a variant of which we follow even when telling ourselves about ourselves. Certain rules come into play about who we think we are or who we think we *ought* to be, even in the self-telling that we do privately. Perhaps, therefore, it is impossible to escape the consequences created by writing for some form of audience (which may only be ourselves).

I can imagine the existence of a tacit *pacte autobiographique* being partly about self-preservation in the sense of being able to come to terms with certain aspects of ourselves. Perhaps it is not possible to tell about ourselves without an awareness (conscious or otherwise) of the other. It also may not be possible, or even desirable, to escape the need to conform to such tacit rules and expectations, even self-imposed ones. Our tacit rules are, by nature, part of who we are. Nevertheless, when I think of my own young children, without so much need to conform to themselves, or to others, I feel a longing for them to remain that way for as long as possible.

In the next section, I share two diary entries, written within weeks of me beginning at the university, as a new mathematics teacher educator. Both entries illustrate how I have used a diary, either to formulate issues, or to generate questions, following a significant moment from my practice. I have chosen to share these two particular diary entries since both of them depict a state of tension, tension that arose within me during different situations but both in relation to me working with prospective mathematics teachers.

1.3 A novice mathematics teacher educator

This entry was written almost immediately after one of my early school visits to a prospective mathematics teacher whilst she was on school placement. Contextual additions are given as footnotes to keep the diary form:

> **Diary entry: 1st March 2016**[3]
>
> *I have just finished a school visit, the third one that I have made on my own. Hayley taught a group of year nines*[4]*. A top set*[5]*. The topic was simultaneous equations and there were lots of good elements of the lesson. I remember thinking what a nice class they were. This was my first time talking to Hayley, we met on recall day a few weeks ago when I was introduced to the whole group. She was quiet and didn't speak during the group session that I remember. This afternoon, we had an hour together immediately after the lesson that I observed, but without the school-based mentor who then joined us after that hour.*
>
> *I wasn't aware of having any expectations about how this particular de-brief might go, but I was surprised when she began by speaking so negatively about the behaviour of the students, perhaps because I had been so aware of my own emotional response to the class, as such a positive one. So, we didn't start with what went well! Which I became quickly aware of and was keen to focus on. She didn't seem to think anything went well, she even said she had thought about it and couldn't come up with anything. I said what I thought was something positive. Perhaps because the mentor was not there. I think I felt a bit awkward about doing this, I suppose because I have been thinking about my role during a lesson de-brief conversation. This was different in that the mentor was not there and in that she was struggling to see anything positive.*

[3] Date of diary entry has been amended for anonymity. A version of this diary entry can be found in Brown, Brown, Coles & Helliwell (2019, pp. 93-94).
[4] Students aged 13-14 years.
[5] See glossary, page 291.

My strategy was to turn to my notes from the lesson[6] and read them out, hoping that she might see a positive in what she had been saying. How much is it my role to state the positive? I am conscious of not casting a judgement, is judgement bad? How much of this is because I have been told judgement is bad? Is there a place for making positive judgements? As a teacher I would offer praise to students, for instance, if they made a valuable contribution to a discussion. What is the place of praise in my new role? I was not sure if she was even looking for praise or whether she believed it.

Was I uncomfortable? At times yes, but not intensely. I was mostly aware of a difference between what I thought I had observed Alf doing, what I intended to do, and what I was actually doing.

We talked about jobs; Hayley had decided to apply for jobs in the independent sector. We talked at some length about her reasons for this. To begin with this seemed to be about the behaviour of the students. I tried to unpick this with her, trying not to be positive or negative about state school vs. independent school, but I was intrigued about the reasons behind her decision. I was also reminded of a teacher that I once worked with who became very negative about the students that he taught and who ultimately left teaching before completing his induction year. I wanted to challenge this negativity towards the students. Through talking, Hayley was able to unpick her negativity and talk about what she did that she would like to have done differently and that the issue of the students' poor behaviour was actually an issue concerning a lack of motivation.

In reading this diary entry, I can still feel the way I felt during that de-brief conversation with Hayley; the awkwardness from not knowing how to behave or respond, the lack of conviction in my actions, a feeling of self-consciousness. Another read, more slowly this

[6] During lesson observations, notes are taken by the university tutor that aim to capture what the prospective teacher says and what the students in the classroom say.

time, I am struck by my concern for doing the *right* thing, but not quite knowing what that might be, neither in the moment nor retrospectively. I have selected the text that best demonstrates this sense of having concern for doing what is right:

> we didn't start with what went well! […] I said what I thought was something positive […] I think I felt a bit awkward about doing this […] How much is it my role to state the positive? I am conscious of not casting a judgement, is judgement bad? How much of this is because I have been told judgement is bad? Is there a place for making positive judgements? As a teacher I would offer praise to students […] What is the place of praise in my new role? […] I was mostly aware of a difference between what I thought I had observed Alf doing, what I intended to do, and what I was actually doing.

I find my need for following rules quite curious, yet I know there is a part of me that wants to do the right thing and would like there to be a set of rules to refer to. As a new mathematics teacher educator, I was looking for rules to follow, for a recipe, for certainty, to be the same as my colleagues and predecessors[7], but of course, as Varela (1999) points out, "such rules will always remain external" since they will always differ from our "internal inclinations" (p. 30). I wanted my intentions to align with my actions (or inclinations), but even my intentions were unclear. I lacked the strength of conviction that I had become accustomed to as a mathematics teacher, and I have experienced this lack of conviction many times since moving to the university, most strongly and frequently as a *new* mathematics teacher educator.

The following diary entry (again with contextual additions given as footnotes) speaks to some of the same issues as the previous one but was written immediately following a different scenario; an interview for potential prospective teachers, led by Alf and myself.

[7] It is important to note, that I work on the same PGCE course that I completed as a prospective teacher.

Diary entry: 18th April 2016[8]

We were interviewing for new PGCE students today. Interviewing is something I have done a reasonable amount of since starting here in February. We are still recruiting for September. I am conscious of the fact that there has been a strong philosophy and approach to the teaching on the course and this begins with the interview.

Recently I seem to be reflecting on everything, alone and with my colleagues. What are the rules?

During the interview, I take notes[9], I try to listen to what is said and capture that on my page. I find this difficult as I can't write quickly enough, and my urge is to watch the body language in this performance. I think I will miss out if I don't watch but notes are what we do. I become aware that I am not sure when it is OK to intervene in the group task, so I pay attention to Alf who I am a little surprised by when he intervenes early on, not just once but a few times. I then feel like I can do the same. I say, "Try and focus on what the triangular number represents."[10] To be honest, I was a little frustrated with the progress that was being made on the problem.

On reflection, I have felt like this before, that sense of not knowing when to intervene and when to just let things take their course. What is the purpose of the group activity? To watch how participants behave in a group? To make sure they can do some mathematics? To find out if they can communicate? To see how they reflect afterwards? If these are the purposes, then why intervene at all? Perhaps because otherwise, I guess we might be there forever.

[8] Date of diary entry has been altered for anonymity. A version of this diary entry can also be found in Brown, Brown, Coles & Helliwell (2021).

[9] One element of the interview for the PGCE course, is an activity where the group of candidates work together on a mathematics task where they are each given a subset of information that is needed to solve a problem. They work together on the problem and as university tutors we observe this group task, writing down what is said to aid the conversation afterwards. The candidates then have a conversation between themselves after the task, to reflect on the process of working on the task together.

[10] Triangular numbers represent a particular piece of information needed to solve the problem.

As was apparent from my account of the de-brief conversation with Hayley, it was also apparent from my account of this interview situation that I was eager to learn the rules, to be the same, to know the reasons. Yet simultaneously, I lacked the conviction to enact those rules, rules such as recording what gets said during the group task, rather than purely watching.

A core tenet of the PGCE course at the University of Bristol, is that "there is not one way of teaching mathematics", these words I have spoken many times during PGCE interviews, but as a new mathematics teacher educator, those words were empty. Even though I understood and appreciated the principle behind those words, my appreciation was inconsistent with what I had come to know through many years of teaching mathematics and developing a mathematics curriculum. I could easily speak the words when we would interview potential prospective teachers for the course, but I was not yet able to speak them with conviction, even though I could rationalise them to others, and not unconvincingly.

1.4 Feeling like an outsider

Soon after visiting each of the prospective mathematics teachers from my tutor group, they finished their main teaching placement and returned to the university (in April 2016) for the next taught part of the course. It was during those PGCE subject sessions (see glossary, page 291) that I was struck by a feeling of not belonging. I felt like a guest speaker and in one sense that is what I was. What did I have to offer the group of prospective mathematics teachers, beyond my own image of mathematics teaching? Both in lesson de-briefs and in taught parts of the course, I was conscious of not wanting to *tell* the prospective teachers how to teach, or even tell them how *I* would teach, but I was unsure what the alternatives might be. I felt unable to be myself during those PGCE subject sessions, I wanted to do the right thing, but I did not know what that meant.

Having come to the university with such strength of conviction about teaching mathematics, I found myself experiencing a sudden sense of loss.

The sense of loss I was feeling led me at times to question my move to mathematics teacher education from what had become a comfortable position in school where I felt both confident and respected. I was determined to learn though, to develop myself as a mathematics teacher educator, to feel that sense of belonging that I had felt in school. I recognised that I needed to work differently from the way I would have worked with the mathematics teachers from my own department when I was in school but knowing this was only the initial step to becoming a mathematics teacher educator. With an awareness of the need to change, I found myself facing what felt like a necessary process of letting go of certain ways of being, that I had previously valued so strongly, to become something other. At times, I wondered if it might have been less problematic to have been *starting from scratch*.

~~~~~
~~~~~

Chapter two

A process of becoming

Chapter one served as an introduction to the problem that is the basis of my study. Through interrogating some of my early experiences as a mathematics teacher educator, I became aware that I did not know how to be a mathematics teacher educator in any of the teacher education scenarios involving prospective mathematics teachers that I had been involved in (such as lesson observations, lesson de-brief conversations, interview situations, and PGCE subject sessions at the university). In examining some of my earliest experiences as a mathematics teacher educator, I realised that my way of seeing the world of mathematics teaching was limited to my own ways of being in the classroom, and that being an expert mathematics teacher was not a sufficient prerequisite to being a mathematics teacher educator. I was compelled to study my practices, to learn to become an effective mathematics teacher educator and to develop a sense of conviction in my actions. I looked to existing research on the development of mathematics teacher educators to get to know the field and to read about the experiences of others. In the early stages of my study (a study that has spanned approximately five years), I had very little sense of what might be possible in terms of an approach to researching my own learning and development, the process of learning seemed to me to be a difficult phenomenon to

observe, capture and report on in a meaningful way, and not something I would be able to easily grasp.

2.1 Finding inspiration in the work of others

I found early inspiration for my own study in the work of two particular authors. Firstly, in Nicol's (1997) doctoral thesis she reports on the problems, tensions, and dilemmas that she experienced as a beginning teacher educator learning to teach prospective elementary teachers to teach mathematics. Nicol's study describes her efforts in designing and investigating a pedagogy of mathematics teacher education, which attempts to place inquiry at the focus of teaching and learning. I found Nicol's thesis a huge source of encouragement, both in terms of knowing that a study existed where the researcher had researched her own learning as a mathematics teacher educator, but also in terms of the candidness she was able to bring to her research. I feel I know exactly what she meant when she said it "takes courage to conduct research as a teacher educator on one's practice and to share such research on learning to teach with others" (p. 81). I have found myself feeling exposed as a novice teacher educator reporting on my experiences to a community of experts. Having presented some of my initial ideas I have received comments such as "the problem is you are researching *yourself*; you need to research *somebody else*". However, as Mason (1998) reminds us, the most significant products of research in mathematics education are the "transformations in the being of the researchers" (p. 357) and I was determined to become an effective mathematics teacher educator even if that meant working "at the edge of incompetence" (Eisner, 1993, p. 10). Nicol (1997) also outlines the ways in which she dealt with the complexity of reporting on a study of her learning, by considering a number of issues (such as time, voice and tense), issues that I myself have encountered and that I consider explicitly within chapter three (sections 3.1-3.5 inclusive).

The second study, where I found early inspiration, was Tzur (2001) who tells the story of his own development as a mathematics teacher educator as a way to abstract notions of more general implication. In doing so, Tzur formulates a "four-foci model of teacher

education" (p. 274) which consists of four levels: 1) learning mathematics; 2) learning to teach mathematics; 3) learning to teach teachers; and 4) learning to mentor teacher educators. Mathematics teacher educators are those who have *developed* from the lowest level (learning mathematics) to a higher level (learning to teach teachers), where development:

> entails a conceptual leap that results from making one's and others' activities and ways of thinking at a lower level the explicit focus of reflection [...] Through such reflection, the developing teacher educator may construct conceptions about learners and learning at the lower level(s), conceptions that become the theoretical ground for one's praxis. (Tzur, 2001, p. 275)

For Tzur, mathematics teacher educators develop through reflecting on "what it means to teach mathematics"; "how someone comes to know how to teach mathematics"; and "how someone's activities promote others' learning of mathematics teaching" (p. 273). Tzur's study demonstrated to me how powerful self-reflection can be and how it can form the basis of rich data about what it might mean to learn as a mathematics teacher educator. I see my research as building on the work from the two studies presented here in that they both centre around the analysis of the researcher's own experiences as a mechanism for researching their learning and development as mathematics teacher educators.

2.2 What constitutes data?

Once I had resolved to research my own learning, one decision I needed to make was how I would go about capturing my lived experiences as a mathematics teacher educator, i.e., what would constitute data in my study, and how I would go about collecting it. Potentially I could draw on any of my lived experiences as a new mathematics teacher educator including my lived experiences preceding my move to the university, experiences that had shaped me as a mathematics teacher.

Nicol (1997) and Tzur (2001) both draw on experiences from their own practice (Nicol uses the term "critical incidents" (Tripp, 1993) and Tzur refers to "experience fragments" (p. 264)) that they weave into a narrative that tells the story (narrative and story are

discussed in section 2.4, page 26) of their learning and development as mathematics teacher educators. Tzur (2001) reflects on fragments of experience that he orders according to the sequence in which his four foci were developed to tell his story. He offers a series of fragments, one at each of the four levels, along with an analysis of each one in terms of its significance, before reorganising the story and developing his four-foci model. Nicol (1997) captures a range of sources of data to inform her study including video recordings of classroom events (from the mathematics methods course where she was teaching prospective teachers), her professional diary and field notes, and the prospective teachers' journals. In capturing herself in-action during teaching scenarios with prospective teachers, she was able to analyse these significant events both from her perspective at the time of the events, and then again, a year later, from a renewed perspective.

From keeping a professional diary, I was accumulating a set of retrospective accounts each of which reflected my perspective at the time of that diary entry being made. These retrospective accounts gave me access to certain significant moments from my practice along with my (almost) immediate retrospective reflections in relation to those significant moments. Like Nicol (1997), I also wanted to capture myself in action as a new mathematics teacher educator, to complement the lived experiences being captured in my diary entries. I also wanted to capture the detail of my being in-the-moment, especially the detail of my interactions with others, since so much of what I was struggling with as a new mathematics teacher educator, was knowing how to interact effectively with the prospective mathematics teachers I was teaching.

As a new mathematics teacher educator, I was fortunate to be able to work with in-service teachers of mathematics, as well as prospective teachers. Ethically, it felt more comfortable to capture (i.e., record) myself in action with in-service mathematics teachers, especially given the perceived power relation between myself as a university tutor and those prospective teachers of mathematics who I was responsible for teaching (and assessing[11]). In terms of working with in-service mathematics teachers, two distinct

[11] My role at the university involves assessing master's level assignments written by the prospective teachers within my tutor group as well as moderating the assessment of teaching (and the subsequent award of Qualified Teacher Status (QTS), see glossary, page 291).

(but related) opportunities presented themselves to me early on in my study, providing me with two different contexts in which to capture myself in action. I was sure that, by studying my learning as a mathematics teacher educator in relation to the learning of in-service mathematics teachers, I would be directly informing my practice with the prospective mathematics teachers and that I would be changed by my study in ways that would make me a more effective mathematics teacher educator.

In the next sections, I provide the detail from each of the two contexts: working with one experienced teacher (context one: section 2.2.1); and a collaborative group of in-service mathematics teachers (context two: section 2.2.2), including an account of how these opportunities came about and some ethical considerations. Data collected from each of these two contexts, along with my diary entries, constitute much of the data for this study.

2.2.1 Context one: Working with one experienced teacher

I first met Sam when I was a mathematics teacher. She had heard about our mathematics curriculum from one of her colleagues who had spent some time on placement as a prospective mathematics teacher in the school where I was teaching. The mathematics curriculum in the school where I worked was often described as "project-based" where students worked for extended periods of time, often for an entire school term (based on six terms per academic year), on a particular project. The projects were designed to incorporate a range of mathematics topics and skills as well as develop the children's mathematical thinking through a framework of conjecture–counter-example–proof, akin to "inquiry–based mathematics education" (Dorier & Maass, 2020, p. 384)[12] where students are supported to work as mathematicians do. Sam was an experienced science teacher and had been head of the science department in her school for many years. She had recently moved over to teaching mathematics as well as leading the mathematics

[12] "Inquiry-based mathematics education (IBME) refers to a student-centered paradigm of teaching mathematics and science, in which students are invited to work in ways similar to how mathematicians and scientists work. This means they have to observe phenomena, ask questions, look for mathematical and scientific ways of how to answer these questions (like carrying out experiments, systematically controlling variables, drawing diagrams, calculating, looking for patterns and relationships, and making conjectures and generalizations), interpret and evaluate their solutions, and communicate and discuss their solutions effectively." (Dorier & Maass, 2020, p. 384)

department having completed a teacher subject specialism course (see glossary, page 291). When she got in touch with me and asked if she could visit me in school, I happily agreed since I enjoyed any opportunity to talk about teaching mathematics and I was curious about her own curriculum plans.

When we met, I was immediately struck by Sam's energy as she expressed her aspirations for the mathematics department that she was now leading. She spoke about her dissatisfaction with the current state of affairs as she saw them at her school, describing a mathematics curriculum that was organised into a sequence of discrete mathematical topics, which the students returned to and repeated year on year. I think her many years of science teaching had given her an interestingly different perspective on how mathematics could be taught. After an extended conversation with me, she asked to take a copy of our scheme of work (a curriculum document detailing the course content and structure) with the purpose of using it to inform the design of her own "intended" (Bishop, 1994, p. 17) mathematics curriculum.

I had no further contact with Sam until I had been working at the university for about seven months. At this point, it was September 2016, the beginning of my first new academic year at the university. As a new mathematics teacher educator, I had wanted to spend time in somebody else's classroom beyond the classrooms of the prospective teachers that I worked with, to experience what it might be like as a classroom observer in a school where I had not taught myself and where I was not in my role as a university tutor. Sam came to mind immediately as the somebody who might be happy to work with me. She had mentioned, when we had met previously, that she would welcome the opportunity to work together in the future.

When I contacted Sam, we spoke on the phone about possibilities. I suggested she think about whether there was something in her teaching that she was wanting to work on, something for us to work on together, providing a focus for me as an observer. I had in my mind our previous conversations about mathematics teaching and curriculum design. She had felt like somebody whose views of mathematics teaching I aligned with. We had similar values which felt promising. She was no longer leading the mathematics department, but instead was now working as a lead practitioner (see glossary, page 291),

supporting newly qualified and in-service mathematics teachers with their teaching, as well as being a mentor to prospective mathematics teachers on placement in her school.

Sam seemed keen to begin a collaboration with me. Soon after our initial phone conversation, I went to visit Sam in her school to continue our conversation and to explore possible areas of focus. Sam talked and I made notes (notes that were sent to Sam following this initial conversation, see appendix 2a, page 295). By the end of this conversation, Sam had articulated the following two questions that she was keen to think about in relation to her teaching practice (by "jump in", Sam was referring to intervening before a student makes an error in their mathematics, e.g., in their written work, or in verbalising their mathematical reasoning):

> Is my tendency to jump in proving to have a deleterious effect on the students' learning?
>
> If I am not going to jump in, how do I build a culture where it is safe to make mistakes and where the students build resilience?

Together we planned for me to make a series of visits (from November 2016) to her school, approximately once per fortnight, to observe her teach a lesson and to talk afterwards. I was interested in developing myself as a classroom observer in an unfamiliar mathematics classroom. I also wanted to learn how to support Sam in developing her own practice, which meant learning how to respond effectively during any post-lesson conversations we might have. We agreed I would audio record the post-lesson conversations (see full ethics application, appendices 4a-4c), and that I would share any of the conversations that I transcribed so that we could refer back to them during subsequent conversations if there was a need to. The first two post-lesson conversations lasted substantially longer (each one lasting approximately 30-40 minutes) than the subsequent ones, due to Sam's changing commitments. I use the second of these audio-recorded post-lesson conversations with Sam (see appendix 2b, page 296, for a partial transcript of this conversation) in the way described in the following chapter (see section 3.6), as the raw data that forms the basis of the story of

chapter four. A further section on ethical considerations in relation to working with Sam (and indeed other teachers) is the topic of interlude one (page 68).

Sam joined the collaborative group of in-service teachers (along with nine other teachers) that became the second context (context two) in which to capture myself in action as part of this study. I stopped visiting Sam in schools in around March 2017 but continued working with her as part of the collaborative group. I give the detail of context two in section 2.2.2.

2.2.2 Context two: A collaborative group of in-service mathematics teachers

I was invited to be the Higher Education Institution (HEI) representative for one of the projects led by the local Maths Hub (see glossary, page 291). Each Maths Hub network includes multiple teacher research groups, work groups and projects. The project I was invited to participate in was a key stage three (students aged 11-14 years) mathematical reasoning project designed to support mathematics teachers in developing the mathematical reasoning of the students in their own classrooms and the classrooms of mathematics teachers from their wider departments. The project involved ten mathematics teachers from five different local secondary schools, a project lead (see glossary, page 291), and myself as the HEI representative. As a group, we met four times over the course of each year for two years. My role as the HEI representative was to support a feedback session at the beginning of each full-day workshop, from the second workshop of the first year, where the mathematics teachers would talk about what they had been doing in schools in relation to the work of the project, and specifically in relation to their gap task (see glossary, page 291). The gap tasks were defined by the project designers from the National Centre for Excellence in Teaching Mathematics (NCETM, see glossary, page 291) and then interpreted by participating mathematics teachers. Having gained participants' permission, I used an audio-recording device to capture each of the feedback sessions. Initially the project was aimed at tracking the development of the teachers in relation to working on their students' mathematical reasoning (see participant consent, appendix 4e). The first recorded feedback session gave me an

opportunity to examine the interactions within the group, including my own contributions, and to consider in what ways these interactions were supporting the learning of the mathematics teachers during that conversation. Prompted by using (with Sam's permission) an extract involving Sam from the first recorded feedback session, at a national conference (see section 8.1.3, page 155, where this is described in detail), the focus of my research shifted away from looking directly at the development of the teachers, to researching how I am becoming a mathematics teacher educator (in relation to the development of the teachers). This change in focus was explained carefully to the participating teachers (hence there being two participant consent forms for context two, see appendix 4e, and see full ethics application in appendices 4d-4f). Some of the further ethical complexities of the study are discussed in section 3.4 (page 44).

The set of audio recorded feedback sessions became the raw data that will form the basis of much of the story of chapters eight and nine. Once I had begun capturing these instances of myself in action, from context one and context two, I faced the challenge of what to do with those instances, which in isolation would not express the kind of *continuous* process of learning that I was hoping to express. I needed a way of knitting these lived experiences together, to fill in the gaps, saying more about my learning than these instances of practice could do on their own. I was looking for more than a method of data analysis, I was looking for an approach to working with my data that supported my learning as a mathematics teacher educator, whilst simultaneously expressing that learning in the form of a research study, and, ultimately, as a written artefact (this dissertation).

2.3 Researching process (as opposed to outcome)

I realised quite early on in my research that I was more interested in the process of learning (how) than the outcome of learning (what). This interest in researching process was motivated by my sense of purpose as a mathematics teacher educator, as supporting the *process* of learning in others (i.e., prospective and in-service mathematics teachers) rather than ensuring those others would reach any particular end state. I had already

learned how to support the learning of others as a mathematics teacher (where I learned to support others in learning mathematics), but for me, learning to teach never involved a systematic approach to researching my *own* learning. The classroom research that I carried out (during my master's study) was focussed on the learning of others learning mathematics. Now that I was a mathematics teacher educator, I needed to learn to support the learning of others (learning to teach mathematics) who were learning to support the learning of others (learning mathematics). Thus, I resolved to examine *how* I was learning within the new educational setting in which I found myself, by researching the process of my learning in relation to the learning of others (i.e., prospective and in-service mathematics teachers).

To research the process of my learning, I needed to develop a methodology that allowed me to demonstrate an ongoing, ever-changing, process rather than a fixed state of being. Thus, I chose to use the term *becoming* (as opposed to *being* which implies a state of having already become). For Etherington (2004), a process of becoming:

> implies movement, agency and continuity, rather than a striving to reach a state at which we have 'become'. It is based upon the notion that we are constantly changing and developing our identities, and that they are never fixed. (Etherington, 2004, p. 15)

As a direct consequence of adopting the notion of becoming as "constantly changing and developing", I formulated the following research question:

How am I becoming a mathematics teacher educator?

With this research question in mind, I needed to design a way of capturing my ongoing process of change: changes in my practice; changes in the way I was viewing the world of mathematics teaching; changes in my ways of being with prospective mathematics teachers; and changes in my in-the-moment responding during the various teacher education scenarios I found myself in.

A turning point for me came during an international conference (the eleventh congress of the European society for research in mathematics education (CERME11)) where, having presented a paper (see Helliwell & Brown, 2019), I was discussing with the group

the fact that I was interested in process (how I am becoming) as opposed to outcome (what I have become) and asked them for their ideas of ways that I might be able to report on a continuous, ongoing process that is my becoming a mathematics teacher educator. One of the members of the group suggested a form of layering in my writing, each new layer acting like a meta-commentary *about* the previous layer, as one way of communicating that I was reporting on a process that was taking place over time. I felt encouraged and was prepared to explore this idea further (for instance, my final chapter, chapter nine, is a meta-commentary on the whole thesis). Then there was a different voice who said:

> *I would like to read a really good story where you show the complexity, you show your struggles. We understand our world through our stories from the past. What stories do you draw from in your telling of the change over time, how do you represent that now?*

Narrative methods had been hinted at to me on a number of occasions prior to that moment. Having collected data in the form of audio-recorded conversations (both from context one and context two) I had dismissed the possibility of using narrative methods, assuming instead that my study would involve a well-established analytical approach such as conversation analysis (Sacks, Schegloff & Jefferson, 1978). Simply presenting the conversations as part of a narrative would be far from analytical enough, I thought, let alone interesting. Yet, in that moment, telling a "really good story" felt like just what I *needed* to do, and the only way that I could express my process of becoming.

According to Bruner (2004), "we have no other way of describing "lived time" save in the form of a narrative" (p. 692). He goes on to suggest a number of other temporal forms that can be imposed on the experience of time (such as a clock or calendar form), yet "none of them succeeds in capturing the sense of lived time" (p. 692). Furthermore, narrative describes "forms of discourse that offer a coherence over time, within space and context, so that: 'narrators make sense of themselves, social situations, and history'" (Speedy, 2008, p. 6). Thus, my challenge was in using narrative methods in order to bring together, in a coherent and meaningful way, those instances of practice that I had been capturing as singular occurrences, in order to tell a really good story. In section 2.4 I

explore the terms *narrative* and *story* before discussing narrative inquiry as an approach to research.

2.4 Narrative and story

According to Riessman and Speedy (2007), the "term *narrative* carries many meanings and is used in a variety of ways by different disciplines, often synonymously with *story*" (p. 426, emphasis original). Sarbin (2004), for instance, uses the term *narrative* as equivalent to the term *story*, being clear to exclude certain types of written or spoken accounts such as certain professional reports and shopping lists. Gallagher (2007) uses narrative in a wider sense than Sarbin, "to include oral and written communications and self-reports on experience" (Gallagher, 2007, p. 65). In this regard, Gallagher assumes narrative to be more basic than story, since for Gallagher, a story is characterised by having "a formal plot structure" (p. 65) which is not the case for all narratives.

Ricoeur (1981) explains that plot is "the intelligible whole that governs a succession of events in any story" (p. 166), a description that "shows the plot's connecting function between an event or events and the story" (p. 166). Plot can therefore be seen as the glue or the web that holds multiple events or fragments of experience together in such a way that they become meaningful (i.e., by becoming a story). Bateson (1979) describes a story as "a little knot or complex of that species of connectedness which we call *relevance*" (1979, p. 13, emphasis original). I see Bateson's conception of story as the "pattern which connects" (p. 13) as complementing the views of Gallagher (2007) and Ricoeur (1981), who both claim that a story consists of a connecting function (that they refer to as the story's plot). Bateson (1979) uses the term connectedness not only in relation to the connectedness of events (or "components in the same 'story'" (p. 13)) but also connectedness between people, in that we "all think in terms of stories" (p. 13). Instead of using the term plot, Bateson (1979) refers to "*context*" and "*pattern through time*" (p. 14, emphasis original) as that which connects the component parts within a story. According to Bateson, "[w]ithout context, words and actions have no meaning at all" (p. 15), so, for any story to be meaningful, there is always a context, since it is "the *context*

that fixes the meaning" (p. 15, emphasis original). In my study, I will be careful to provide enough context so that my stories really are stories and nothing less, stories that tell of my becoming a mathematics teacher educator.

Throughout this study, I use the term narrative in relation to an approach to research, that is, narrative inquiry (that I discuss in more detail in section 2.5). As you read this thesis, you will read a series of stories: stories about teaching mathematics; stories about teaching teachers of mathematics; and stories about aspects of my practices as a mathematics teacher educator that are of particular relevance to this study. I view this thesis in its entirety as a longer story. It is the story of how I am becoming a mathematics teacher educator. Hence, I see story both as the object that this thesis has now become, that is, an expression of my becoming, and as the process of my becoming, since telling this story is inseparable from the process of my becoming.

Bateson (1979) points to an important distinction between the stories that we live (our behaviour) and those that we tell when he says, "[c]ontext and relevance must be characteristic not only of all so-called behaviour (those stories which are projected out into 'action'), but also of those internal stories" (1979, p. 14). Throughout this thesis you will be given access to the stories that I tell, and to an extent, those I live, through my process of inquiry. I return to the distinction between living and telling stories in section 2.5 where I explore the meaning of narrative inquiry as "the study of experience as story" (Connelly & Clandinin, 2006, p. 477) and discuss the way in which my study is framed as a narrative inquiry.

2.5 Narrative inquiry

Telling stories is a fundamental human activity in that we "shape [our] daily lives by stories of who [we] and others are" (Connelly & Clandinin, 2006, p. 477). When we note something of our experience, we do so "not by the mere recording of experience over time, but in storied form" (Clandinin & Connelly, 1998, p. 154). Narrative inquiry, as a well-established research methodology in the social sciences, is thus premised on the

notion that we live our worlds through our stories, and that our experiences are made meaningful through the process of telling stories:

> Narrative inquiry, the study of experience as story, is first and foremost a way of thinking about experience. Narrative inquiry as a methodology entails a view of the phenomenon. To use narrative inquiry methodology is to adopt a particular view of experience as phenomenon under study. (Connelly & Clandinin, 2006, p. 477)

Narrative inquiry is the study of experience as story, where the term narrative refers to "both the phenomenon and the method" of study (Clandinin & Connelly, 2000, p. 18). Extending this two-fold conception of narrative, Freeman (2015) highlights the "necessity of narrative understanding in comprehending certain fundamental features of the human realm" (p. 21), describing this necessity on *three* distinct fronts (method, theory, and practice):

> Narrative can be, and often is, a *method*, a mode of inquiry into the human realm. In addition, the idea of narrative can be employed in the context of *theory* about some aspect of the human condition, for instance cognition or personal identity. Finally, it can be considered in the context of *practice*, that is, the various human "doings" that are part of everyday life. (Freeman, 2015, p. 22, emphasis original)

I view my approach to this study as consistent with this three-fold conception (narrative as a mode of inquiry; narrative as a theoretical construct; and narrative as a way of being) but emphasise narrative as a mode or *process* of inquiry that frames both the way I have approached my research problem ("how am I becoming a mathematics teacher educator?" (section 2.3, page 24)) and the way in which I report on my research.

In terms of narrative as a process of inquiry, Clandinin and Connelly (2000) tell us that "in the construction of narratives of experience, there is a reflexive relationship between living a life story, telling a life story, retelling a life story and reliving a life story" (p. 71). These four stages of narrative inquiry are neatly summarised by Clandinin (2013) as "*living, telling, retelling,* and *reliving*" (p. 34, emphasis original). I offer the following as my interpretation of these stages that I view as always being in relation to one another,

where narrative inquiry is the ongoing process of moving back and forth between and across the four stages:

> we *live* out our stories,
> we *tell* the stories we have lived,
> we inquire into the stories we have told (through *retelling*),
> (and in doing so) we *relive* our stories.

Writing stories is a form of telling stories. Richardson (2000) guides us not to think of writing solely as a way of *telling* about the social world, but that writing can also be "a way of 'knowing'—a method of discovery and analysis" (p. 923). Writing/telling stories as a way of researching is thus a way of finding out, a way of making connections, and a way of uncovering aspects that would otherwise go unnoticed. Researching through telling stories is therefore not a process of recounting an already internalised story (like a well-rehearsed response to a frequently asked question), researching through telling and retelling stories is a generative process, one consequence of which is that "possibilities emerge for reliving in more thoughtful and responsive ways in the future" (Clandinin, Huber, Steeves, & Li, 2011, p. 34). The process of retelling requires that we go beyond regarding stories as fixed objects or entities. It is "the retelling of stories, that allow for growth and change" (Clandinin & Connelly, 2000, p. 71), akin to a process of becoming.

Throughout this thesis, you will encounter stories, stories told of stories that I have lived. You will also encounter a number of *retellings* of those stories. When a retelling is offered, I make this process of retelling explicit, referring to the story as a retelling that consists of a reseeing of the original story told. This explicit retelling, of previously told stories, is one way that I have addressed the problem of how to express changes in myself in relation to my practices. The process of retelling is itself part of the process of my becoming a mathematics teacher educator. Throughout this thesis, I draw on existing theories (the role and use of theory in this thesis is discussed in more detail in the following chapter, see section 3.2, page 40) that allow me to retell stories from a new and different perspective, and thus through telling and retelling stories, I am becoming a mathematics teacher educator.

2.5.1 On two types of narrative inquiry and doing both

Clandinin et al., (2011) acknowledge a distinction, within the narrative tradition, between the process of *telling* stories (where the emphasis is on discovery through telling and retelling) and the process of *analysing* stories (where the story or stories are treated as objects of analysis). Telling stories, or as Morris (2001) puts it, thinking *with* stories, "is a process in which we as thinkers do not so much work on narrative [… but allow] narrative to work on us" (p. 55). Thinking with stories opposes and modifies (but does not replace) "the institutionalized Western practice of thinking *about* stories" (p. 55, emphasis original). Clandinin et al. (2011) suggest that thinking with stories is what it means to think relationally (p. 34) as a narrative inquirer, opposing the dominant focus on thinking about stories as objects. The emphasis for thinking with stories is therefore on process, rather than outcome, a process of discovery, akin to exploring. Thinking with stories is about finding out; it is a way of knowing.

Polkinghorne (1995) draws on Bruner's (1986) two modes of thought to emphasise the similar, but subtly different, distinction between two distinct types of narrative inquiry. The first type, based on Bruner's "paradigmatic mode" (1986, p. 13) of thought (which operates by recognising elements as members of a category), is referred to by Polkinghorne (1995) as "paradigmatic analysis of narratives" (p. 12), or paradigmatic-type narrative inquiry. This form of narrative inquiry involves the collection of stories, usually from across multiple research participants, which are then analysed in terms of form and content to identify general themes, categories, or concepts. The second type of narrative inquiry is "narrative analysis" (Polkinghorne, 1995, p. 12), or narrative-type narrative inquiry, which is derived from Bruner's "narrative mode" (Bruner, 1986, p. 13) of thought (which operates by combining elements into an emplotted story) that consists of gathering events and happenings as data and then organising that data into a story (with a plot that unifies that data). The story created is a narrative explanation of the phenomena under study.

I see my own study as beginning with living out my stories (Clandinin & Connelly, 2000; Clandinin, 2013), emphasising thinking *with* stories (Morris, 2001), and the process of *telling* stories (Clandinin, et al., 2011). This emphasis is demonstrated most strongly in

the way that I work on and with my recorded conversations with Sam (context one, see section 3.6, page 50, for a description of the process). However, I also draw on both forms of narrative inquiry described by Polkinghorne (1995) in the way that I work on and with my recorded conversations with Sam (context one, see section 2.2.1, page 19, and section 3.6, page 50, for a description of the process of analysis) and the group of in-service mathematics teachers (context two, see section 2.2.2, and section 7.2, page 131, for a description of the process of analysis). For me, the two processes of paradigmatic-type analysis and narrative-type analysis have become inextricable. For instance, the process of analysing the recorded conversations with the group of teachers (context two) consisted of me simultaneously recognising elements as members of certain categories and combining those elements into an emplotted story, the story of my becoming a mathematics teacher educator. Hence, the two processes of paradigmatic-type analysis and narrative-type analysis were not distinct stages as such, rather, they happened together so that the story and related categories emerged simultaneously.

2.5.2 Narrative approaches within mathematics education

There are only a modest number of studies within mathematics education that use narrative approaches to research. According to Chapman (2020), the focus within the majority of mathematics education studies that employ a narrative approach tends to be on analysing stories/thinking about stories rather than placing emphasis on telling stories/thinking with stories. These narrative studies in mathematics education (e.g., Chapman & Heater, 2010; Cox & Harper, 2017; Lutovac & Kaasila, 2014; Nardi, 2016; Oslund, 2012; Suazo-Flores, 2017) usually involve using interviews in order to collect the stories of mathematics teachers, stories that are then analysed for content, form, themes and so on, where mathematics teacher educator researchers use narrative methods to inquire "about the other" (Chapman, 2020, p. 22).

There are a smaller number of cases where mathematics teacher educator researchers have used narrative approaches to learn about themselves and their practices. For example, Chauvot (2009) and Bailey (2008) both use narrative inquiry as an approach to

researching their knowledge of teaching or curriculum as mathematics teacher educators. These self-based narrative inquiries use "stories to construct and tell a larger narrative of new learnings, meanings, or understandings about themselves and possibly others" (Chapman, 2020, p. 23). Nicol (1997) is the earliest example of a self-based narrative inquiry within mathematics education that I have been able to identify (though Nicol never uses the term self-based narrative inquiry, she does refer to a "self-investigation" (p. 79)).

2.6 My self-based narrative inquiry

As a researcher, researching how I am becoming a mathematics teacher educator, I am engaged in a self-based narrative inquiry. In self-based narrative inquiries, researchers learn through the process of writing stories about themselves, their practices, and their values, which includes the significant process of determining which particular aspects of themselves and their practices to reveal. This is a process of writing autobiographically, a process where new insights get developed.

In his discussion on autobiographical understanding in narrative inquiry, Freeman (2007) concludes that the writing and interpretation of the personal past is not a "dispassionate process of reproducing what was" (p. 137) but is instead:

> a product of the present and the interests, needs, and wishes that attend it. This present, however–along with the self whose present it is–is itself transformed in and through the process at hand. Indeed in a distinct sense, a new self is fashioned via this very process; dimensions of being are disclosed that literally would not have existed, would not have reached articulated form, had the autobiographical process not taken place. (Freeman, 2007, pp. 137-138)

This generative function of writing autobiographically comes with some important considerations. For instance, Derrida and Prenowitz (1995) describes the creation of historical accounts, including autobiographical ones, as a process of archiving that is not simply a process of "*secondary recording*, to the printing and to the conservation of the history" (p. 17, emphasis original), but, instead, one that literally transforms history

"from top to bottom and in the most initial inside of its production, in its very events" (p. 17). I take from Derrida and Prenowitz that the process of archiving is more than the production of an exact copy (in fact, it can never be an exact copy) of the past, but the generation of history itself: "the archivization produces as much as it records the event" (p. 17). Bruner (2004) recognises a similar phenomenon when he observes that "the culturally shaped cognitive and linguistic processes that guide the self-telling of life narratives achieve the power to structure perceptual experience" (p. 694) so that ultimately "we *become* the autobiographical narratives by which we 'tell about' our lives" (p. 694, emphasis original). In becoming our autobiographies (which for me is identical to 'reliving' of narrative inquiry), through the recursive process of retroactively and selectively writing about past experiences, we shape our ways of being such that there are undeniable implications for our present and *future* lives and the way we *experience* "problems, situations, the world […] in relation to which [we] are acting" (Marton & Booth, 1997, p. 111):

> Accordingly, a capability for *acting* in a certain way reflects a capability [for] *experiencing* something in a certain way. The latter does not cause the former, but they are logically intertwined. You cannot act other than in relation to the world as you experience it. (Marton & Booth, 1997, p. 111, emphasis original)

Thus, through the process of writing autobiographically, I am learning *about* past experiences of teaching mathematics and teaching teachers of mathematics whilst simultaneously changing the way I will experience and act in future situations as a mathematics teacher educator.

There are important (and deliberate) methodological implications of writing autobiographically. For instance, I am deliberately transforming my practices through the process of researching how I am becoming a mathematics teacher educator. Thus, writing autobiographically as a process of coming to know, both about the past and in relation to future knowing (and actions) can be justified. Consequently, I acknowledge that everything I do as part of this study, this autobiographical writing process, will impact on my future practices, everything I do will do something, "even refraining from

all action is a meaningful behaviour" (Hutchins, 2010, p. 708) and this is an ethical responsibility that I take very seriously.

Having framed this study as a self-based narrative inquiry, I use the next chapter to explore a range of important methodological issues in relation to the process of writing autobiographically (specifically: truth; the role of theory; perspectives (voice, time, and tense); authorship; and who is this research for?). I outline how I deal with each of these issues in my research.

~~~~~
~~~~~

Chapter three

Methodological meanderings

Having established, in the previous chapter, that my study is a self-based narrative-inquiry that aims to report on the *process* of my becoming a mathematics teacher educator, there are a variety of methodological implications that I address in this chapter. The five methodological issues I address are:

1) Truth
2) The role and use of theory
3) Perspective (voice, time and tense)
4) Authorship
5) Who is research for?

Each of the five methodological issues arose over the course of my study, causing me to pause for significant amounts of time, as I grappled with implications, both philosophically and practically. None of these issues have been trivial, and they overlap to a certain extent. Sometimes an issue that was not easily resolvable would arise through the process of writing, which would send me off in a different direction, seeking a solution to the problem I had stumbled across. For instance, it was only when I came to thinking about how I might use the names of people close to me, that the broader issue of

authorship struck me. Similarly, as I began writing stories, I did so without considering how and where I would use existing theory. Only when theory began to creep its way into my stories did I need to review the role of theory in its entirety.

After considering each methodological issue in general and in relation to this study, I then describe the way I analyse, in chapter four, one particular conversation with Sam (context one: section 2.2.1, page 19). Having described my analytical approach to working on the data from context one (see section 3.6, page 50), I then set out and respond to four appropriate criteria for evaluating this study (section 3.7, page 52).

3.1 Methodological issue one: Truth

The main objection from critics of narrative research, especially self-based narrative research, is around claims of truth. The basis of this objection is that unlike scientific facts or historical facts, there can be no claim to the status of truth since autobiographical writing is a product of the autobiographer's *imagination*, where imagination is a form of fantasy. As Gough (1994) suggests, however, any claim to truth depends on the particular ontological perspective, so historical facts, for example, are truthful in the world that historians imagine, so the same could be said for autobiographical truths, that they are truthful in the world that the autobiographer imagines. Eakin (1985) claims that "autobiographical truth is not a fixed but an evolving content in an intricate process of self-discovery and self-creation" (p. 3). What is real or truthful leads immediately on to debates around fact and fiction, which many critics of narrative research take as in binary opposition. Gough (1994), however, sees fact and fiction as mutually constitutive, recognising "that facts are not only important elements of the stories we fashion from them but also that they are given meaning by the storytelling practices which produce them" (Gough, 1994, p. 53). Thus, it would be impractical (and impossible) for me to present exactly where the boundary lies between what some might consider fact and fiction:

> If there is a line between fact and fiction, it may by necessity be a winding border that tends to bind these two categories as much as it separates them, allowing each side to dissolve occasionally into the other. (Young, 1988, p. 52)

Bruner (2004) distinguishes between external and internal "criteria of rightness" (p. 693) on the self-reporting of a life. External criteria (or facts), such as whether the autobiographer did or did not visit a certain place at a certain time, are perhaps more easily verified. However, even in cases where aspects of the account are *untrue* (such as the precise date an event took place), the self-report could still be considered as *right* under certain criteria of narrative adequacy. Internal criteria, relating, for example, "to how one felt or what one intended" (p. 693), are more demanding even though they are not necessarily subjected to verification. Bruner points to the complexity and challenge of judging "rightness" in relation to autobiographical writing arguing that verificationist criteria are simply not appropriate given the instability of self-reports. He reminds us, however, that telling is only one function of writing autobiographically and it is the very instability of these accounts that makes writing them such a powerful tool in self-transformation. As Bruner (2004) reminded us in the previous chapter (section 2.6, page 32), writing about our lives can have profound effects, changing our personal life narratives, in that we can literally "*become* the autobiographical narratives by which we "tell about" our lives" (p. 694, emphasis original). Given the purpose of my study is to research how I am becoming a mathematics teacher educator, then the transformative function of writing autobiographically is an important aspect for me to consider.

Freeman (2010) introduces a similar distinction to Bruner's internal and external criteria between what Freeman calls first–order and second–order autobiographical memory. First–order autobiographical memory "entails a reference to the existence of what had once been sensorially present" (p. 111), which he describes as an "anchor" of actuality. With second–order memories the anchor is gone, yet we often experience them as if they are first–order even though they originate through others' first– (and second–) order autobiographical memories, through stories that we are told. Consequently, what we experience as memories can be suffused with all kinds of "extraneous" matter, matter

that we may not have experienced ourselves, and that have been shaped by the experiences of others and our history of interactions with the world:

> "Memory", in this context, becomes a curious amalgam of fact and fiction, experiences and texts, documentary footage, dramatizations, movies, plays, television shows, fantasies, and more. (Freeman, 2010, p. 101)

Importantly, Freeman (2010) explains, memory is thoroughly conditioned by one's ever-changing vantage point at the present moment of remembering and narrating. For an autobiographical study like this one, written from a retrospective point of view, I do not pretend that I will be able to disentangle my first– and second–order autobiographical memories, but I draw on my different perspectives; diary entries and the audio recordings of conversations (from context one (section 2.2.1, page 19) and context two (section 2.2.2, page 22)), as a way of grounding my memories in actualities as far as possible.

Freeman's (2010) response to issues of associated biases that accompany writing in hindsight seems to centre on the limitations of what we are able to understand, in the immediacy of the moment, that realisations or "narrative connections" (p. 26) are so often made after-the-fact. Gusdorf (1980) proposes that autobiographical understanding is attained *only* with the remove that comes with hindsight, since in the moment of any experience the "agitation of things ordinarily surrounds [us] too much for [us] to be able to see it in its entirety" (p. 38). We experience a *delay* in our understanding of things. So, it follows that hindsight can "rescue us from the oblivion that so often characterizes the human condition" (Freeman, 2010, p. 26). This rescuing from oblivion may be pleasurable or painful or something else, depending on the realisations made. Autobiographical understanding can thus be viewed as a second reading of experience that is "truer than the first" (Gusdorf, 1980, p. 38):

> The past that is recalled has lost its flesh and bone solidity, but it has won a new and more intimate relationship to the individual life that can thus, after being long dispersed and sought again throughout the course of time, be rediscovered and drawn together again beyond time. (Gusdorf, 1980, pp. 38-39)

Autobiographical understanding, according to Gusdorf, is not a simple distortion of the past, it is the truest version of experience, "truer than the first because it adds to experience itself consciousness of it" (Gusdorf, 1980, p. 38).

In her ground-breaking work on the theory of art, Langer (1953) describes memory as "the great organiser of consciousness" (p. 262), the *process* by which past experiences take form and character. She suggests that memory is "a normal and familiar condition which shapes experience into a distinct mode, under which it can be apprehended and valued" (p. 262). Memory as a process is the means by which "the meanings of *then* become the meanings of *now*, the feelings of *then* become the feelings of *now*" (Rosen, 1998, p. 102, emphasis added), a fitting description of my own experience in returning to my diary entries (section 1.3, page 9):

> To remember an event is to experience it again but not in the same way as the first time. Memory is a special kind of experience because it is composed of selected impressions, whereas actual experience is a welter of sights, sounds, physical strains, expectations and minute undeveloped reactions. (Langer, 1953, p. 263)

In Langer's sense, memory is not a thing, it is an activity, it is not something that we possess, it is something that we do. Rosen (1998) describes memory as the "sifting" process that "teases out a pattern from the sheer muddle of events" (p. 103). Akin to Bateson's (1979) conception of story as a "connecting pattern" (p. 16), Rosen's (1998) description of memory confirms the idea from narrative inquiry that when we note something of our experience, we do so in storied form.

As with Langer's (1953) sense of memory as activity, Sarbin (2004) views imagining as "an active process, something that the imaginer does" in contrast to the substantive imagination that "suggests a thing-like entity, or a property of the mental apparatus" (p. 9). For Sarbin, "imaginings are storied sequences of actions" (p. 11), they are "emplotted narratives carrying implications of causality and duration" (p. 11). In short, Sarbin tells us that "imaginings are storied constructions" (p. 11) but in an effort to avoid architectural connotations of the word constructions, he suggests the alternative *poetics* as a term that more aptly reflects the process of story making. It would seem, therefore,

that the act of remembering past events and the act of imagining storied sequences of actions have the same experiential quality, they feel the same, they share "a similar phenomenal structure" (Kerby, 1991, p. 25).

In this study, truth is not something that is viewed as being out there in the world, external to the way we experience it. I am interested in studying my practices, i.e., using my interpretations of past and current happenings. I do not expect others to interpret these happenings in the same way as I do, but I endeavour to give as close an account of my experiences as is possible, by combining the process of telling stories with using data in the form of recorded conversations with mathematics teachers. Thus, I use memory and imagination as a way of probing beneath the surface of what is possible to see in the data alone.

3.2 Methodological issue two: The role and use of theory

In considering the role and use of theory in my study, I have found myself musing over what Clandinin and Connelly (2000) describe as their "consideration of life at the boundary between thinking narratively and thinking formalistically" (p. 40). Clandinin and Connelly use the metaphor "life at the boundaries" (p. 21) to describe a place of tension, "where narrative inquiry thinking comes into the intellectual territory of another way of thinking" (p. 21). They suggest that one of the central tensions at the formalistic boundary, is the place of theory in a narrative inquiry. Having, to an extent, come myself from a formalistic background, from which my familiarity with a more conventional approach to presenting research has developed, the formalistic boundary is indeed a personal place of tension, a tension that has led me to think at length about how, for example, to present any literature that has meaningfully shaped my becoming at various points in time along the way.

According to Clandinin and Connelly (2000), narrative inquirers frequently forsake formal literature review chapters and instead "weave the literature throughout the dissertation from beginning to end in an attempt to create a seamless link between the theory and practice embedded in the inquiry" (p. 41). A further tension for me, at the

formalistic boundary, is created by the conceivable ideals of potential examiners who may object to the lack of theory in the form of stand-alone chapters. I find some of these objections within myself, but for this dissertation to be both a representation and a source of my becoming a mathematics teacher educator, I have remained committed to a process of telling stories and looked for ways of integrating theory in a way that best demonstrates how I have used theory in my process of becoming.

Part of the challenge for me, in writing this dissertation has been figuring out, from the array of possible moments or fragments of lived experience, which of those moments to select and how these moments can coalesce in a meaningful and coherent way that tells the story of my becoming. I now realise that this process of figuring out is exactly the process of theorising that I hear being described in Winter's (1988) words when he says that telling a story is "a skilful process of organizing material to make it 'interesting'" (p. 232) and that the "aesthetic structuring of material can constitute a method for theorizing, rather than merely a format in which theory (created by some other process) can be presented" (p. 233). This is the process of theorising through telling stories, creating the "pattern which connects" (Bateson, 1979, p. 13) the selected instances of practice.

In their consideration of the place of theory within research, Barone and Eisner (2012) use a familiar pairing (pairing also used by Mason (2002), see section 8.2.6, page 190), "between theory as an effort to *account for* some set of empirical relationships and theory as an effort to give an *account of* the world as it is" (p. 156, emphasis original). This distinction is essentially that of the two functions of theory within research, which are to describe (to give an account of) and to explain (to account for). Winter (1988) points to a related distinction when he says that within research:

> the aim is both a description, which will seem plausible because of its richness of 'contingent' detail derived from the situation itself [...] and the creation of a theory, which we may take to be the structure that in some sense lies behind those details and gives them a pattern and a significance in relation to other situations. (Winter, 1988, p. 232)

For a narrative inquirer, the pattern, or selectivity (that we could call plot) that determines the structure behind the story being told is equivalent to the creation of a theory.

The process of conducting this self-based narrative inquiry has involved many iterations: adding details, removing details, reordering, reemphasising, and so on. This re-writing and re-structuring was not about fabrication, or even clarification, but a process of aesthetic structuring (i.e., theorising), that led to me finding new meanings and developing new insights. At times it felt like this iterative process could go on forever, that there would always be a new seeing from each and every iteration, so at some point I had to make the conscious decision to stop and settle on what the object has now become. One guiding principle throughout the process of writing this dissertation, which reflects a commitment to expressing my becoming as a *process* rather than a fixed or finished state, is to show the progression of theoretical ideas as they have become meaningful for me over time and in relation to the story of my becoming. By showing a progression of ideas, I aim to express changes in my ways of seeing the world that integrating new theory has enabled along the way. I did not set out on this journey with a range of theoretical ideas already firmly established, as might be the case in doctoral studies where the theoretical constructs are set up from the beginning of the dissertation. Instead, I began with living and telling stories before bringing theory to bear on those stories as and when it was meaningful to do so. I have aimed to show a development in theoretical ideas throughout the dissertation from start to end. You, the reader, will thus find theory woven throughout each of the chapters. Certain chapters are more heavily theoretical (such as chapters five and six) and you will also find interludes where, for example, a theoretical construct is introduced that needs exploring more thoroughly.

3.3 Methodological issue three: Perspectives (voice, time and tense)

In autobiographical studies, one criticism is often centred around the dependence on a single perspective, that is the perspective of the researcher alone, yet this need not necessarily be the case. During the course of this study, I have utilised my *own* multiple

perspectives through the processes of telling and retelling stories, and performing multiple analyses (during distinct phases, see section 7.4, page 133) of the recorded conversations with teachers of mathematics (from context one (see section 2.2.1, page 19) and context two (see section 2.2.2, page 22)) as well as actively seeking the perspectives of others (e.g., by sharing extracts of recorded conversations with members of the research community, see section 8.1.3, page 155).

One way of demonstrating multiple, evolving perspectives is to vary narrator proximity, a term used within narrative theory to refer to the temporal gap, or lack of, between experience and narration of experience, the "distance from the past-present of experience" (Freeman, 2010, p. 22) to the articulation of that experience. My diary entries, for example, were narrated in close proximity to the event being narrated about. When there is "narrative immediacy" (Rossholm, 2017, p. 85), there is no apparent temporal gap between an experience and the narration of that experience, and the reader is given *direct* contact with what is happening in the story, rather than experiencing the sense of distance that may come with reading stories that are written retrospectively.

There is, therefore, a delicate methodological balance to be reached between providing the reader with an immersive experience and the need to demonstrate a range of perspectives. An immersive experience for the reader (and thus an increased likelihood of developing insights about their own practices) can come through narrative immediacy where the reader is given direct contact with what is happening in the story. In my study, narrative immediacy is more likely to be experienced if my stories are told in the present tense (as they are, for example, in my conversation with Sam in section 4.1, page 57). On the other hand, there is the need to make explicit the distinction between my past and present perspectives, thus pointing explicitly to retellings of previously told stories (which I do on multiple occasions throughout the dissertation). Through telling and retelling, I aim to create a sense of temporal distance, in order to clearly express the way in which past perspectives have changed over time through my becoming a mathematics teacher educator.

These tellings and retellings are communicated using my multiple perspectives (then, now and places in between), which are made apparent through varying narrative

proximity, through the use of associated *tenses* (past–, present–, and future–tense). I have access to my past perspectives (rather than my present perspectives on my past perspectives), in the form of diary entries that were written in close narrative proximity to the events accounted for in those entries. I have access to past instances of my practices in the form of audio-recorded conversations with teachers of mathematics. By reading my diary entries and by listening to the recorded conversations, I am given access to my past perspectives, not solely from the meanings that are communicated in the words themselves, the accounts given in the diary entries, for instance, but through the active process of memory and imagining that are evoked through the process of reading and listening. I also have access to my past perspectives through my actions in the moment of my conversations with others. These observed changes in perspective feature explicitly in the stories that are told, as an expression of my becoming a mathematics teacher educator. When a range of perspectives (and associated tenses) are utilised, I have endeavoured to be explicit about this in advance.

I also use different *voices* throughout this dissertation. There is my leading narrative voice, that is the voice I am using now, and is always written from my current (at the time of writing) perspective. This voice seems consistent, yet it evolves in terms of perspective as you move through the chapters. There is a slightly more formal-analytical voice that I use during interludes when I am exploring a particular issue or set of theoretical ideas. There is my diary voice from the time of the diary entry being made and also my voice in the conversations with teachers speaking words that were spoken at the time of those conversations.

3.4 Methodological issue four: Authorship

What if the stories we tell as our own, are actually the stories of others? Freeman (1993) addresses the question, "is autobiography itself – the telling of 'our own' life story – really possible?" (p. 79) and in response he argues:

> There is no other way, no other means of capturing the 'really real', outside of language and culture, simply because the 'really real', in all its

> multiplicity and changeability, is constituted as such *inside* language and culture, inside that 'world' we keep on referring to" (Freeman, 1993, p. 79, emphasis original).

Roth (2005) points out that autobiography as written text must always use "words which are not entirely our own, but always also belong to the Other" (p. 12). I accept that my words will always contain the words of others, after all, as Bakhtin said:

> The word in language is half someone else's [...] it exists in other people's mouths, in other people's contexts, serving other people's intentions: it is from there that one must take the word, and make it one's own" (Bakhtin, 1981, pp. 293-294).

I fully acknowledge that the stories I tell contain the words of others, both in the literal sense (through directly referring to sources of literature), but also in the sense that my stories contain the stories of others, especially those more experienced others with whom I work closely with. There are, therefore, ethical dimensions of originality and authorship to be considered, concerns that have troubled me greatly at points during this study that I would not have been able to predict from the outset.

I suppose I had naively thought that an autobiographical study would be more straightforward, that is, in terms of *ethical* considerations. I was wrong and not only because my autobiographical study does indeed involve the collection and use of primary data from a number of participating mathematics teachers (see section 2.2.1, section 2.2.2, and interlude one for explicit ethical considerations, as well as appendix four for full ethical applications). As my study has been unfolding, including the shift of focus towards my own practice, so have the complexities of the associated ethical considerations. The specific issues I am referring to here, concern authorship and originality. Although I was prepared, from the beginning of study, to consider more standard issues of participant anonymity, what I did not foresee was how to deal with the almost contradictory issue that is dealing with the *acknowledgment* of those people closest to me and my work, specifically my two doctoral supervisors, given our extended and entangled histories (see notes on how to read this dissertation, page xii). Rather than ensuring anonymity (which, in the case of my supervisors, Laurinda and Alf, would be

impossible even if sought after), I was struggling with how to express the strength of influence they have had in terms of my past and present practice and their undoubtable shaping of my becoming a mathematics teacher and a mathematics teacher educator. The selection of what gets told and what does not get told in my story became an ethical issue in itself, so many important conversations not included *explicitly*. How could I explicitly acknowledge those individuals closest to me? I got to a point in this study where I needed to write about the issue, so I wrote a letter to Laurinda and Alf (initially not knowing whether I would send it):

> **Letter 6th June 2019**
>
> Dear Laurinda and Alf,
>
> There is a lot to say…. I wonder if you will read this letter, whether I will be brave enough to show it to you.
>
> As I go further down the road of writing my dissertation, in its autobiographical form, I have become increasingly aware of how and when you feature in it… and how and when you do not.
>
> When I write of the many experiences I have had, you should know you are there in all of them, though I fully appreciate that may not be made clear when you are reading my work. If I was to be explicit in my writing each time you came to mind, I am not sure how much would be left that felt like it was me, and therein lies the difficulty… when do I call you out, and when do I not, what is you, and what is me… The people closest to me are so ever present it is difficult to make these distinctions; you are so part of my being a mathematics teacher and teacher educator that I can no longer discern you. So familiar… you just are.
>
> When I call out others that I have encountered along the way, it might seem like that means something more than it does, I am mindful every time I do this how that might make you feel.
>
> Learning to teach, working together in school and now at university… everything gets blurred, there are so many

> *significant moments it is difficult to pull them apart, to locate where something began, to point to one conversation amongst the many. I hope some of this is making sense. It is important to me that I don't cause you any pain or sadness. I know I would not be here if not for you being there.*
>
> *Perhaps what I am getting at here is the reason why so many researchers steer away from autobiographical narratives… not only can the process of telling stories be painful for the person telling them but also for the reader reading them. I am acutely aware in this moment of the potential impact that leaving things out might have, and in some respects, this outweighs the impact of writing things in.*
>
> *It dawns on me that what I am speaking about here are issues of ethical concern, issues that are directly related to my closest friends and colleagues. This is rather alarming; how did I not foresee this. Of course, writing about myself would involve writing about others, I realised this, but I had not quite realised the magnitude of the issue, especially for myself and those close to me.*
>
> *I would value any thoughts you might have, both on a personal level and in terms of how I deal with these issues within autobiographical writing.*
>
> *With all my thanks,*
>
> *Tracy*

Pragmatically, I decided (after multiple conversations) to acknowledge Laurinda and Alf by name at various points throughout my dissertation, including a commentary of my history with both of them in one of the introductory pages (see notes on how to read this dissertation, page xii). Of course, Laurinda and Alf's words are not always explicitly acknowledged. My history of interactions with them both are ever-present in the words I write. Although I use the words of others, this is "not done so without remainder" (Freeman, 1993, p. 80). Accordingly, in telling my story, I hope to give more than I have

taken, by offering a different perspective, making the words my own, by populating "it with [my] own intention" (Bakhtin, 1981, p. 293). I hope that those people closest to me can hear their voices in my stories, even when they are not pointed to explicitly.

3.5 Methodological issue five: Who is research for?

As an early career research academic, I have struggled a little with giving credence to the way that a lot of research gets communicated. For example, it concerns me that many teachers of mathematics (unless they are actively engaged with academic research/study) do not have access to academic journal articles about mathematics teaching that could inform their own practice. I am convinced by the importance of making a contribution to knowledge, and I hope that my research makes a useful contribution, but what I feel most strongly about is that research is useable.

I have come to see the pursuit of my life "in narrative form, as a 'quest'" (Taylor, 1989, p. 52), a quest to better understand my practice as a mathematics teacher educator, and as Greene (1995) puts it, a quest for "a better state of things for those we teach and for the world we all share" because "it is simply not enough for us to reproduce the way things are" (p. 1). As Mason (1998) suggests, the most significant products of research in mathematics education are the "transformations in the being of the researchers" (p. 357). Rather than doing research so that I can learn from that research, I view researching, doing narrative inquiry, as (a way of) learning. I am convinced that my research will have a direct impact on those I teach, through changes I make within my practice. Although I do not pretend to know what exactly these changes might be, I feel some assurance from knowing that my research is an exercise in becoming more attentive to those I teach.

I am also interested in how I can pay attention to the way my writing is experienced by others. The reader's experience has been at the forefront of my decision making with regards to the form of my dissertation. For example, if you are a mathematics teacher educator, or a mathematics education researcher, I hope that my research helps you to see something in your own practice that you have not seen before. This is more likely to

happen if you engage in critical reflective thinking about your own professional practice while you read.

De Freitas (2007) draws on the motives of Maclure (2003) and Lather (1991) which she describes as a desire to "breach the usual authority of educational research texts and to generate a more critical reading habit on the part of those they aim to reach" (de Freitas, 2007, p. 336). For Lather, this breach is about diffusing "the power of the author, and thus improv[ing] the 'democracy' of the findings" (Lather, 1991, p. 92). This shift from thinking about the researcher as the sole owner of the research outcomes, to the reader as "actively construct[ing] possible counter-interpretations" (de Freitas, 2007, p. 336), has been a particularly powerful idea, helping me to shape my own approach to research. This active construction by the reader is what de Freitas calls "reading *otherwise*" (p. 336, emphasis original) and she argues that it is through an approach to narrative research, where literary, fictional, or poetic strategies are employed by the researcher, that readers are most likely to engage in the act of reading otherwise. If, for example, you disclose to the reader that a text is (partly at least) a work of fiction, or partly based on memory and imagination, they will literally read that text differently, since they are acknowledging the distinct claim about the nature of truth that the author is making.

Lincoln and Denzin (2000) claim that qualitative researchers:

> care less about our 'objectivity' as scientists than we do about providing our readers with some powerful prepositional, tacit, intuitive, emotional, historical, poetic, and empathic experience of the Other via the texts we write (p. 1058).

"As we identify ourselves with the protagonist of a story, *we* live his or her feelings and actions without having to act ourselves" (van Manen, 1997, p. 70, emphasis original). I want to give my readers the opportunity of gaining insight into certain aspects of my professional life, to explore the meanings that emerge for them as they read and have the freedom and permission to do so. I want the reader to find what resonates for them in my story and to consider what that might mean for their own practice.

3.6 Creative analytical practices

In the previous chapter (see section 2.5.1, page 30), I explained the fact that I use paradigmatic-type narrative analysis *and* narrative-type narrative analysis (Polkinghorne, 1995) in the way that I work with and on audio-recorded conversations with teachers of mathematics. Accordingly, I draw on "creative analytic practices" (Bochner & Ellis, 2016; Ellis, 2004; Richardson, 1999; Richardson & St. Pierre, 2018) in working with certain recorded conversations (from context one: section 2.2.1, page 19, and context two: section 2.2.2, page 22). In so doing, I combine the process of telling stories with analytical techniques more associated with traditional data analysis. In this section, I will discuss the nature of creative analytic practices before outlining my use of them to analyse a conversation with Sam (context one) that is the basis of the following chapter (section 4.1, page 57).

One lens through which to view the world is with creative arts and another lens is scientific/formal analytical, yet these need not be contradictory ways of seeing as some might have it. In fact, we see more clearly with the two lenses at play, "we see best with both lenses focused and magnified" (Richardson, 1999, p. 666). Creative analytic practices are not alternative or experimental, rather, creative analytic practices are creative *and* analytical:

> they are in and of themselves valid and desirable representations of the social. They display the process and the product as deeply intertwined; both are privileged. The product cannot be separated from the producer or the mode of production or the method of knowing. (Richardson, 1999, p. 661)

In the previous chapter (section 2.5.2, page 31), I described a limited number of studies within mathematics education that explicitly draw on self-based narrative methods. Creative analytic practices are also uncommon in mathematics education research but one source of inspiration for my study is Hannula's (2003) creative approach to working with transcript from a mathematics classroom where he presents his raw data alongside what he calls the inner monologue of the student, Helena, "as a stream of consciousness" as a way of creating "connections that do not exist in the original data" (Hannula, 2003,

p. 32). In Hannula's use of creative analytic practices, he manages to go beyond what is possible to communicate with transcript alone. Though it could be argued that the "fictional inner monologue" (p. 32) that Hannula presents is some form of invention or fantasy, I would argue that he portrays a vivid and valid representation of the particular context and setting within that mathematics classroom through his use of memory and imagination, based on what he had learned during the numerous interactions he had with Helena both before and after the scene depicted. I would suggest that, in using his creative analytical practices, Hannula was able to give the reader an experience close to his own experience of being in that classroom, in a way that formal analytical methods alone would not be able to achieve. For instance, a teacher of mathematics who feels a strong resonance in terms of their own classroom and who can empathise with Helena through reading Hannula's classroom episode, is more likely to consider their own teaching practices as a result. I would say, in this scenario, that the research is powerful indeed and I hope that my research has a similar impact (impact, as criteria for this study, is discussed in section 3.7, page 52).

In the following chapter (section 4.1, page 57), I present one conversation with an experienced teacher called Sam (context one, see section 2.2.1, page 19). Specifically, I present sections of transcript (see appendix 2b, page 296) from one audio-recorded conversation between myself and Sam, which I present as an augmented dialogue by including my own inner voice, "as a stream of consciousness" or "inner monologue" both as a way of communicating my (then) perspective from the time of the conversation and as a way of exposing hidden biases and assumptions that otherwise would not have surfaced (a key methodological principle of this study). I present this augmented conversation using the present tense so that, as you read, it is as if the conversation is taking place. I use the present tense to give direct access to what is happening, as it is unfolding. I distinguish my then perspective (in the present tense) from my newer perspective (my perspective at the time of analysing the conversation) that appears as retrospective reflections *about* the conversation in which I draw out issues as they arise. This newer perspective is where I make explicit use of theory in order to think *about* what is happening in the conversation (or to retell). It is important to note that my inner

monologue (my then perspective) was written at the time of analysing the conversation and not at the time of the conversation originally taking place. In order to access my then perspective, I listened repeatedly and carefully to the recorded conversations with Sam. As I listened, I imagined myself back in the classroom where that conversation took place; I read and re-read diary entries and field notes from the time of the conversation; and I allowed my memory and imagining to shape this inner voice. Before presenting my analysis of one conversation with Sam, I explore appropriate criteria for evaluating a study that takes a narrative approach and specifically one that utilises creative analytic practices.

3.7 Criteria

Bochner (2000) suggests that, as social science researchers, we can get preoccupied with rigour and should worry less about defending the validity and reliability of our research and be more concerned about whether the work is "useful, insightful, or meaningful–and to whom" (p. 267). He warns of criteria being used to stifle our creative energy and that as a research community we have paid too much attention to criteria that pose as "something beyond culture, beyond ourselves and our own conventions, beyond human choice and interpretation when, of course, they are not" (p. 267). Criteria for evaluating research thus differs depending on the researcher's ontological and epistemological perspective, so it is important to use the appropriate conceptual apparatus in any process that involves evaluating research.

Richardson and St. Pierre (2018) offer four criteria used for evaluating research studies that utilise creative analytic practices, namely: *substantive contribution; aesthetic merit; reflexivity; and impact* (p. 823, emphasis original). I take each of these four criteria in turn, presenting the detail as described by Richardson and St. Pierre, before giving a brief description of how I respond to that criteria within my own research. These are the criteria against which I suggest my work is evaluated.

Substantive contribution:

> Does this piece contribute to our *understanding* of social life? Does the writer demonstrate a deeply grounded (if embedded) social scientific perspective? Does this piece seem "true"–a credible account of a cultural, social, individual, or communal sense of the "real"? (Richardson & St. Pierre, 2018, p. 823, emphasis original)

Issues of truth have already been discussed at the beginning of this chapter (under methodological issue one, section 3.1, page 36) along with some of my responses to those issues in relation to the design of this study. Beyond that discussed already, it is worth pointing explicitly to the *layered* nature of my text when considering the credibility of my research. According to Richardson and St. Pierre (2018), a "layered text" is a "strategy for putting yourself into your text and putting your text into the literatures and traditions of social science" (p. 834). They suggest that one means of producing a layered text is to write a narrative about an event that has been especially meaningful to you before stepping back to look at the narrative from your "disciplinary perspective" (p. 834), which involves inserting, into the narrative, relevant analytical statements. Throughout this dissertation, I have used theory in the ways articulated earlier in this chapter (under methodological issue two, section 3.2, page 40). Specifically, as I tell stories that have been meaningful to me both as a mathematics teacher and as a mathematics teacher educator, I often take a step back and offer a theoretical perspective on the experiences being described. Furthermore, in my explicit retellings of stories previously told, I do this retelling from a new and different theoretical perspective than any I had available to draw on in the original telling. The building of theoretical ideas throughout this thesis is a key feature of producing this layered text. One consequence of this layering of text is that many key theoretical ideas do not appear in early chapters, rather, they appear at the point where the theory can add usefully to the story being told.

Aesthetic merit:

> Does this piece succeed aesthetically? Does the use of creative analytical practices open up the text and invite interpretive responses? Is the text

> artistically shaped, satisfying, complex, and not boring? (Richardson & St. Pierre, 2018, p. 823)

I hope that this study invites interpretative responses as discussed already in this chapter (under methodological issue five (section 3.5, page 48)) through inviting the reader to read otherwise, particularly in parts of the thesis where I use creative analytical practices by presenting augmented conversations that include my inner consciousness from the time of the conversations.

Reflexivity:

> How has the author's subjectivity been both a producer and a product of this text? Is there adequate self-awareness and self-exposure for the reader to make judgments about the point of view? Does the author hold himself or herself accountable to the standards of knowing and telling of the people he or she has studied? (Richardson & St. Pierre, 2018, p. 823)

Reflexivity in this study is brought to bear through engaging in the four stages of narrative inquiry: living, telling, retelling, and reliving, stages that are viewed as always being in relation to one another in the ongoing process of moving back and forth between and across the four stages.

According to Etherington, reflexive methodologies are those that remain "close to the hearts and minds of practitioners who value using themselves in all areas of their practices (including research) and who also value transparency in relationships" (Etherington, 2004, p. 16). A key feature of my research has been about uncovering those hidden assumptions and biases that limit what it is possible for me to see in the world of mathematics teaching, a process that is necessarily self-exposing. Much of the theoretical premise of this study is around establishing a discipline that supports the development of self-awareness (see discussions in interlude three, page 105, interlude four, page 108, and section 8.1.8, page 169); a central feature of my becoming a mathematics teacher educator is the development of self-awareness that allows me to attend to others (i.e., prospective, and in-service teachers of mathematics).

Impact:

> Does this piece affect me emotionally or intellectually? Does it generate new questions or move me to write? Does it move me to try new research practices or move me to action? (Richardson & St. Pierre, 2018, p. 823)

An important feature of my study is the attention that I pay to the other. What I mean by this, is that I consider my becoming a mathematics teacher educator always in relation to those who I am working with. In later chapters, where my focus is on using audio-recorded conversations with a collaborative group of mathematics teachers (context two), what materialises, through combining telling stories with more analytical techniques, is methodology for becoming a mathematics teacher educator (chapters eight and nine) on three distinct (but interrelated) fronts (summarised in table 9.3, page 268):

1) In relation to the process of learning to teach mathematics and mathematics teachers.
2) In relation to *researching* how I am becoming a mathematics teacher educator.
3) In relation to a way of working with mathematics teachers.

Thus, in terms of impact, given the methodological nature of my study, I hope this research is useful for mathematics teachers, mathematics teacher educators and mathematics teacher educator researchers. One goal of my study is to provide practical and tangible ways for mathematics teacher educators to work on themselves (both in terms of their learning and in terms of researching their learning) as well as in their work with mathematics teachers. In the following chapter, I draw on data from context one (section 2.2.1, page 19) in the way described in the previous section (section 3.6, page 50), as a way of retelling the story of one of my experiences as a new mathematics teacher educator, as a way of exposing hidden assumptions and biases.

~~~~~
~~~~~

Chapter four

A (not so) hidden agenda

In the previous chapter, I discussed a series of methodological issues in relation to a self-based narrative-inquiry that aims to report on the *process* of my becoming a mathematics teacher educator. I also described (in section 3.6, page 50) how I utilise creative analytical practices to analyse one conversation with Sam (context one: section 2.2.1, page 19), an analysis that forms the basis of this chapter. Having listened, multiple times, to each of the recorded conversations with Sam, I made the decision to focus my analysis on one particular conversation. The conversation I chose was an early conversation (the second one following a lesson observation) that was both substantial (in terms of length, which was not the case for the majority of the conversations) and indicative of the issues that have become central to my study. Although every conversation with Sam included opportunities for me to explore my inner biases and assumptions, the conversation I use presented me with the greatest opportunity to uncover what was behind my comments during the conversation. The conversation could therefore be described as paradigmatic in that it is representative of a central issue that is fundamental to my study. The issue I am referring to will become apparent over the course of the chapter.

I transcribed those sections of dialogue that I have used directly in the augmented conversation that follows (section 4.1), as well as some additional sections that can be accessed (see appendix 2b, page 296, for the partial transcript of this dialogue) for further context. The sections of dialogue presented in the following augmented conversation do not always follow directly on from one section to the next. When there is a gap in the dialogue that is not presented, I make this clear throughout, including marking (in square brackets) each new section with the time stamp from the audio-recording as referenced in the original transcript (appendix 2b, page 296). The original transcript is in black text, augmented with my stream of consciousness using purple italic text.

4.1 One early conversation

It was the second lesson that I had observed Sam teaching her year 10 class (students aged 14-15 years). I told myself before going in to observe Sam teach that I was not there to somehow change her so that she fitted with my image of teaching mathematics, the image that I had developed as a mathematics teacher. Instead, I was there to support her in working on issues arising from her own teaching practice, that she herself had identified and that were meaningful to her.

The lesson had begun with students being asked to calculate the five-number summary (see glossary, page 291, and figure 4.1, page 57), needed for constructing a box plot, from a dataset consisting of ninety pieces of data. Our conversation took place immediately after the year 10 lesson, and it began by me asking Sam for her initial reflections. She began by talking about how she would usually teach box plots and described what she had been teaching in the lessons preceding the one she had just taught, all related to statistics and handling data. The conversation moved to teaching statistics more generally which led to Sam's strong statement [00:09:58] that took me quite by surprise:

Sam: I think we teach statistics really badly.

This seems to come quite out of the blue. I am not quite sure what she is getting at. I take a guess based on my appreciation that teaching the many similar, but ever so slightly different, statistical techniques can lead to a dependency on memorisation, rather than any sense of purpose, especially when techniques are taught in isolation.

Tracy: You mean as a process?

Sam: Totally!

Her enthusiasm is palpable.

I don't think we teach statistics with enough real data. So, when it comes to a question where the students have to analyse two different data sets, like they were doing in the lesson today, the data doesn't mean anything to them.

I feel a strong resonance with what she is saying. I certainly valued giving students real data sets in my classroom, allowing them to explore the data, sometimes to the point where it became challenging to refocus the students. Sometimes I would ask the students to collect data for themselves, about themselves and one another, but more often I would give them an existing data set from a real source[13]*. Over time, I developed a kind of structure when working with students and data. I would almost always ask students to hypothesise about potential relationships or patterns from the data before planning an approach to test these hypotheses. This stage would involve me offering a number of example hypotheses as many of the students would seem to find the idea of developing hypotheses quite challenging (something I learned from a number of unsuccessful lessons). There also seemed to be something powerful to me about working with the same set of data over an extended period of time, so that the students were more likely to develop an understanding of the purpose of the different statistical techniques, in relation to their hypotheses.*

[13] For example, I used to use comparative data from countries across the globe (characteristics such as life expectancy, gross domestic product, population growth, literacy levels, etc).

The first box plots we ever looked at as a class were the boys' mock exam results versus the girls' mock exam results, and it really meant something to them. The upper whisker for the boys' box plot was huge [Sam is pointing to a diagram of a box-and-whisker plot similar in form to the one in figure 4.1, the upper whisker refers specifically to the line that extends from the upper quartile (Q_3) to the maximum], the top quartile for boys was really spread out, whereas the same section for females was much shorter, and not as far along the scale.

She is really animated; I can feel her enthusiasm, which I find encouraging. She seems to be enjoying our conversation, like it's an opportunity for her to formulate and develop some of her ideas.

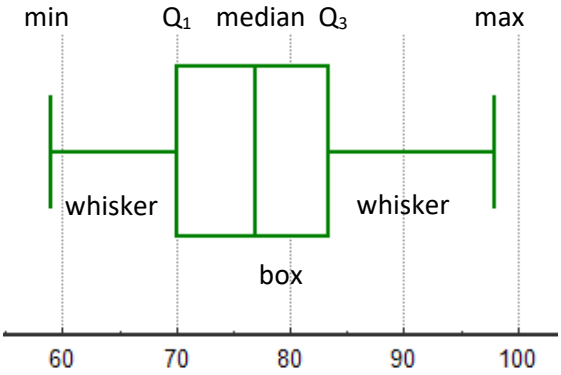


Figure 4.1: Box and whisker plot.

I still hear Sam's enthusiasm as a marker of something significant, though not so much as a result of our conversation. Instead, I recognise her energy as an indication of something she values in her classroom, using "real data". One reading of the situation is that Sam views real data as consisting of data that is directly connected to the students themselves. Another reading is that she sees real data in the broader sense, in the same way that I had been imagining. I could have sought some further clarification rather than assume we were sharing a common image. It now feels important to me, in conversations that I have with teachers of mathematics, that we establish what it is that we are each talking about, to avoid making assumptions about meaning. I had not known Sam for long at this point, having only had a limited number of interactions before this particular conversation. I think I made some assumptions because they suited me. It felt comforting to hold on to

my teaching practice in this way, through somebody else. I think I went into that conversation assuming we shared a view of teaching, but I am not sure this can ever really be the case, even with those you have worked closely with. We are all different and we bring our different experiences and values to our mathematics classrooms. What feels important now, as a mathematics teacher educator in conversation with mathematics teachers, is that I get as close as I can to understanding the different images of mathematics teaching that these teachers have. I could have asked myself, "What is Sam's image of teaching mathematics?" and been curious about that or "What does she mean by what she has just said?" rather than making comfortable and appealing assumptions. We continued talking for a couple of minutes, the subject of outliers came up and we discussed how to identify these in a data set, before I brought the focus back to the lesson [00:13:29]:

Tracy: So why did you choose the data that you used today? The ninety pieces?

Sam: Oh. Because it was just there, so it was quick.

She sounds flat suddenly, perhaps she thinks I'm judging her or doubting her decision given what she just said about using real data. I liked the ninety pieces of data though, it may not have been real, but the students were presented with a complex situation that they had to work at making sense of. In my classroom, I would sometimes begin from a more complex starting point than a textbook or scheme of work might suggest, rather than presenting ideas as small steps up to some end point where it all comes together. I would begin almost with the end point, though not always successfully. I suppose that was in the belief that complexity does not have to be complicated. I assumed that mathematical relationships would only really become apparent to children if they were presented with situations where those relationships could be made sense of. I was also convinced that, by becoming accustomed to dealing with complex situations, children would more easily cope with whatever a formal assessment might throw their way.

Tracy: But I think it was nice having that level of complexity.

I say, partly hoping to reassure her but I really do mean it.

Sam: I always have in mind, what is going to push those two [Sam is pointing to where two students were sat together during the lesson] a little bit further, so when I saw it was point two seven five [one of the values needed for a calculation that the students needed to use to draw a box plot], I thought yeah let's give it a go. That is going to sort that table [indicating the same students as before] out.

I feel a little uncomfortable. I want to help Sam see that it is not just the children in that particular group who can benefit from a challenging task. The complexity presented to the students by this relatively large set of data seemed worthwhile for all of the students in that classroom. I had noticed many of the students getting stuck in, having productive conversations with one another about the detail of their solutions.

Tracy: I really liked the ninety pieces of data. I think the only thing a few of them seemed to struggle with was finding the forty fifth and forty sixth pieces, just because they were trying to count off the board [the set of data (consisting of 90 separate pieces) was displayed on the board as a list], so they were getting lost in the list of numbers.

It is easy to make assumptions in the moment, to jump to conclusions. I think I made an unhelpful assumption here, and rather than testing it out, or probing for a little more clarity, I allowed myself to make a judgement based on little information. I decided that Sam was limiting her expectations of the other students. It might have been, for example, that the two students that Sam referred to acted as a gauge for her, a way of helping her to determine her starting point, not to the exclusion of the other students. I could have explored this with her further.

I was keen for Sam to appreciate the benefits of students being allowed to find things challenging: that it was OK for them to experience some confusion; and that more connections can be made when presented with complex situations. She said the question

she had used (with the 90 data points) was "harder than anything the students [would] have to do in the examination" [00:14:48] which I took positively. So, I was disappointed in Sam's response when I asked her if she would return to that dataset again:

> Sam: I don't know, the range goes from thirty to one thousand four hundred and forty-one so it would be quite difficult to draw that accurately on their exercise-book paper. I think I will do one tomorrow with nice traditional whole number values for all those things [Sam is referring to the relative positions of the five number summary (see figure 4.1: min, Q1, median, Q3, max)]; then I will do one that is a little bit more complicated but where they [Sam is again referring to the relative positions of the five number summary] are all halves; and then I'll go back to that data I think.
>
> *Oh, that's not what I was expecting. I guess I understand the reassurance that an easier example can bring. The data was messy, but data is messy, and I don't doubt the importance of students being given plenty of regular exposure to past examination questions. I'm not sure I would have wanted to simplify so quickly though, but maybe the example from the lesson has run its course and Sam seemed particularly keen on using data about the students themselves.*

It was not my place to be disappointed of course. They were not my students, and it was not my classroom. But I was disappointed, nonetheless. I think, to an extent, I was longing to be back in my own classroom, teaching children mathematics, so my decision to approach Sam, as a new mathematics teacher educator, was not made without a certain promise, promise of a classroom that I would feel comfortable in, that I would feel was close to my own. Yet, unsurprisingly, Sam's classroom was different to mine, something I was not yet ready to accept [00:15:44]:

> Tracy: I saw a nice lesson on box plots a few years ago where the teacher had got some data from the school on rewards and he had it organised by year group or by house. The students had a connection with the data, like you were saying with your mock

exam data yet they didn't need to worry about collecting it for themselves. I realise they need to know how to do that as well, but actually the sophisticated bit is forming the statements about what is different about the distributions and why, which you talked about in your lesson. This teacher had a series of box plots, without any scale, and asked them what's the same, what's different.

It strikes me that there was a perfect opportunity in Sam's lesson to get the students discussing the box plots that they were being presented with, using a same-different prompt, so that everybody would have been able to get involved.

It seemed to generate a really rich discussion. He might even have asked them to suggest possible datasets that were represented by the box plots, before revealing to them what they actually were.

I had remembered that particular lesson vividly because I had been involved in the planning of it and I had been delighted to observe the students being given the opportunity to share their ideas and formulate their own arguments. I chose, however, not to share my close involvement in that lesson with Sam. I had developed a strategy fairly early on as a new mathematics teacher educator, which was to avoid talking directly about my own teaching, as if there was a little bird, sitting on my shoulder…

4.1.1 "There is not one way of teaching mathematics"

… I would hear the bird say, a principle from the course I teach on for prospective mathematics teachers, a principle that I was well aware of as a new mathematics teacher educator. What I was not aware of, however, was how to make that principle *real* in my actions, as part of my practice, as part of me. To be honest, I think I denied it a little bit, but I knew that had to change. What else did I have to offer beyond stories about my own teaching? As a result of the bird that sat on my shoulder, I found that when I spoke to teachers about teaching, I would talk about the classrooms of others, just like I did during this conversation with Sam, sometimes knowing full well that I was very much communicating my own values, just indirectly.

My experience of classrooms beyond my own were limited to those of the teachers with whom I worked, in a department where I had been responsible for the development of the mathematics curriculum; not exactly far removed from my own way of doing things. I may as well have talked about my own teaching and been open about that with Sam. At the same time, I could have worked on being open to hearing new and different possibilities. Sam was not a prospective teacher, we had things in common and she had expressed a desire to teach *mathematicians*, that is, to support her students in working as mathematicians do. I guess during that conversation, I thought that I could help her to achieve her aim, that I could provide her with some solutions.

Becoming a mathematics teacher educator is evidently not as simple as learning how to avoid talking directly about one's own classroom. Avoiding talking directly in this way was something that I learned to do, something that was different to what I had been doing as a mathematics teacher and as a curriculum leader. More recently, becoming a mathematics teacher educator has involved finding myself in many different classrooms. I now have numerous stories that I can draw on in conversation with others that really are not stories involving me and my teaching. Having said that, I suspect I still choose to tell those stories that demonstrate some of my own values. I wonder how I might know whether that is indeed the case and how much it matters. As a mathematics teacher educator, I have come to feel uncomfortable when talking about my own classroom, which can feel indulgent and self-centred, although less so maybe when I am reporting on past struggles or perceived failures rather than successes. Maybe that is about empathy and being human. I do, on occasion, still share stories from my classroom. I think it is about the particular context of sharing and the purpose of the story. If only becoming a mathematics teacher educator was as simple as ruling out certain behaviours or sticking to a set of well-defined rules. As a mathematics teacher educator, I feel I probably give much of myself away through my teaching, regardless of the stories or experiences that I do or do not share. When I teach a group of prospective teachers, I am still that mathematics teacher, but there are differences. That is partly what this study is about, understanding what those differences are and how they come about.

Sam and I continued talking for a few more minutes. She mentioned a "human boxplot" (when students position themselves as if they were a piece of data (e.g., by height) to show the distribution of the data as a boxplot would) that somebody from her department had tried that she quite fancied trying for herself. She mentioned some ideas that had come from her experiences of teaching science. I asked her at one point [00:20:56] if there was anything she would do differently and she talked about using an "easier set of data", which I questioned. It seemed like she had not made the connections to whole-class discussion as I had intended her to make through my description of the lesson I had observed. Then there was a natural pause, I prompted again [00:24:48]:

Tracy: So, is there anything *else* you would do differently?

Sam: Um, well, I would like to think I could find a way to talk less but have the same opportunity for discussion before they actually get into a question.

This is promising. I am aware of Sam's voice having been dominant. She had talked for a significant amount of the time, relative to the students in any case. I have always valued whole-class discussion in my own teaching, I want to hear what the students think; I want them to hear one another; and I want Sam to experience that in her own classroom. It is important to allow students to be able to articulate their own understanding and to make sense of one another's ideas. I could support Sam in developing a culture where the students are happy to openly discuss their ideas. I spent years working on exactly this issue after all.

Tracy: Why do you want to talk less? Is that a general thing?

Sam: Well, I don't know; I think it all links to getting them working harder than me.

That is interesting, and not really where I am coming from. I have frequently heard a related question around in school though, along the lines of, "Who is it that is doing the work?" During class discussions, I

would certainly be working, working really hard, to make sense of what was being said by the children, to reframe, to respond, to repeat, and so on. "Who is doing the work?" does not feel like it is in quite the right place. Equally, the students in Sam's classroom were working hard.

Tracy: Right, OK.

Sam: But then, I think it's quite an easy thing to criticise how long a teacher talks for. There are some people in this school that have a bit of a thing about this. We have been told recently that the teaching should all be done in the first ten minutes of the lesson.

Oh dear, this is somebody else's issue, not Sam's.

Sam continued to describe her interpretation of what was being presented to her, and the rest of the staff in her school, as a general issue in classrooms around teacher talk and the potential for students being passive. I had wanted to know what issues Sam wanted to work on, not somebody else's [00:27:10]:

Tracy: And what do *you* think about that?

Sam: I don't think my kids look switched off: I think they engage with the white board; I think they've got pens in their hands; I think they are taking notes; when I ask them a question, they have all listened to the question, they might not have an answer but that's not because they haven't been listening. So, I think it is quite an easy thing to say, "If you talk for less, your lessons will be better". Actually, if you take what happened when I said, "Now do question one and question three", with the exception of three groups who just didn't read the question properly, they all knew what they were doing. They got on with it; it was quite a good level of challenge. So, in one sense you could argue I judged the amount of teacher talk just right.

I am not sure if Sam is trying to convince me. I hope I haven't given her the impression that I agree with the ten minutes teaching time

nonsense. I guess I am pleased she is happy to voice her opinions to me. I am hoping that there is still something in what she was saying, though, about talking less and opportunity for discussion, beyond her exploring what sounds like an issue that might have come from an external source, rather than from her.

Tracy: So, can you unpick what you meant by finding a way to talk less, because I had felt that this was something that had come from you. You mentioned wanting the students to work harder than you. So, is it the amount of talk, or the type of talk, because I think what you said was, something about discussion, getting them to discuss more?

What Sam actually said was that she had wanted the students to "have the same opportunity for discussion before they actually got into a question", which can be achieved in all manner of ways and does not necessarily require less teacher talk, or a whole-class discussion (which, during that conversation, was what I had wanted to steer Sam towards). I had had "getting them to discuss more" as a criterion of my own, something I valued (and still value) in my own teaching. Sam had, during our initial conversation, mentioned that she has a "tendency to jump in" before students make mistakes. It would have been interesting to explore links between her image of jumping in and her sense of offering the students the opportunity for discussion. From my current perspective, the two things seem connected, but in the moment of the conversation I had lost track of issues that had already been raised. I think I was searching for something that I was confident I could offer a solution to. Of course, things are not that simple.

~~~~~
~~~~~

Interlude one: On the *ethics* of teacher change

I was experiencing what Pimm (1993) so powerfully ascribed to my "teacher-educator-lusts" (p. 31). As Pimm put it, "mathematics education is a discipline prone to the lure of single solutions" (p. 30). I was mortified by the suggestion from Sam's colleague that the solution to more effective teaching was to limit teacher talk to ten minutes, yet, in that same moment, I had my own set of solutions in mind for Sam. As Pimm warns, the danger is that a "layer of complexity" is confounded by an "externally imposed layer of control" (p. 30). One such lust, "thou shalt change" (p. 31), is the yearning that a teacher educator possesses to change the teachers that they work with:

> Their change is not our business; how, when and if they change is surely their concern alone. It is a continuation of the dangerous idiocy of assessing teachers (under the name of "accountability") through their students' results. If I as a teacher educator can only feel successful if the teachers I work with change (and in ways I want them to), I am setting up both myself and the teachers I am working with quite dramatically. I believe it is dangerous to lose sight of how difficult personal change can be - and we should not talk lightly or glibly about it, let alone expect or demand it. (Pimm, 1993, p. 31)

Pimm is right. Personal change can be extremely difficult. It can involve grieving for the way things were or the way we had imagined things to be. I am a mathematics teacher educator who is working on myself, on developing my practices. Many teachers want to do the same, for the benefit of those they teach. As Mason (2003) puts it, "personal change is a personal matter. Only I can make changes to my practice, wittingly or not" (p. 283). The desire for change needs to come from within, not simply from an external source or force. We must want to change for ourselves: to feel a need; to imagine a different way of being; and to believe it is possible. There are certainly external influences on that change, for instance, a novice teacher observing a more experienced teacher, can change their own practices by observing the classroom practices of others, but only if they want to make those changes. The way that you experience external influences will depend on your own desire to be different.

Mary Boole (born in 1831) introduced the antecedent to teacher educator lusts, in describing "teacher lusts" as she saw them:

> First, the teacher wants to regulate the actions, conduct and thought of other people in a way that does no obvious harm but is quite in excess both of normal rights and practical necessity. Next, he wants to proselytise, convince, control, to arrest the spontaneous action of other minds, to an extent which ultimately defeats its own ends by making the pupils too feeble and automatic to carry on his teaching into the future with any vigour. Lastly, he acquires a sheer automatic lust for telling people "to don't", for arresting spontaneous action in others in a way that destroys their power even to learn at the time what he is trying to teach them. (Boole in Tahta, 1972, p. 11).

Boole's powerful description of a teacher's wants and desires will still resonate with mathematics teachers of today. Her account certainly resonates strongly for me. As Boole implies, teacher-lusts are not ill-intended, rather, as Tahta (1991) alludes, they exist because of our personal desire to understand. For those who teach, the desire to understand is "also accompanied by the need to get others to understand" (p. 240). For Boole, this desire can translate, sometimes unknowingly, into "control" and the suppression of "spontaneous action".

It could be argued that it is the responsibility of all mathematics teachers to teach so that students *understand*. Why else would the mathematics teaching community spend so much time talking about *teaching mathematics for understanding* or more recently, at least in England, *teaching mathematics for mastery*? Why would we use international comparisons tests such as the Programme for International Student Assessment (PISA, see glossary, page 291) or Trends in International Mathematics and Science Study (TIMSS, see glossary, page 291), as a way of finding out the most effective ways of teaching for understanding? Teaching for understanding, however, is not what Boole, Pimm and Tahta seem to be questioning. What these authors are bringing to our attention is the lust for children to understand mathematics in a certain way, in *the way we think that we understand it*.

Teacher lust has been described as an affliction. Kilpatrick (1987), for example, claimed that mathematics teachers are especially likely to be afflicted with teacher lust, as, "having just asked a student to explain something, they often jump in, with scarcely a pause, to provide a clearer explanation themselves" (p. 1). Interestingly, Sam used exactly the phrase, "Is my tendency to jump in proving to have a deleterious effect on the students' learning?" (see section 2.2.1, page 19) during our initial conversation before I observed her teaching. Sam's awareness of this desire to jump in aligns with Tyminski's (2010) conception of teacher lust as something that is *experienced* by some teachers of mathematics as a kind of tension that arises from an apparent gap between intention and action. For Tyminski (2010), teacher lust is an internal impulse to act in a way that could remove the "opportunity for students to think about or engage in mathematics for themselves" (p. 295). Tyminski claims that teacher lust is only *experienced* however by those teachers whose desire "for students to learn and to understand mathematics as the instructor does" is "in conflict with [the teacher's] intended pedagogy" (p. 298). A little like Cobb's (1994) "pedagogical dilemmas" (p. 14), experienced by teachers who are trying to navigate the seemingly contradictory view of mathematics learning as "enculturation" and at the same time as "individual construction" (p. 14). Consequently, for Tyminski (2010), teacher lust is only conceptualised for a mathematics teacher whose explicit goal is to move away from direct instruction towards a "focus on facilitating student learning" (p. 299).

If teacher lusts are connected to our desire for others to understand mathematics in the way that we as mathematics teachers imagine we understand mathematics, then teacher educator lusts could be described as our desire for others to teach mathematics in the way that we as mathematics teacher educators imagine teaching mathematics *should* be done. Mason (2003) described teacherly lusts as the desire for others to have experiences that are similar to the experiences that we have had. Is this so wrong?

> One of my guiding principles is that desiring other people to change says a great deal more about me and my desires than it does about them. What am I hiding in myself, or from myself, by focusing on others? Is it a wishful or blaming sentiment of
>
> "if only they would . . .",

> by which I try to pass responsibility onto others? If so, then the real task is to work on myself. Is it an evangelistic sentiment of
>
> "I really enjoy, got benefit from… so others could (should?) do likewise",
>
> trying to urge my experience upon others? What is this teacherly lust [...] this desire for others to have experiences similar to mine? If this is the force, then the real task is to invite and inveigle rather than force others to become interested in what interests me. […] trying to force other people to change in specified ways is ultimately unproductive, and even ethically dubious. (Mason, 2003, p. 284)

Much of Mason's work has been focussed on how teachers can go about changing their practices. In *Researching your own practice: The discipline of noticing*, Mason (2002) sets out a systematic approach to developing teaching practices based on the central idea of "noticing". In fact, he states that the discipline of noticing provides "a method of working on issues to do with practice, at every level and in any context" (2003, p. 281), yet he is careful to make no claims about what is worth noticing and warns of the ethical issues associated when an external force attempts to get teachers to "adopt practices that are deemed valuable or even necessary" (2003, p. 282). External stimuli can be useful in initiating or affecting teacher change, but only, Mason warns, if the teacher wants to work at changing their practices and recognises issues in their practice that they feel are within their power to change. Ultimately, change in practice depends on the teacher's own desire to change, which can be enhanced by the support of others: "Only *I* can make changes to *my* practice, wittingly or not" (2003, p. 283, emphasis original). What we notice as practitioners reveals something of our own value system or set of criteria, which, if unchallenged can lead to a self-reinforcing cycle of matching our expectations. That is why support and challenge from others is essential for professional development. Mason promotes the discipline of noticing as therefore placing importance on "seeking resonance in an ever-expanding community of sceptics" (2003, p. 282). In looking beyond our own criteria, beyond the criteria of our local "communities of practice" (Wenger, 1998, discussed in section 5.1, page 80), as practitioners, we can extend and sharpen our capacity to notice. Mason brings into question an often-unchallenged criteria, that is "whether learning is being or has been enhanced" (Mason, 2003, p. 282), but who specifies whether this criteria has been met?

Pimm (1993) suggests that "[t]o go to conferences where teacher education is being discussed is to fall in with the change merchants" (p. 31). I have considered his claim at some length. Curriculum reform movements, often motivated by a change in a country's central government, appear to be a familiar feature of the mathematics education landscape, so, in that respect, change is always on the agenda. I remember well a conversation I had at an international conference with a mathematics teacher educator researcher called Anika, who described herself as feeling "torn". She was working in a context where "reform mathematics" was the expected model of teaching mathematics promoted by the government and by mathematics teacher educators. She was expected to advocate a certain way of teaching mathematics in primary schools in line with the reform mathematics agenda. She talked about some of the prospective primary school teachers that she was teaching, some of whom were reluctant to teach mathematics in the way that Anika was trying hard to encourage. Anika said she knew that reform mathematics was the most effective and socially just way of teaching mathematics but not all of her prospective teachers were convinced. In some cases, when the prospective teachers got into school for their teaching practice, their school-based mentors (see glossary, page 291) offered them different models of teaching mathematics in comparison to Anika's. I asked Anika why it mattered to her so much that she could change the teachers that she worked with when they seemed reluctant, and, in some cases, determined not to change. Anika described feeling powerless. She said the schools that her prospective teachers were placed in were more influential in terms of the adopted teaching practices of the prospective teachers and she did not know what to do about that. She said that, in some cases, there was a divide between the theory taught at university and the teaching practices within schools and that a few of the school-based mentors actively reinforced this divide. I really felt for Anika. By that point, I had spent a lot of time contemplating my own desire to change the practices of the teachers that I worked with, in the way that I had wanted the practices of those teachers to change. I offered Anika a little of my own story, by no means was it the same as Anika's but there were resonances. She was looking for a solution to a problem: what could she do with school-mentors so that they were more open and supportive of reform mathematics teaching? I suggested

she could think about the problem from a different angle, that she could consider how she might adapt her course to be more accepting of the different ways of teaching mathematics; where the prospective teachers can develop their images of teaching mathematics without feeling a disconnect with the 'theory'. That would not have to mean abandoning the promotion of reform mathematics if that were the context of teacher education in Anika's country. There was something inevitable to me about what can happen if you only offer one model of teaching. What happens if prospective teachers are not able to fit this model, even if they try to make it happen in their classrooms? Anika could indeed work with school-based mentors, but Pimm (1993) was right, "[t]heir change is not our business; how, when and if they change is surely their concern alone" (p. 31). Perhaps I had fallen "in with the change merchants" (p. 31) at this conference, whom it was not my business to change! But there is a subtlety in what Pimm's words say to me that I have only recently been able to appreciate. I can *offer* ways of teaching, new potential ways of being in a classroom, but this does not have to equate to a lust for change. So long as I let go of the *need* to change teachers (and teacher educators) "in ways I want them to" (p. 31) and that I am sensitive and open to the needs and values of those teachers with whom I work, then there is still a place for mathematics teacher educators within teacher education (much of what I have been doing is questioning my purpose).

> [M]y desire becomes an obstacle, an impediment, as soon as I desire specific practices, specific changes, specific pleasures. I consider it to be ethically sound to wish for others that they have continued opportunities to choose to participate in activities through which they may experience something fresh, some expansion of their current awareness. But I am adamant that at every moment I respect their choice to opt out. I do not, however, respect a choice to revert to (or even worse, to subside into) mechanical and unreflective behaviour. I consider that to be unprofessional (Mason, 2003, p. 284)

Personal change is indeed very difficult, and "we should not talk lightly or glibly about it, let alone expect or demand it." (Pimm, 1993, p. 31).

~~~~~
~~~~~

4.1.2 The conversation continues

I continued my attempts to steer Sam towards the idea of supporting whole-class discussion. Sam was not taking my steer, however, and instead she began to describe two lessons where she had set up a group of students to teach some new content to the rest of the class. The group had spent the first of the two lessons teaching themselves iterative methods (see glossary, page 291) from a set of textbooks and other resources. She went on then to describe the moment from the following lesson when the group of students began teaching the rest of the class [00:30:35]:

Sam: As soon as they put an example on the board I thought, oh I can see where this is going, they didn't know though, and the example did exactly what I thought it would do and it went really badly. A lot of kids were saying, "I don't get it", "you're rubbish at explaining", kids that wouldn't say that to me. So, I ended up having to teach it again anyway. How can you be responsible for the quality of the initial teaching of the topic whilst still letting them be involved?

I have often shied away from this type of activity in the classroom, getting children to teach the rest of the class. It feels different to sharing ideas through conversation, where children can ask questions and offer their own understandings.

Tracy: So, what other strategies are there, where you are still teacher if you like, but you're not doing all the talking or, or questioning?

I find it a little shameful to return to some parts of this conversation with Sam, knowing I am exposing myself to you, the reader, but it feels important to do so. I have in mind the voice of another little bird on my shoulder who reminds me that "there are no *shoulds*". By sharing one possibility, I am opening myself up to new and different possibilities. It seems so clear to me now that I was playing a game of guess what is in my head, but without necessarily an awareness of doing so. If I had explicitly offered whole-class discussion as a different possibility it would at least have been open and honest. Sam

could then have decided if she wanted to work on establishing more whole-class, discussion-based activity in her classroom or whether she did not, and I would have then known.

Interestingly, the issue of whether or not to *tell* has been an issue that I feel I have been grappling with since the beginning of my teaching career. I think 'telling' has always had negative connotations for me, perhaps now that is explained by Cobb's (1994) pedagogical dilemmas or by Tyminski's (2010) conception of experienced teacher lust. This conversation with Sam, however, demonstrates to me, firstly, that this issue is still something I am well aware of, and, secondly, that telling does not need to equate to behaving wrongly. What does feel unethical is to act on the pretence of not telling, but then try to steer somebody towards a solution that I deem of value or even that I deem necessary, especially in the context of a mathematics teacher educator working alongside a mathematics teacher. A challenge for me is to become more awake to moments where my desire to fix a problem, as I see it, then influences my actions as a mathematics teacher educator. What this study is partly about is understanding how to support mathematics teacher learning and development (and change) without that being about me and my desires.

Rather than discussing alternatives to getting students teaching one another, as I had prompted, Sam suggested that it was her strategy that was the issue, rather than the principle itself, and that she had not spent enough time with the group before they began to teach the rest of the class. She went on to describe other lessons she had taught [00:31:35], that she had viewed as successful and attributed the success of them to being "student-led". She concluded by saying, "I guess the moral of the story is, if you are going to do it, you can't skimp on the time that it takes, I think the learning they got out of it was significant because it wasn't from me". I am not sure exactly what she meant by any of her uses of "it" since I did not ask her. I made my assumptions and carried on so that eventually I managed to engage Sam in some discussion about the merits of whole-class discussion [00:35:40]:

Sam: I think well managed whole-class discussion is an invaluable tool for the teacher, but is a really hard skill, because you've got to have all of your behaviour management sorted out; you've got to be prepared to think on your feet, so, what happens if somebody asks me a question that I don't know? So, there's all that subject knowledge, all that pedagogy, and I think it is easier for somebody to say whole-class discussion is not a useful tool, than to say, whole-class discussion is a useful tool when it is skilfully managed, but it's not often skilfully managed, so, think really carefully about whether you want to do it or not. It's almost like, let's get rid of those things because it's easier to say, there's too much teacher talk in lessons, too much group discussion, not enough individual work. It's easier to say that than to actually unpick the fact that it can be an extremely powerful tool, if it's well used. That balance of too much teacher talk; I think that particular person might possibly say there was too much teacher talk in my lesson.

We seem to be stuck in a bit of a loop, I'm not sure what the important issue is for Sam. She seems to want to talk about what somebody else has asked her to change in her classroom, which is not where I wanted this conversation to go. I know from working with other teachers that issues need to belong to the teacher themselves.

Tracy: But what do *you* think?

Sam: Well, I'm not sure there was too much teacher talk because once I set them off, and they got on with the questions, their focus was phenomenal, you know, they were talking about the maths. So, was there too much? Probably not, but that depends on who you ask, yes, if you don't want more than ten minutes teacher talk at the beginning of the lesson then there was too much.

Tracy: But I'm interested in what you want, because you brought it up yourself, there does seem to be something there for you.

Or was I imagining that… perhaps I am confused with what was there for me, what I wanted to be there for Sam.

Sam: Maybe there is something there, or I guess I could ask what my reason is for wanting to talk less. Is it because I feel I ought to, because we are being told to talk less? One of the things I pride myself on, with my kids, is that they will get the absolute best of me for the fifty minutes I'm in there. They might not get their books marked particularly brilliantly. I suppose in some ways I feel that I am the best person to teach this, so maybe it is me who does all the talking.

Sam needed to vent; I am happy she felt she could do that. She is an experienced teacher and with her experience comes a certain sense of conviction. In her exploring of this issue, summed up by Sam as "being told to talk less", Sam seems to me to be demonstrating exactly what Mason (2003) alerts us to:

> [P]eople are much more likely to want to work at changing something if they feel a need for change, so the most effective fulcra for leveraged change are issues which teachers recognise and feel are within their reach, within their power to change. On the whole, this means their own actions, for the actions of others are very hard to influence. (Mason, 2003, p. 282)

We continued to talk for a while, before bringing the conversation to a close. We did not really get to identify an issue in Sam's teaching that day, not one she was compelled to work on in any case. We did learn about one another though. Over the forthcoming few months, I continued to visit Sam, to observe her teach and to talk about the lessons.

4.2 An emerging and fundamental issue

It is so interesting to return to this conversation from my current perspective, to have viewed the dialogue with the benefit of hindsight. There is so much more to see than I was able to during the conversation itself. How ironic that so much of our conversation was in relation to Sam feeling coerced to make changes in her practice by others in her

school, and really, I was not so far away from coercion myself, perhaps my agenda was just slightly more hidden, even from myself. Now, in every turn I hear myself, my own way of seeing the world, my needs, my solutions. As an early mathematics teacher educator what did I have? I had my convictions from the classroom, convictions that developed over time. In some cases, that development began early on in my teaching career.

Over thirteen years as a mathematics teacher and curriculum leader I developed a lens of sorts, a lens with which I observed all mathematics lessons through, especially my own. I call it a lens, since it feels like a fitting metaphor for an observer, but really what I am describing is a set of criteria, criteria against which all of my classroom observations were set, with which to observe my own classroom and the classrooms of others. Having an internal set of criteria in a department where I was responsible for the mathematics curriculum allowed me to support the teachers which whom I worked to develop a similar set of criteria to my own. I am sure as teachers we all form our own criteria.

There are direct implications of having an internal set of criteria. As a classroom researcher doing research in classrooms, it feels important that I am aware of my own criteria, to keep returning to this awareness, to try to be aware in the moment. That must be what it means to be a reflexive practitioner. In the same way that as a mathematics teacher educator my own way of teaching mathematics taints how I work with prospective teachers (which includes observing them), my research will also be limited by my ways of seeing. How do I open myself up to seeing more as a mathematics teacher educator, as a researcher? Researchers develop frameworks for analysis and theory to bring to bear, but surely these are never completely impartial, neutral, or objective?

In the following chapter, I draw on enactivism as a theory of cognition in order to explain the issue that I have described here as being limited by some internal set of criteria. I use enactivist theory to consider why problems arose for me as a new mathematics teacher educator and to explicitly retell my story of working with Sam. Through the process of retelling, I am able to reframe my research problem.

~~~~~
~~~~~

Chapter five

Everything is said by an observer

In the previous chapter, I presented a layered analysis of one conversation with Sam, using creative analytical practices with an initial telling of the story (through the conversation itself) along with two retellings (firstly through my inner monologue, and then through episodes of retrospective analysis). Engaging in this analytical process of telling and retelling allowed me to uncover a central issue in my becoming a mathematics teacher educator; what I was seeing as a fixed internal set of criteria that was limiting what it was possible for me to see in mathematics classrooms and hear in conversations with mathematics teachers. I was left after that process of analysis asking, "How do I open myself up to seeing more as a mathematics teacher educator, as a researcher?" (page 78).

In this chapter, I begin by interrogating the issue as framed above, by considering my becoming as a mathematics teacher as a way of further formulating the problem I was facing during my conversations with Sam, as a new mathematics teacher educator. I then turn to the enactivist theory of cognition in search of a theory that both describes and explains what I was experiencing during my conversation with Sam and more generally as a new mathematics teacher educator. Towards the end of the chapter (section 5.3.1,

page 96), I then offer another explicit retelling of my story as a new mathematics teacher educator, which results in me being able to articulate the research problem, "How am I becoming a mathematics teacher educator?" (section 2.3, page 24), in more nuanced terms.

5.1 An internal set of criteria or…?

During the course of my teaching in secondary schools, my idea of what constituted effective mathematics teaching became more certain, it narrowed. I became less open to newness, though I would not have wanted to admit that at the time. I worked hard on developing my practices as a mathematics teacher. I studied mathematics education at master's level, early in my career. In doing so, I think my image of mathematics teaching hardened. I have previously described this hardening as a process of developing conviction (Helliwell, 2017a).

During my master's, I was exposed to a body of literature that comprised a diverse set of ideas. I read as widely as I could, but I chose to believe some ideas and not others, those ideas that did not conflict with my own experiences. The first school that I worked in (for eight years) was the school where I had taught mathematics as a prospective teacher. There was a culture in that school that I felt part of, both within the mathematics department and within the mathematics classrooms. Cobb (1994) would possibly describe my learning in that school from a sociocultural perspective, as "enculturation into established practices" (p. 13), a process by which cultural norms, values and practices are learned. In this school, mathematics teaching bore little resemblance to the mathematics teaching I had myself experienced as a student of mathematics. Rather than rejecting a different way of teaching, to that I had myself experienced, I embraced this new possibility. In becoming a mathematics teacher in that school, I experienced a sense of belonging that I welcomed with my arms wide open.

Lave and Wenger (1991) conceptualise a special type of community, a "community of practice", a concept expanded on by Wenger (1998) who describes a "community of practice" as necessarily consisting of three characteristics: mutual engagement; a joint

enterprise; and a shared repertoire (see figure 5.1, taken from Wenger, 1998, p. 16). As I look at the words Wenger uses to describe each of these characteristics, I am particularly drawn to "rhythms", "stories", "actions" and "doing things together" all of which speak to my experiences of becoming a mathematics teacher in those early years.

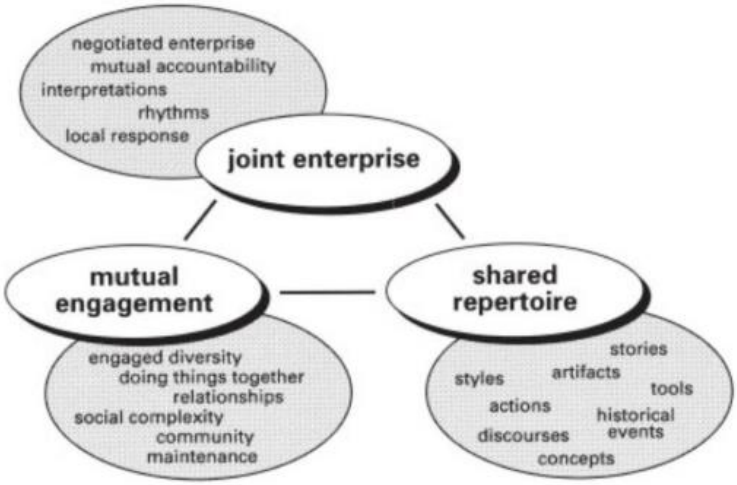


Figure 5.1: Characteristics of a community of practice (Wenger, 1998, p. 16).

Learning as "situated" acknowledges the relationship between learning and the culturally constructed social and material settings in which that learning takes place. For some scholars, learning is necessarily situated in that it is "a product of the activity, context, and culture in which it is developed and used" (Brown, Collins & Duguid, 1989, p. 32). For others, learning is not seen so much as a *product* of activity within a context or environment, rather, learning *is* activity, an *active process*, viewed as dynamic, local and emergent (e.g., Maheux & Proulx, 2015; Reid & Mgombelo, 2015).

As a new mathematics teacher educator, my view of the world of mathematics teaching was determined by my own history of experiences and interactions, just as it is now, and just as it will always be. During my early visits to observe Sam teaching, my history of experiences had accumulated over years of becoming a mathematics teacher. Davis (1996) brings together the ideas I have discussed so far in this section when he says that "[e]ach of us carries not only the history of our personal experiences, but the accumulated experience of the culture in which we are embedded" (p. 48). I find myself now, as a university mathematics teacher educator, becoming embedded in a new and different culture from that of school, which has meant, to an extent, learning to let go of

certain ways of being that were established in my previous settings. Returning to that conversation with Sam, uncomfortable as it has been, has highlighted to me how fixed my ways of seeing mathematics teaching had become. Though I had intended to be open, to listen and to allow Sam to identify her own issues, I seem to have been limited by my own predispositions; I heard Sam's issues as my own and felt that I could offer her the practices and ways of working that I had myself spent years establishing. Surely there was more to explain this problem than having transitioned from one community of practice to another. I (thought I) knew I needed to surrender my own feelings and ideals, yet I brought all of my criteria and judgements with me.

5.2 In search of explanation

When I started my doctoral study, I understood that I would need to articulate my ontological perspective and my corresponding epistemological position. These theoretical constructs would need to be coherent and inform my methodology and research design. I asked myself, "What do I know about the nature of reality?", replying, "I know that I am alive, that stuff happens and that some stuff that exists is probably stuff that you cannot directly perceive, like my feelings." I really had no idea about how to identify an epistemological position, surely that was the whole point of my research, to understand my own becoming, my own learning. How could I pin down a particular theory of learning before the learning that I was trying to theorise had begun?

During my master's study, I decided to write about constructivism, not really knowing too much about what the implications of that were, but I needed a theory! I understood constructivism to be a theory of learning where it is recognised that learners actively construct their knowledge out of their experiences. Rather than knowledge being seen as some sort of internal representation or construction of an external world, "constructivism claims that we have no access to an objective truth and that all knowledge is subjective and dependent on the learner" (Proulx, 2008, p. 14). Constructivism describes human cognition in terms of "*fitting* to, and compatibility with, the experiential world" (p. 14, emphasis original) as opposed to more positivist theories

that assume a direct relationship or one to one correspondence between an objective reality and our internal representation of that objective reality. The existence of an external reality that is independent of the knower is neither acknowledged nor denied by constructivists, rather, it is deemed inaccessible and therefore 'existing' purely within the domain of experiences. During my master's research, I could see no objection to constructivist ideas, even though much of what I researched and observed were students' use of gestures as they worked on mathematical problems. I took those gestures as evidence of the students accessing the mental imagery contained within their heads, which seemed to cohere with my sense of constructivism. I did not consider these gestures could be in any way *constituent* of the students' images, rather, I saw these movements more as the product or outcome of some mental processes. At that time, I had not read about enactivism or embodied cognition (see interlude two, page 84). Even if I had, the ideas may not have become meaningful to me during that study. So, I wrote my methodology chapter beginning from a constructivist position and that was that… until I presented some of my ideas at my first national mathematics education conference (the British Society for Research into Learning Mathematics (BSRLM, see glossary, page 291) was being held in November 2007) and had it pointed out to me that there may be other, more *fitting*, frameworks that I could have considered using.

Rather than beginning with a theory of learning, I have instead used narrative methods in order get a grasp of my experiences in storied form. The use of narrative methods and creative analytical practices (Bochner & Ellis, 2016; Ellis, 2004; Richardson, 1999; Richardson & St. Pierre, 2018) have enabled me to inquire into my conversation with Sam, to get a sense of what was happening, to uncover the phenomenon under investigation. Only now am I ready to explore the enactivist theory as a way of describing and explaining the uncovered phenomenon, to further inquire into my story as a new mathematics teacher educator and to support a further reseeing of the research problem.

~~~~~
~~~~~

Interlude two: On cognitive theories and enactivism

In this substantial interlude, I explore a range of ideas. Specifically, I explore *cognitive theories* before moving to focus on one particular cognitive theory, *enactivism*, which includes discussions on *structure and organisation; all knowing is doing;* and *learning*. I have marked the beginnings of these sub-sections using italics.

Cognitive theories: Cognitive theories seek to describe and explain the organisation of cognitive systems. Whereas representationist theories have conceptualised cognition as mental processes that occur within the brain, other cognitive frameworks extend to incorporate the body, such that the cognitive system is bounded by the individual cognising agent and the processes within. Furthermore, some cognitive theories are more inclusive and extend to beyond the body of the individual cognising agent to include the relationships and interactions with other cognising agents and the environment.

A cognitive theory (e.g., cognitivism) that places the cognitive system within a purely internal mental environment (i.e., the brain), separated from the external world, regards cognitive development as the acquisition of knowledge and subsequent effects on the mental structures within the mind. In this model, human intelligence so "resembles computation in its essential characteristics that cognition can actually be defined as computations of symbolic representations" (Varela, Thompson & Rosch, 1991, p. 40). The emphasis for cognitivists is on information processing where "the mind is thought to operate by manipulating symbols that represent certain features of the world" (Capra, 1996, p. 258). The computational model of the mind has been contested by a number of post-cognitive thinkers, primarily in response to two well-acknowledged deficiencies, specifically:

> The first is that information processing is based on sequential rules, applied one at a time; the second that is it localized, so that an injury to any part of the system results in a serious malfunction of the whole. (Capra, 1996, p. 259)

Capra, a proponent of a more inclusive model of cognition, contends that, in relation to the first point, even the tiniest of insects is able to complete complex cognitive activities

much more rapidly than would be possible if those cognitive activities were completed sequentially. In relation to the second point, he notes the well-recognised resilience of a brain that has been damaged without compromising all of its functionality.

Another critique of cognitivism comes from Hubert Dreyfus, who, along with his brother (Dreyfus & Dreyfus, 1986), argued that a computational model of the mind fails to account for human understanding as a skill "akin to knowing how to find one's way about in the world" (Dreyfus & Dreyfus, 1986, p. 4). They used this argument to explain why artificial intelligence endeavours of that time were destined to be unsuccessful. Hubert Dreyfus took inspiration from the phenomenological tradition, particularly the works of Merleau-Ponty and Heidegger, whose work focused on the meanings of everyday experiences, concluding that perception could not be explained using a set of objective principles in the way that an information processing model relied upon.

A particular objection from Varela et al., (1991) was the cognitivist hypothesis that "a system acts on the basis of internal representations" (p. 134) that they saw as carrying "heavy ontological and epistemological commitments" (p. 135). These commitments include the existence of a pregiven world or external truth that is out there, features of which can be represented by the cognising agent for that cognising agent to act accordingly in this pregiven world. Varela et al., reject the notion that cognition is "the representation of a pregiven world by a pregiven mind" (p. 9) where the mind is a mirror of nature, instead taking an enactive approach, where cognition is viewed as "the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs" (p. 9). Enactivism, as a theory of cognition, is explored in more detail later on in this interlude.

Hutchins (2010a; 2010b; 2014) uses the term "distributed cognition" to describe *all* cognitive systems where a central assumption is that "all instances of cognition *can be seen as* emerging from distributed processes" (Hutchins, 2014, p. 36, emphasis original). To take a distributed perspective, Hutchins explains, is not to make claims about the nature of the world but to look at the world in a certain way, by selecting a unit of analysis such that "wholes are seen as emergent from interactions among their parts" (Hutchins, 2014, p. 36). Furthermore, distributed cognition does not depend on some pre-

determined boundary, rather, the limits of a distributed cognitive system should "be responsive to the nature of the phenomena under study" (Hutchins, 2010a, p. 426). For certain phenomena, the appropriate unit of analysis would be a particular neural circuit, for others, the brain, or the body, or, more inclusively, the unit of analysis could extend to systems that are greater than an individual cognising agent. From an inclusive distributed view, where cognition is not purely a feature of the individual, cognitive processes are seen as distributed across individual cognising agents, artifacts, materials and tools within a culturally organised setting.

Distributed cognitive theories, which view cognitive systems beyond the limits of the individual, are by necessity *situated*. Situated cognition theory has been described as the genus of a number of inclusive cognitive theories all of which look beyond the boundaries of the individual and assume the now familiar brain-body-world formulation, where, in the case of cognitive systems involving humans, the world consists of culturally constructed social and material settings where culture itself can be considered a complex cognitive system within which human cognition is embedded (Hutchins, 2014). According to Smart, Heersmink & Clowes (2017), the particular species of situated cognition theory are inclusive of (but are not limited to): the *embedded* cognition thesis (which claims that "our cognitive processes are sometimes shaped but not constituted by bio-external resources" (p. 256)); the *embodied* cognition thesis (which claims that "cognition depends on, and is sometimes constituted by, the human body" (p. 256)); and the *extended* cognition thesis (which claims that "cognitive states and processes, under certain conditions, are distributed across embodied agents and cognitive artifacts or other bio-external resources" (p. 256)). Another so-called situated cognition theory is the *enactive* approach to cognition already referred to (Di Paolo, 2009; Varela, et al., 1991). The commonality, across all of these inclusive, non-Cartesian views of cognition, is that they are all concerned with the way a cognitive agent (such as an individual) is situated in the environment. My focus now turns to this last species of situated cognition, that is enactivism.

Enactivism: Enactive approaches to cognition (first labelled "enactive" by Varela et al., 1991, but also known as the Santiago theory of cognition (Capra, 1996)) are inspired by phenomenological philosophy and have been linked to Bateson's (2000) *ecology of mind*, where organisms and their environment are not viewed as independent objects that interact independently, rather, organisms and their environment are viewed as one complex system. Cognitive ecology points to "the web of mutual dependence among the elements of a cognitive ecosystem" (Hutchins, 2010b, p. 705). The study of cognitive ecosystems looks to "trace the course of human thinking to bidirectional coupling of persons and 'things'" (Cowley & Vallée-Tourangeau, 2017, p. 10). As in any ecosystem, everything is connected to everything else, or as Davis and Samara (1997) put it, "[e]verything is inextricably intertwined with everything else" (p. 111). It follows that in order to understand cognitive phenomena it is vital that we consider the "environments in which cognitive processes develop and operate" (Hutchins, 2010b, p. 706). Furthermore, within a cognitive ecosystem, the individual cognising agent is not viewed as situated *within* the environment, rather, the cognising agent is *part of* the situation or context (Davis & Sumara, 1997). This within-part of distinction is an important one, since for a cognitive system to be considered ecological, like any ecosystem, it must consist of mutually dependent, co-evolving elements all of which constitute, or are part of, the constantly changing environment, situation, or context involved in cognitive processes. By situation, enactivists do not only refer to our tools and technological resources, or our social and cultural practice and institutions, but "it also includes *us*" (Gallagher, 2017, p. 59, emphasis original). Thus, within a cognitive ecosystem, the common-sense divisions drawn "among individuals and between "persons" and "contexts" must be abandoned" (Davis & Sumara, 1997, p. 116).

For enactivists, this abandonment of common-sense divisions does not equate the absence of boundaries between person and context or "unity (entity, object)" (Maturana & Varela, 1998, p. 40) and "medium" (Maturana, 1988a, p. 9), in fact, it is exactly the achievement of a boundary between a unity and its medium that creates the biological conditions for cognitive processes to emerge. As human beings, we perceive an object as separate from its background, or in enactivist terms, we *observe* a unity as separate from

its medium, by "making an *act of distinction*" (Maturana & Varela, 1998, p. 40, emphasis original). According to Maturana (1988a) the "basic operation [that we perform] in the praxis of living is the operation of distinction" (p. 5). Each and every time we refer to something (implicitly or explicitly) "we are specifying a *criterion of distinction,* which indicates what we are talking about and specifies its properties as being, unity, or object" (Maturana & Varela, 1998, p. 40, emphasis original). Though we observe the distinction between unity and medium (figure and ground), enactivism maintains that "we can only perceive and interpret [the individual's] action by attending to the conditions of their existence" (Davis & Sumara, 1997, p. 111). In other words, organism and environment or unity and medium are simultaneously defined. Enactivists view cognition as an active process, which is neither located within the individual cognising agent (for example, as *a product* of an interaction), or within the environment, but emerging with and existing in the ongoing interactions between all elements of the cognitive ecosystem.

Enactivism offers a biological theory of cognition, which is understood from an evolutionary standpoint in that all organisms evolve simultaneously with their environments. In this view, "cognition is the very process of life" (Capra, 1996, p. 257). An important enactivist concept here is "natural drift" (Maturana & Varela, 1998, p. 115), a concept that originated from Darwin's theory of evolution. Natural drift suggests a process of co-evolution as opposed to evolution (Capra, 1996), where rather than an organism progressively adapting to their environmental world, organism and environment co-adapt to each other through their interactions. Species and environment co-evolve through an ongoing process of natural drift. Maturana and Varela's concept of natural drift only becomes possible because of the "dynamic structure of the organisms and the dynamic structure of the medium in which these exist" (Maturana & Verden-Zöller, 2008, p. 26). To understand more fully the idea of natural drift, Maturana and Varela (1998) introduce the notions of "structural determinism" and "structural coupling" (pp. 95-99). Before these two notions can be explored it is important to firstly establish what is meant by structure within this context.

Structure and organisation: An important distinction in enactivist theory is between organisation, "those relations that must exist among the components of a system for it to be a member of a specific class" (Maturana & Varela, 1998, p. 47) and structure, the "components and the relations between the components that constitute the system" (Maturana, 1987, p. 73) that make its organisation real. Organisation is invariant, common across all members of a particular class (examples of classes: humans, cats, chairs, tables, trees, etc.), whereas the structure of a unity is always unique to the individual (e.g., no two human beings have the same structure). As observers we have "distinguished the living system as a unity from its background and have characterized it as a definite organization" (Maturana & Varela, 1998, p. 95). In this sense, a living being is considered "*operationally distinct*" (p. 95, emphasis original) from its medium. Medium and living being consist of their own distinct structural dynamics, such that the two structures are operationally independent of one another. This operational independence does not equate to separation in the sense of adaption, however, since organism and environment co-evolve through the process of natural drift.

Natural drift is made possible because all living organisms are "structure-determined systems" (Maturana, 1987, p. 73). To say that all living organisms are determined by their structure means that it is the structure of the organism or system that determines how it responds given any interaction with and in its environment. What happens to a living system is determined by that system's structure and not by the interactions that system undergoes. An organism can therefore not be *instructed* in the direct sense since "nothing external to it can specify what happens to it" (Maturana, 1988b, p. 29). The interactions between organism and its environment can only *trigger* changes, the changes themselves are determined by the existing structures of the systems that are interacting. For instance, two different mathematics teachers could respond completely differently to the same stimuli, depending on their history of interactions up to that point.

Ontogeny is the "history of structural changes in a particular living being" (Maturana & Varela, 1998, p. 95). The structure of a living being is what "conditions the course of its interactions and restricts the structural changes that the interactions may trigger in it" (p. 95). As an organism and its environment interact, they experience a mutual history of

evolutionary changes. This co-evolving happens through a process of "*structural coupling*" (Maturana & Varela, 1998, p. 75, emphasis original). This mutual adaptation means that *fit* between organism and environment is not fixed but is also co-evolving. Natural drift, therefore, can be seen as a "process in which organism and environment remain in a continuous structural coupling" (Maturana & Varela, 1998, p. 115).

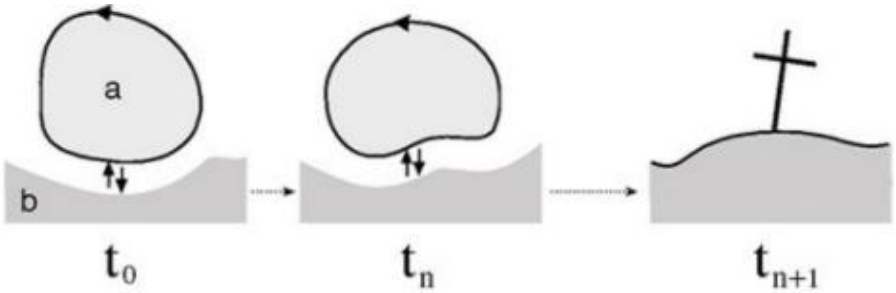


Figure 5.2: Diagram to illustrate process of natural drift (Maturana & Verden-Zöller, 2008, p. 26).

In figure 5.2, the organism (a) and the medium (b) exist in "recursive interactions beginning at time t_o and continuing through time t_n" (Maturana & Verden-Zöller, 2008, p. 26). These interactions result in a history of mutual congruent structural changes, that is, a process of structural coupling (as indicated by the changes in both shapes a and b). Over the course of time, each organism will specify its own individual pathway of structural changes, until time t_{n+1} at which point the organism dies. These specified structural changes are acts of cognition. The organism not only specifies these structural changes, it also specifies which interactions from the environment trigger them. Interactions that trigger structural changes are known as *perturbations*. By specifying which interactions from the environment trigger the changes, the system "brings forth a world" (Capra, 1996, p. 260), so that cognition is not seen as a representation of an independently existing world, but rather "a continual *bringing forth of a world* through the process of living" (Capra, 1996, p. 260). This sense of bringing forth a world is expressed by one of the key enactivist aphorisms, "everything is said by an observer" (Maturana, 1987, p. 65), or, more fully, "everything said is said by an observer to another observer that could be him or herself" (Maturana, 1988a, p. 5).

All knowing is doing: Capra (1996) points out that not all changes in an organism are acts of cognition. Some physical changes, for example, that may have been caused by an

accident or illness, "are not changes of choice and thus not acts of cognition" (p. 261). Some of these imposed changes, however, will be accompanied by other structural changes that are acts of cognition (for example, a response from the immune system). Conversely, not all interactions trigger a response in the organism's structure, in other words, not all interactions are perturbations. As human beings, for example, we can only act in the way our structure, the manner in which we are "embodied" (Varela et al., 1991, p. 172), allows us to act. That is, our "sensory and motor processes, perception and action, are fundamentally inseparable" (p. 172). This idea is best summarised by another of Maturana & Varela's key aphorisms: "*All doing is knowing, and all knowing is doing*" (Maturana & Varela, 1998, p. 26, emphasis original). Thus, we can only perceive those phenomena that having a human body with its various sensorimotor capacities allows us to perceive. For example, humans are only capable of hearing a certain range of frequencies. There would be no perturbations to trigger a response to those interactions that are outside of our perceivable range. What we can respond to in the environment is therefore limited by what is possible to perceive as embodied beings.

The idea that the manner in which we are embodied (our structure) determines the way it is possible for us to act includes what is possible for us to imagine. As a child, I used to dream that I could fly, in fact, it was a regular feature of my childhood dreams. In all of those dreams of flying, the experience was precisely as it was to swim, I would swim through the air, feeling the resistance of the air as if it were water. I assume that I could not imagine what it was like for a bird to fly. In his well-known paper, *What is it like to be a bat?*, Nagel (1974) argued that while a human might be able to imagine what it would be like for a human to be a bat (or indeed a bird), it would be impossible to imagine what it is like for a *bat* to be a bat:

> Our own experience provides the basic material for our imagination, whose range is therefore limited […] In so far as I can imagine [what it is like to be a bat] (which is not very far), it tells me only what it would be like for *me* to behave as a bat behaves. But that is not the question. I want to know what it is like for a *bat* to be a bat. Yet if I try to imagine this, I am restricted to the resources of my own mind, and those resources are inadequate to the task. (Nagel, 1974, p. 439)

What is possible for a human being to imagine is thus determined by our own sensorimotor capacities and our history of experiences. In other words, we can only imagine those phenomena that having a human body, and corresponding history of structural changes, allows us to imagine.

Furthermore, we only notice what concerns us, which is conditioned by our existing conceptual structures and our cultural context, meaning that many stimuli that could *potentially* trigger a response go unnoticed, that is, they are not perturbations. For that reason, "each living system builds up its own distinctive world according to its own distinctive structure" (Capra, 1996, p. 262), which is what is meant by bringing forth a world of significance. Individual beings are also part of one another's worlds. In a cognitive ecosystem that comprises of multiple individual cognising agents and their environment, "there is an ecology of worlds brought forth by mutually coherent acts of cognition" (Capra, 1996, p. 262).

Learning: As an organism interacts with and in its environment, it does not *react* to the external stimuli, rather, it *responds* with structural changes through a process of structural coupling. The "range of interactions a living system can have with its environment defines its 'cognitive domain'" of which "emotions are an integral part" (Capra, 1996, p. 262). Kieren (2001) suggests that it is the continuous occurrence and re-occurrence of interactions between organism and environment that allows for an increase in the cognitive domain, "or the domain of possible knowing" (p. 3). Maturana and Varela (1991) refer to a living system's "niche", which constitutes "the classes of interactions into which an organism can enter" (Maturana & Varela, 1980, p. 10). In other words, the niche of a living system is "the feature of the medium it actually *encounters* in its interactions" (Maturana, 1988a, p. 12, emphasis added); it is the space of perturbations. The niche is not fixed, it is in constant flux while the organism "slides through the medium in continuous structural change" (Maturana, 1988a, p. 9).

As I have already explained, living systems are in constant flux (i.e., they are *always* changing). Thus, as human beings, we cannot *not* change since we are dynamic systems continuously interacting in and with our environment. When we walk, our structure

changes, when we read, our structure changes, even when we breathe, our structure changes. Changes are unavoidable. Certain structural changes, triggered in response to physical actions, are necessary for the survival of the living being. The process of cognition (the process of life) is the adaptive process that we might call "adequate conduct" (Maturana, 1987, p. 74) or "effective action" (Maturana & Varela, 1998, p. 29). Since *all doing is knowing, and all knowing is doing*, it follows that "a living system is necessarily always engaged in adequate [conduct] in the domain in which it is distinguished as a living system" (Maturana, 1988a, p. 17). In other words, an organism will survive in its medium only whilst it operates adequately. In Maturana's (1987) words, *learning* takes place when an organism, which "operates adequately to its need, can undergo a continuous structural change such that it goes on acting adequately in its medium, *even though the medium is changing*" (Maturana, 1987, p. 74, emphasis added).

What is considered adequate conduct or effective action for me as a mathematics teacher educator is not really a matter of survival in its literal sense. What I am interested in are those structural changes that lead to an *observable change in behaviour* and the ways in which those structural changes are triggered. As a mathematics teacher educator who is undergoing continuous, and unavoidable, structural change, I am in search of new, observably different, adequate conduct. The problem I am interested in is the problem of showing how the structure of a living system "changes in a manner such that we see a particular adequate conduct that we did not see before," (Maturana, 1987, p. 74). It is when habitual responses are no longer adequate that the system must develop new and different responses. The need for different and varied responses can arise particularly when the environment changes quite profoundly (like, for example, when the temperature of the environment rises significantly, or when a living system moves from operating adequately in a mathematics classroom to teaching teachers of mathematics).

Davis (2004) offers this related and powerful description of learning:

> [L]earning seem[s] to be more about expanding the space of the possible and creating the conditions for the emergence of the as-yet unimagined, rather than about perpetuating entrenched habits of interpretation […] learning [is] not about convergence onto a pre-existent truth, but about

> divergence–about broadening what is knowable, doable, and beable. The emphasis is not on what *is*, but on what might be brought forth. Thus learning comes to be understood as a recursively elaborate process of opening up new spaces of possibility by exploring current spaces. (Davis, 2004, p. 184, emphasis original)

Davis (2004) seems to be talking about creating the conditions that expand the cognitive domain, that is, the range of interactions a living system can have with its environment. By expanding the range and differentiation of interactions, those interactions, which a living system actually encounters (i.e., perturbations) that trigger structural changes, will also expand. In return, new ways of acting and interacting in the future are made possible. As human beings, what is possible to notice in the environment is literally enhanced, as is the potential for new and different responses, triggered by future interactions, opening new ways of acting up in a recursive process of learning. A living organism can continue to act based on its habitual responses to environmental triggers, but only whilst these habitual responses are deemed adequate to its changing needs and to the changing needs of the environment. In a changing environment, the organism's entrenched habitual responses may no longer continue to be necessarily adequate; the organism must therefore adapt.

Consistent with Davis' view of learning, Brown (2015) offers this description:

> [L]earning is seeing more, seeing differently, in a recursive process linked to actions in the world giving feedback leading to adapted actions until the behaviours become effective. (Brown, 2015, p. 192)

Our structures determine what is possible to see. Only through changes to our structure can we ever see more or differently. These structural changes are triggered in a recursive process, linking to our actions and interactions in and with the environment. The way our structure changes is determined by our existing structure, hence we can become entrenched in habitual ways of seeing the world, that is, our habitual ways of making distinctions. To see more and differently requires us to open our habitual ways of seeing the world up to question, what I think Davis (2004) meant by "creating the conditions for the emergence of the as-yet unimagined" (p. 184). Learning, therefore, requires a change

to our ways of making distinctions, a change in the way we see the world that we bring forth.

~~~~~
~~~~~

5.3 Retelling as reseeing

I want to retell parts my story, but in light of the theory just presented. By retelling my story, by re-framing it in enactivist terms, I hope to say more than was possible to say the first time around; to understand why problems arose as a new mathematics teacher educator and to explain what happened during that conversation with Sam. I explicitly retell my story, from the perspective of an observer, which of course is unavoidable since everything is said by an observer, but to emphasise this principle I take the role of an outsider looking in and tell my story from a third-person perspective. I now see the process of retelling as a process of reseeing (or seeing more and differently). By integrating a different theoretical perspective, taking an outsider position, I will see more than I was able to see in previous telling and retellings from a new and different perspective:

> It's never enough to just tell people about some new insight. Rather, you have to get them to experience it in a way that evokes its power and possibility. Instead of pouring knowledge into people's heads, you need to help them grind a new set of eyeglasses so they can see the world in a new way. (Brown, 1991, p. 109)

5.3.1 The story retold

Tracy was a teacher of mathematics. Her classroom was a complex system, in which she was one cognising agent amongst many. She was young when she started teaching and, as a new teacher, she knew how to make very few distinctions. Her cognitive domain was limited by what her existing structure allowed, meaning what was possible for Tracy to notice in her classroom was determined by her history of structural changes up to that point. As a new teacher, Tracy did not know how to see in mathematics classrooms. When she observed others teaching, she was not able to make many distinctions, yet, over the years, Tracy developed her own unique ontogeny, her history of structural changes through an ongoing process of structural couplings with and in her classroom and the classrooms of others. Tracy found that her structure evolved in harmony with her

classroom, and she developed news ways of seeing and new ways of responding in the moment.

Each of Tracy's classes was different, so she adapted her actions in each of these different and changing environments. Each different group of children co-evolved along with Tracy through a process of natural drift until they found an effective way of working together. Tracy developed certain habitual responses that were needed in a classroom environment, allowing Tracy to be triggered by more and different types of interactions that required more than a habitual response, such as listening to the mathematical reasoning of children. Over time, Tracy's cognitive domain grew, she encountered many more interactions through a process of structural coupling. The range of distinctions that Tracy was able to make in the moment of teaching expanded.

When Tracy studied for her master's degree, she went through a period of accelerated structural changes as she committed much time to interacting with ideas, and with others, through reading, writing, and discussion. Some of the interactions with these ideas triggered responses in Tracy and her structure changed, as did the structure of the ideas in an ongoing structural dance. Other interactions she had did not trigger responses, they were outside of her cognitive domain, she did not notice them, or they made no sense or were of no relevance. All of the time she was interacting with the ideas, and with others, she was teaching in her classroom, interacting with the mathematics and the children, with the tools and the resources. The complexity of this structural dance increased as did the complexity of her own structure. All of these structural changes, these acts of cognition, led to Tracy developing a new set of distinctions and adapted actions in her classroom.

After many years of teaching mathematics, Tracy's conduct in the classroom became mostly adequate to her needs, the needs of the children and the school where she worked. Her interactions eventually became such that fewer and fewer perturbations were created. Tracy's structure stabilised. She became less open to seeing differently. She developed an almost rigid set of distinctions. She more or less knew, now, how to teach mathematics in the school she was in.

Then she moved to a profoundly different environment.

When Tracy was a new mathematics teacher educator, she went with her more experienced colleague on a school visit to co-observe a lesson taught by one of the prospective mathematics teachers from the course and to observe the lesson de-brief conversation (see glossary, page 291). At this point, Tracy's structure was such that she had been conditioned to see mathematics teaching from her own teacherly perspective. Her structure had not yet evolved through her interactions as a mathematics teacher educator. Just as she did not know how to see as a new mathematics teacher, Tracy did not know how to see as a new mathematics teacher educator. When she observed her colleague during that lesson de-brief conversation, she was not able to make many distinctions, other than those she had been conditioned to see.

It soon came to Tracy doing her own school visits. One of these visits was to a prospective teacher called Hayley. During the lesson de-brief conversation with Hayley, Tracy experienced many moments where her actions were not deemed as effective and were felt as emotional responses. Her interactions with Hayley, within that unfamiliar environment, gave her feedback that her conduct was not adequate. She had noticed her lesson notes, turning to read them, since this was something she knew to do as she had done this as a school-based mentor (see glossary, page 291); it was part of her existing structure. By dwelling in the detail of the notes, new ways of seeing were activated both in Tracy and in Hayley, yet, on reflection, Tracy came to know that there was a problem. She did not know how to be a mathematics teacher educator.

Tracy decided it would be good experience for her to work with an experienced teacher. Having already met Sam when she was still a teacher of mathematics herself, she thought of her immediately. When Tracy had met Sam in school, she expressed a view of mathematics teaching that Tracy had assumed was similar to her own, as if their cognitive structures were almost identical. When Tracy went to visit Sam, to observe her teach mathematics and to talk after the lesson, Tracy was only able to observe Sam's mathematics lesson in a way that was determined by her own structure, the structure that had evolved through her years of teaching mathematics. Though Tracy told herself that she needed to be open to Sam's ways of seeing the world, this principle had not

become integrated into Tracy's structure. Tracy was stuck with her own way of seeing the world of mathematics teaching, her entrenched habits of interpretation. She was limited by what was possible for her to see. This unfamiliar classroom environment was not one that Tracy had co-evolved with, this was Sam's classroom and she and her students had co-evolved with one another in their own process of natural drift. When Tracy and Sam spoke about the lesson, neither could really hear one another, since their structures determined what was possible for them to hear in each other's words. Everything is said by someone, but that someone can never specify what gets heard. Tracy's way of making distinctions meant that she saw in Sam's classroom behaviours that would not have been deemed adequate in her own classroom, yet Sam's actions had evolved to be effective in her own environment.

During their conversation, Tracy made several attempts at instructing Sam, but instruction is impossible. Nothing external to Sam would be able to specify what happened to her, like Sam's colleague who wanted to change Sam's behaviour, Tracy could at best trigger changes, but only Sam could specify how she would change:

> we *can* learn from being told things, but only if we are able to integrate what we hear; we *can* learn from experience, but only if triggers are set up to alert us in the future; we *can* learn from others, but only what we mark and try out for ourselves. (Mason, 2002, p. 197, emphasis original)

5.4 The problem reframed

Becoming a mathematics teacher educator becomes a process of adapting to my new and changing environment by expanding the range and differentiation of perturbations, to expand the possibility for responding differently in any given moment. Becoming a mathematics teacher educator involves a recursive process of my acting in the world of mathematics teacher education, with teachers of mathematics, giving feedback leading to adapted actions until my actions are effective in that context.

By opening myself up to new ways of seeing the world of mathematics teaching and mathematics teacher education, I have arrived at a new framing of the problem. What was originally, "How am I becoming a mathematics teacher educator?" (section 2.3, page 24) has become:

> *How can I explore past and current spaces to see more and see differently as a mathematics teacher educator?*

Having established the research problem in more nuanced and technical terms, the purpose of the following chapter (chapter six) is to provide the theoretical basis on which to develop a methodology for seeing more, seeing differently as a mathematics teacher educator in the context of working with a collaborative group of mathematics teachers (context two: section 2.2.2, page 22). Throughout chapter five, I have used the terms *habit* and *habitual* on several occasions but without any real sense of what I am referring to when I use those terms in the context of my own becoming. Thus, in the following chapter, I explore the nature of habit and habitual behaviour in the context of my becoming since my conceptualisation of these terms is central to any methodology that looks to modify these behaviours.

~~~~~
~~~~~

Chapter six

Developing expertise

In the previous chapter, I explored the enactivist theory of cognition in order to retell my story as a mathematics teacher and as a new mathematics teacher educator. This retelling led to a new articulation of my research problem, in more technical terms, as: "How can I explore past and current spaces to see more and see differently as a mathematics teacher educator?" The purpose of this chapter is to provide the theoretical basis on which to develop a methodology for seeing more, seeing differently as a mathematics teacher educator (chapters seven, eight, and nine) and to conceptualise the process of developing expertise as a mathematics teacher educator. I consider different modes of behaviour, before moving on to considering the process by which it is possible, based on these different modes of behaviour, to develop expertise.

6.1 A paradoxical situation?

I had ventured into the unknown as a new mathematics teacher educator. As an experienced teacher, I developed adequate behaviours, effective actions in the classroom, many of which, at some point along the way, became automatic, that is,

without the need for rational deliberation. Over time, an increasing number of teaching behaviours became automatic, for instance, knowing the children's names, writing and drawing on the whiteboard and operating other tools fluently (including technological ones), listening to children's mathematical contributions, following established routines around classroom organisation and so on, and I needed to be doing all of these things (and more) simultaneously and without the need for constant conscious deliberation to stop me in my tracks. My range of automatic behaviours expanded over time in the process of becoming a teacher of mathematics. I needed to form habits to teach mathematics effectively. As more and more behaviours became automatic, I was able to direct my attention to things I had not been able to direct my attention to previously, to see more than I had been able to see before, such as the detail in the children's mathematical conversations and doings. Developing automatic behaviours was essential to becoming an effective teacher of mathematics, yet not all habitual behaviours are necessarily effective.

6.2 A story of ineffective habitual behaviour?

As a newly qualified teacher, teaching a class of students, aged 15-16 years, that I found particularly challenging, I developed a habit of never addressing the class as a whole for more than a few moments. Each time I attempted to speak to the class for a longer period, several students would talk over me, triggering me to speed up. This *mutually* habitual behaviour (me addressing the whole class, several students interrupting me, me talking more quickly) continued so that, over quite a short period of time, I found that I was only saying the bare minimum to get the lesson started. I would get the students seated in groups and then proceed to visit each one in succession to set up what they were going to be working on. As you can imagine, this habitual behaviour resulted in several students regularly being unable to work productively until I had spent some time with each group to give them the support they needed. In each lesson, I would get myself ready to address the whole class for a more substantial amount of time, but, somehow, I would give up as soon as I had communicated the bare minimum and move automatically, almost compellingly, to visit the first of the succession of groups.

It is clear from this account that certain habitual behaviours need calling into question, either because those habitual behaviours are ineffective (as they were in my story), or because a wide range of behaviours that were initially effective have, over time, become completely habitualised to the point of complacency. After years of developing a range of automatic behaviours as a teacher of mathematics, I had to work hard not to let myself exist purely as a creature of habit in the classroom, especially with an increasing number of other things going on that took my energy (such as having children, dealing with grief, taking on additional responsibilities at work, starting a doctorate, and so on…). Developing automatic behaviours were essential for me in becoming a mathematics teacher and coping with the complexity of the mathematics classroom, yet equally, I found myself having to work hard to avoid operating exclusively at a habitual level. The ability to continue acting habitually in certain ways yet not allowing myself to behave purely as a creature of habit, required a certain level of effort and discipline.

In situations where habitual responses are ineffective, for whichever reason, it is essential to call these behaviours into question to bring about change. The process of calling habitual responses into question, however, is not necessarily a straightforward one, yet it seems to me to be a fundamental part of the process of learning to teach and in becoming a mathematics teacher educator. According to Mason (1998), "the trouble with habits is that they sink below the level of awareness, and then they are unavailable for inspection" (p. 372). In the same paragraph, Mason acknowledges how essential it is to automate certain responses so they can "flow automatically without agonising all the time over making choices" (p. 372), suggesting he could equally have claimed "the *great thing* with habits is that they sink below the level of awareness!", otherwise, how could anybody possibly cope in a complex environment such as a mathematics classroom (without developing a range of automated responses and behaviours). If, as Mason claims, habitual behaviours do indeed sink below the level of awareness, regardless of whether those habitual behaviours are effective or not, then either:

1. My behaviour, as a newly qualified teacher, was not in fact, *habitual* (as framed in my telling of the story), or

2. My story as a newly qualified teacher *is* one concerning habitual behaviour, but not the *type* of habitual behaviour that Mason is referring to, rather, a type of habitual behaviour that is consistent with an acute (and almost painful) awareness (and if this is the case, then the assumption is that there are multiple *types* of habitual behaviour).

Determining which of these scenarios holds true, is a necessary step towards uncovering the process by which it is possible to modify my habitual/automatic behaviours, a process I see as fundamental to my becoming a mathematics teacher educator. By unearthing any potential distinctions, between, for example, *habitual* behaviours and *automatic* behaviours (which up to this point I have been seeing as synonymous, and using interchangeably), I may be able to answer (or in fact discount as no longer valid) the question:

> *What is the process by which it is possible to modify habitual behaviours, if they are indeed below the level of awareness and unavailable for inspection?*

Resolving this seemingly paradoxical situation is a key issue for me in developing an effective discipline/systematic approach, to working on my practice and becoming a mathematics teacher educator.

~~~~~
~~~~~

Interlude three: On modes of behaviour

In this substantial interlude, I explore a range of ideas. Specifically, I consider the nature of *habitual behaviour,* before moving on to explore different *modes of behaviour*, with a focus on *the link with language*. I have marked the beginnings of these sub-sections using italics.

Habitual behaviour: Bateson (2000) confirms the necessity of habit formation, a process he describes as "a sinking of knowledge down to less conscious and more archaic levels" (p. 141). For Bateson, habit is thus associated with the *economics* of the living system. He goes on to explain that the unconscious contains many matters, some that are so familiar to us that we have no need to interrogate them. Bateson (2000) also makes it clear, however, that there are certain types of knowledge that "must be kept on the surface" (p. 141), that is, those types of knowledge that do *not* "continue to be true *regardless of changes in the environment*" (p. 142, emphasis added). In enactivist terms, only behaviours that remain effective, regardless of a continuously evolving environment, can remain buried below the level of awareness. It is the "pragmatics of particular instances" (p. 142) that must be kept accessible, within the conscious, so that behaviour can be modified for every potential instance. In the mathematics classroom there are certain behaviours that can safely sink to the realm of the unconscious (like writing on a whiteboard). These behaviours, based on my reading of Bateson, are truly habitual, they are behaviours that stay true regardless of the changing environment. Then there are other behaviours (automatic ones) that need to be constantly modified, in a mindful way (although without rational deliberation), depending on the situation at hand. For instance, in working alongside a classroom full of children, there would be a need for me to adapt my responses to their changing needs as learners of mathematics. Returning to my story as a newly qualified teacher, based on my reading of Bateson (2000), my behaviour in that classroom (of not addressing the whole class, and instead addressing multiple smaller groups) would not be considered habitual since I was aware (retrospectively at least) of what was happening, yet, in the moment, it was *experienced as automatic,* and it remained unchanged. Therefore, there are behaviours that we

experience as automatic yet would not be considered habitual. One reading of the situation in my classroom is that my behaviour was good enough for my behaviours not to evolve (in fact they had already evolved in coordination with my evolving environment), although maintaining this good-enough behaviour caused me a certain amount of discomfort. I am interested in the mechanisms that support changes in behaviours, particularly those experienced as automatic.

A different, but related model may be helpful to consider here and comes in the form of the well-known *competence model*, the origin of which is most widely attributed to Broadwell (1969) who described "four levels of teaching" (p. 2). Specifically, Broadwell referred to "unconscious incompetent" (p. 2), where a teacher is unknowingly ineffective; "conscious incompetent" (p. 2), where a teacher is ineffective but knows this is a problem (a description that fits my own experience of teaching the group of 15-16 year old students as a newly qualified teacher in section 6.2); "conscious competent" (p. 2), where a teacher is effective and knows why this is the case; and "unconscious competent" (p. 2), where a teacher is effective but does not know the reasons for this. At all four levels, Broadwell suggests a separation in terms of the way a teacher *behaves* in the classroom and what the teacher *knows* about this behaviour. For the unconscious competent, the teacher behaves in a certain way, yet is unable to rationalise those behaviours. For the conscious incompetent, the teacher behaves in certain way, but knows better than to do this, suggesting a form of rational deliberation that opposes the teacher's behaviours.

Bateson (2000), whose ideas are closely linked with enactivist theories of cognition, would reject this decoupling of knowing and doing, yet in relation to habit and behaviour, he seems to be describing *types* of behaviour in relation to levels of consciousness. In Broadwell's (1969) model, the assumption is that there are two distinct modes of being: there is behaviour (what teachers do); and there is reason (what teachers know). To develop expertise (i.e., from incompetent to competent), there needs to be a change in what teachers know (i.e., from unconscious to conscious) suggesting that it is only behaviour that we are conscious of that can be modified. My reading of Bateson (2000) would suggest that we are able to modify a range of behaviours, including automatic behaviours *without* the need for rational deliberation. Then there will be some

behaviours that, in order to be modified, will trigger a process of rational deliberation, either in-the-moment or retrospectively, and certain behaviours that are rational by nature (such as deciding on a particular course of action given a classroom incident).

Modes of behaviour: Behaviours that we have access to modify, without the need for rational deliberation, are neither *habitual* (those behaviours that sink below the level of awareness and are unavailable for inspection) nor *rational* (those behaviours that need conscious deliberation to perform). This means that there must exist a whole set of behaviours that sit between these modes, behaviours that we *experience* as automatic (without triggering conscious deliberation) yet could be made available for retrospective analysis. There are multiple labels I could potentially assign to this intermediate mode of behaviour: intuitive; integrated; embodied, all needing some more research to feel confident in my use of them. I offer table 6.1 as an initial set of distinctions based on the discussion and my thinking so far and figure 6.1 which presents the three levels of behaviour as a continuum across two dimensions (availability for inspection and dependency on adaptiveness of the environment). The distinctions in table 6.1 are slightly artificial since, in my experience, a range of behaviours can be performed simultaneously (in the case of habitual and intuitive behaviours) in an *interdependent* way, hence the continuum model presented in figure 6.1. For instance, as a mathematics teacher, I found that I could listen to children's mathematical reasoning and simultaneously capture this reasoning on a white board whilst scanning the room for reactions. As a mathematics teacher educator, when observing a prospective teacher teaching, I can hear what is being said in the lesson, whilst simultaneously making verbatim notes, and conversing with the school-based mentor. I experience all of the behaviours listed here as automatic, almost all of the time. Hence, I recognise some common *characteristics* across the habitual mode of behaviour and the intuitive/integrated/embodied mode (e.g., both are experienced as automatic). Having said a range of behaviours can be performed simultaneously, this is not the case with rational behaviours, which, according to Kahneman (2011), can only be performed in isolation from any other behaviours, since they take our full attention. Kahneman describes "two systems of the mind" (p. 20) which he denotes as System 1 and System 2.

For Kahneman, System 1 "operates automatically and quickly, with little or no effort and no sense of voluntary control" (p. 20) whereas System 2 "allocates attention to the effortful mental activities that demand it […] operations of system 2 are often associated with the subjective experience of agency, choice and concentration" (p. 21). Kahneman describes System 2 as, on occasion, "taking over" (p. 21) the smooth functioning that is associated with System 1, an experience I have described as "a breaking down of the usual flow" (table 6.1). Accordingly, it is "impossible to conduct several [System 2 behaviours] at once" (p. 23). It seems to me, that by combining Bateson's (2000) ideas within Kahneman's two systems I am extending two systems to three, by separating those System 1 behaviours into those that I am calling habitual and others that I am referring to as intuitive/integrated/embodied (all of which are experienced as automatic).

Mode of behaviour	**Associated characteristics**	**Example**	
Rational	These behaviours require rational deliberation in order to perform. These behaviours are constantly modified depending on the particularity of the situation at hand. These behaviours correspond with a breaking down of the usual flow. These behaviours are available for in-the-moment analysis.	Deciding on a particular course of action given a classroom incident	Realm of awareness
Intuitive Integrated Embodied	These are behaviours that we have access to modify, but without the need for conscious deliberation. These behaviours are constantly modified depending on the particularity of the situation at hand. These behaviours are experienced as automatic. These behaviours are available for retrospective analysis.	Listening to students' mathematical articulations and responding	
Habitual	These are behaviours that fall below the level of awareness. These behaviours remain effective in a continuously adapting environment. These behaviours are experienced as automatic. These behaviours are unavailable for analysis.	Writing on the whiteboard	Realm below awareness

Table 6.1: Modes of behaviour (initial conceptualisation).

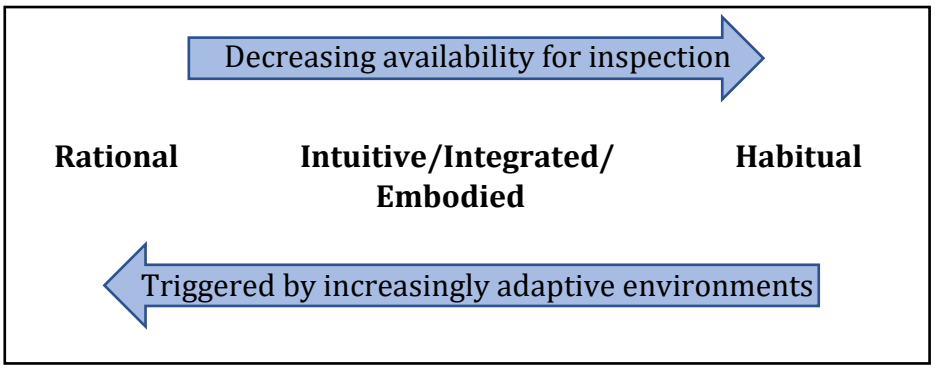


Figure 6.1: Modes of behaviour as a continuum across two dimensions.

Table 6.1 also includes common *characteristics* between the intuitive/integrated/ embodied mode of behaviour and the rational mode of behaviour (e.g., both are constantly modified in relation to the particularity of the situation at hand). There are certain characteristics however that only pertain to particular modes, for instance, habitual behaviours are the only behaviours that are unavailable for analysis. Rational behaviours, on the other hand, are the only behaviours that are available for in-the-moment analysis, in fact, by nature, rational behaviour is in-the-moment analysis and as Kahneman (2011) says, are often associated with the experience of agency, choice and concentration.

In answer to my driving question which motivated this interlude: "What is the process by which it is possible to modify habitual behaviours?" (see page 104). The focus has now shifted slightly, since I now see habitual behaviours as those that persist regardless of changes within the environment (e.g., from mathematics classroom to mathematics teacher educator situation). For me, that is those behaviours that have remained effective regardless of my move from teaching mathematics to teaching teachers of mathematics. So, I need to look beyond what is habitual to the vast array of intuitive and embodied behaviours, many of which, for me, were no longer effective having moved to the university. To inform a methodology for educating my intuitions as a mathematics teacher educator, I need to further explore the distinctions between the intuitive mode of behaviour and the rational mode, so I turn my attention to these two modes, and the relationship between them, for the remainder of this interlude.

Teaching is an *ethical* endeavour since the primary concern of teaching is for the other. Teaching is predominantly an activity involving personal interaction, it is about being human and being with others, that is why, for me, it is such a special thing to do. In his essays concerning ethical behaviour, Varela (1999) refers to our spontaneous responding as "immediate coping" (p. 5), the "immediacy of perception and action" (p. 4), that requires no rational deliberation and what Dreyfus and Dreyfus (1991) similarly refer to as "spontaneous coping" (p. 240). According to Varela, our spontaneous action stems from an "*immediate coping* with what is confronting us" (Varela, 1999, p. 5, emphasis original) and he urges us to critically examine the "immediacy of perception and action" (p. 4), to closely consider what it is that we do spontaneously when presented with any given situation. Varela describes what he distinguishes as *two* different cognitive modes (p. 18); immediate coping being the most predominant cognitive mode, at the level of what is directly perceivable, what Dreyfus and Dreyfus (1991) refer to consistently as our "*intuitions*" (p. 241, emphasis original), and rational deliberation and analysis that are often associated with *higher* cognitive levels, being the other cognitive mode, mirroring Kahneman's (2011) distinction. Mandelbaum (1955) in his discussion of moral experience, acknowledges spontaneous ethical conduct when he notes "I sense the embarrassment of a person, and turn the conversation aside; I see a child in danger and catch hold of his hand; I hear a crash and become alert to help" (p. 48) and though ultimately Mandelbaum focuses his arguments around willed and deliberate action he does concede that much of what we do in our everyday living consists of "reacting directly and spontaneously to what confronts [us]" (p. 48):

> [I]t is appropriate to speak of "reactions" and "responses," for in them no sense of initiative or feeling of responsibility is present […] [W]e can only say that we acted as we did because the situation extorted that action from us. (Mandelbaum, 1955, pp. 48-49)

Interestingly, Mason (2010) differentiates between reacting and responding. For Mason, reacting in an unthinking, automated action (as Mandelbaum (1955) has it) but maintains that "to respond is to make an intentional, conscious, considered choice of action" (Mason, 2010, p. 37). Mandelbaum's (1955) sense of a situation *extorting*

particular actions from us is reminiscent of Varela's (1999) description of immediate coping. For Varela:

> we are *always* operating in some kind of immediacy of a given situation. Our lived world is so ready-at-hand that we have no deliberateness about what it is and how we inhabit it (Varela, 1999, p. 9, emphasis original).

Mandelbaum ("we acted as we did because the situation extorted that action from us" (1955, p. 49)) and Varela ("the situation brought forth the actions from us" (1999, p. 5)) both seem to be describing a sense of being compelled into action, without conscious deliberation, the difference being, for Varela, our actions also bring forth a world, suggesting the compulsion is two-way, a coupling between us and our environment. The situation *triggers* us to act (or react) impulsively, without conscious reflection, which simultaneously triggers a change in the environment and hence the situation.

These kinds of unreflective actions (or reactions) do not only concern our ethical behaviour. Dreyfus and Wrathall (2014) suggests there are a wide variety of situations where we act without conscious deliberation that relate to us being in the world. He gives us examples that include "skillful activity" such as skiing or playing a racket sport; "habitual activity" like brushing our teeth; "casual unthinking activity" such as rolling over in bed or gesturing as we speak; and "spontaneous activity" (Dreyfus & Wrathall, 2014, p. 83) such as tapping the steering wheel of a car when a song we like comes on the radio. In relation the three modes of behaviour presented in table 6.1, Dreyfus and Wrathall's (2014) four categories of activity would all belong below the rational layer, and some ("habitual") potentially below the embodied layer as behaviours that stay true regardless of the changing environment (although, even teeth brushing may need to be modified if, for example, I have had some treatment done to my teeth). All of these human activities are only made possible from having a human body, from our embodied "being-in-the-world" (Merleau-Ponty, 1962, p. 94).

From an enactivist perspective, all actions, including these unreflective actions, are acts of cognition, and in that sense, we can refer to our different cognitive modes as a type of *knowing*. Varela (1999) refers to the spontaneous and pervasive type of knowing as

"*know-how*" as opposed to "*know-what*" (p. 6, emphasis original), a distinction acknowledged as originating from Ryle (1945). According to Dewey (1922) we "*know how* by means of our habits" (p. 177, emphasis original) yet I would extend the notion of know how to include all intuitive, non-rational, embodied ways of knowing (that include our habitual behaviours). It is only when we experience a "breakdown" (Varela, 1999, p.11), or what Dewey (1910) describes as a "*problem*" (p. 9, emphasis original), an interruption that needs to be accounted for, that we become deliberate in our actions, as Kahneman (2011) describes as "taking over" (p. 21) the smooth functioning that is associated with system 1. In these moments of breakdown, we "become like beginners seeking to feel at ease with the task at hand" (Varela, 1999, p. 18) and resort to deliberate, intentional analysis or what Dewey (1910) refers to as a process of "reflective thought" that involves "[*a*]*ctive, persistent, and careful consideration*" (p. 6, emphasis original). Varela does not deny the importance of rational deliberation and analysis, but rather he advocates the necessity to "understand the role and relevance of both cognitive modes" (Varela, 1999, p. 18).

A range of authors have explored the distinctions between the two cognitive modes distinguished by Varela (1999) and in doing so make use of a range of different words for these modes. Table 6.2 summarises some of the key authors that I have encountered in this context along with the words that they use to denote the rational and non-rational forms of knowing or related constructs. Not all authors use a consistent pair of terms for both modes and in that case, I have used the most commonly used terms or phrases. I have then listed associated words used by the range of authors in describing and characterising the two modes. It should be noted that the categorisation is my own. Each author has their different take on these different modes, or related ideas and their different emphases. It is also important for me to be clear that I do not see each idea as being synonymous to but certainly related and connected to the cognitive modes described by Varela (1999) and Dreyfus and Dreyfus (1991) in the preceding discussion:

Cognitive mode 1	Author	Cognitive mode 2
Immediate coping	Varela (1999)	Rational deliberation
Intuition	Claxton (2000)	Reason
Tacit	Polanyi (1966)	
Unformulated	Davis (1996) Taylor (1995)	Formulated
Tactfulness	van Manen (1991)	Thoughtfulness
Know-how/Knowing how	Ryle (1945) Dreyfus & Dreyfus (1986; 1991) Varela (1999) Dewey (1922)	Know-what/Knowing that
System 1	Kahneman (2011)	System 2
Concrete, embodied, physical, practical, action, emotion, unspeakable, without language, unconscious, gut-feeling, heart, instinctive, automatic, spontaneous, immediate, practical, impulsive, skill, routine, ritual, irrational, non-rational, art, flow, implicit, creative, skillful, habitual, fluency, fast, effortless, involuntary, practice, integrated, sensitive, inclination, soul, unthinking, egoless, responding, selfless, coping, low-level, sensing, behaviour, good	Abstract, mental, intellectual, knowledge, rational, articulated, language based, conscious, head, higher-level, cognitivism, deliberate, thought, reason, reasoning, attentive, science, reflective, awareness, explicit, considered, slow, effortful, intentional, theory, analysis, interrogation, deliberate, planning, mediate, logical, ego, initiative, responsibility, willed, self, goal-driven, causal, judgement, justification, choice, calculation, right	

Table 6.2: Various terms used in relation to cognitive modes and related constructs.

It is stressed by a number of the authors that their own categorisations should not be seen as dichotomies. For instance, Claxton (2000) remarks that "simplistic polarization [of reason and intuition] is neither psychologically accurate nor professionally productive" (p. 34). He goes on to suggest, that instead, we could examine in which situations "non-intellectual ways of knowing" (p. 34) may be of benefit, and to consider the relationship between the implicit and the explicit in such settings. Claxton suggests there are ways that some kinds of analytical, articulate reason can work productively, in tandem with certain types of intuition. Equally, some authors are careful not to position one cognitive mode as of more importance or significance than the other. What has been articulated by some is the lack of attention that non-rational forms of knowing are given in researching human cognition and the importance of considering both modes and the relationship between them.

The link with language: One corollary of intuitive, embodied ways of knowing, is that they can never be fully *articulated* since know-how is not commensurable with explicit knowledge. The term "tacit" knowledge is attributed to Polanyi to describe the type of knowing that cannot be explicated, what both Davis (1996) and Taylor (1995) have referred to as "unformulated":

> I shall reconsider human knowledge by starting from the fact that *we can know more than we can tell*. This fact seems obvious enough; but it is not easy to say exactly what it means. Take an example. We know a person's face, and can recognize it among a thousand, indeed among a million. Yet we usually cannot tell how we recognize a face we know. So most of this knowledge cannot be put into words. (Polanyi, 1966, p. 4, emphasis original)

Such implicit, intuitive knowing "embodies observations, distinctions, feelings, perceptual patterns and nuances that are too fine-grain to be caught accurately in a web of words" (Claxton, 2000, p. 36). We can of course try to articulate *how* we know something, yet it would be impossible to capture the full extent and texture of any situation where we are operating intuitively. Needless to say, as complex systems, it is still possible to continuously modify our intuitive behaviours in response to our changing environments, without the need for language.

Polanyi (1966) maintained that not only is there knowledge that cannot be adequately articulated by use of language, but that *all* knowledge is rooted in tacit knowledge, emphasising the *connection between* cognitive modes. Claxton (2000) claims, in a similar way, that intuition (in some varieties) "is clearly the bedrock on which all other ways of knowing are constructed" (p. 48) and that intuition "provides the 'glue' that holds together our conscious intellect and our intelligent action" (p. 36). He also warns it can be deleterious to consciously monitor what you are doing, or to constantly think about what you are doing as you are doing it and he points to the loss of fluency in becoming too aware of and reflective about our actions in the moment and the potential paralysis that comes with certain degrees of self-awareness. Claxton goes on to suggest that even post hoc rumination on intuitive action could potentially be damaging. Mason (2002) addresses some of Claxton's legitimate concerns about bringing intuition into

consciousness and in doing so presents a useful distinction between self-awareness and self-consciousness, with self-consciousness being the dysfunctional state of awkwardness that can lead to a lack of flow and even restricted vision whereas "self-awareness or 'being awake' […] is a positive state of heightened awareness and sensitivity to what is happening" (pp. 223-224).

The key enactivist aphorism "*all doing is knowing, and all knowing is doing*" (Maturana & Varela, 1998, p. 26, emphasis original) highlights the emphasis that enactivists place on non-rational forms of knowing and a view of cognition as "effective action" (Maturana & Varela, 1998, p. 29). This emphasis on non-rational forms of knowing has led to criticisms of enactivism as a theory of cognition for its inability to account for "higher-level" cognitive skills that we associate with consciousness (e.g., imagining, memory, reflecting, analysing). It is a problem known as the "scaling-up" (Gallagher, 2017, p. 187) problem. In response to the scaling-up problem, Gallagher (2017) makes use of an extended conception of actions:

> I want to argue that an enactivist account of such cognitive activities [as remembering or imagining] should focus on the fact that in the kind of activities that we are considering, these activities are just that–activities, or *doings.* When I am remembering or imagining something, I am doing something, I am engaged in some kind of action, whether for the purposes of solving a problem […] gathering some information, constructing some account […] To think in this way is to focus on the continuity that exists between different cognitive activities–perception, action, memory, imagination, and more specialized cognitive activities that we are capable of. (Gallagher, 2017, p. 191, emphasis original)

So rather than thinking of more abstract cognitive activities as at a higher-level on a hierarchy of cognitive acts, Gallagher argues that to remain consistent within the enactivist account of cognition, these abstract cognitive acts should be considered as integrated with perception and action "in an ongoing dynamical pattern […] or figure-ground relation" (2017, p. 191). Gallagher goes on to argue that reflective thinking, contra to Dreyfus and Wrathall's (2014) conception, is akin to skillful activity in the same way that skiing is skillful. Dreyfus would argue that when an expert skier is in flow, there

is no *reflective* element, and as soon as there is, the skier is no longer an expert. Gallagher (2017), however, sees reflective thinking as a form of embodied coping or effective action. Reflective thinking is not disconnected from the skier's performance, rather, it is part of it. In this argument, reflective thinking is a dimension of flow. Thus, in enactivism, our embodied actions are foregrounded, and rational deliberation arises out of an awareness of action.

Capra (1996) reminds us also that a complex system such as a human being "couples structurally not only to its environment but also to itself, and thus brings forth not only an external but also an inner world." In humans, "the bringing forth of such an inner world is intimately linked to language, thought, and consciousness" (pp. 262-263). Moreover, conscious thought could be conceived of as "bringing intuition into awareness via language" (Atkinson & Claxton, 2000, p. 6). Maturana (1988a) claims that self-awareness or conscious thought can *only* take place in language, where language is what distinguishes us, as humans, from other animals and is the basis of the phrase "[e]verything said is said by an observer to another observer that could be him or herself" (Maturana, 1988a, p. 5).

~~~~~
~~~~~

6.3 In search of a methodology for developing expertise

In becoming a mathematics teacher educator, I am interested in how I can work purposefully on developing my intuitions, my immediate and spontaneous behaviours that, as an early mathematics teacher educator, were either ineffective or simply absent. I came to the role of mathematics teacher educator packed with habitual behaviours, behaviours that meant I could smoothly manage the running of a classroom and so on, but I lacked the intuition to respond to individuals and groups of teachers in a variety of mathematics teacher education scenarios, including de-brief conversations (see glossary, page 291) with prospective teachers (like Hayley, section 1.3, page 9), and conversations with experienced teachers (like Sam, section 4.1, page 57). In PGCE subject sessions (see glossary, page 291) with the group of prospective teachers, I have struggled to know how to effectively manage discussions with the group around a range of teaching issues. I had *my way* of teaching mathematics along with all of my intuitions that had developed as a result. I would frequently get stopped in my tracks, not knowing what to say or do, resorting to rational deliberation which often coincided with an acute awareness of not knowing how to respond.

Developing intuitive behaviours is often acknowledged within literature as the process of developing expertise. Whether this involves embodying rationalised behaviours (such as intentions) or calling into question current intuitions by making them available for analysis (either retrospectively or in the moment), the process itself needs determining. I am in search of a methodology for developing expertise, a methodology that will ultimately inform my methodology for researching how I am becoming a mathematics teacher educator (expressed in chapter seven). To inform this methodology, I will first explore some of the literature concerning expertise in relation to the modes of behaviour already discussed.

~~~~~
~~~~~

Interlude four: On becoming expert

Varela (1999) draws heavily on Hubert Dreyfus' work on the phenomenology of skills and their ethical importance and in doing so there is a common (or at least very similar) language that both Varela and Dreyfus use to describe and explain the phenomenon of expertise. Dreyfus and Dreyfus (1991) associate expertise with the spontaneous responding to complex situations without the need for rational deliberation, that is, they suggest that experts encounter minimal breakdowns. It is not, according to Dreyfus and Dreyfus, that expert deliberation is *inferior* to expert intuition, but neither is expert deliberation "a self-sufficient mental activity that can dispense with intuition" (p. 240). An expert may deliberate about the appropriateness of their intuitions when faced with a *familiar* yet problematic situation. It is when an expert encounters a *novel* situation in the absence of intuition, that a breakdown is experienced and they resort to *detached*, principle-based deliberation, what Varela (1999) refers to as "*know-what*" (p. 6) or "knowledge *of* and *about* things" (Dewey, 1922, p. 177), in the same way that somebody who describes themselves as consciously incompetent (Broadwell, 1969) might do.

Claxton (2000), writing specifically about expertise as a *variety* of intuition, refers to "the smooth, unreflective mastery of complex but familiar domains – such as a classroom" (Claxton, 2000, p. 35) and claims that "such [expert] performance is described as 'intuitive' [...] when it is unpremeditated and unselfconscious" (p. 35). The expert mathematics teacher, in dealing with a complex yet familiar classroom environment, for example, may get to the end of a lesson having not once been consciously deliberate in their actions, yet they have made an inordinate number of decisions along the way, constantly responding yet unaware (at the level of consciousness) of the interactions that have triggered these responses, and thus unable to articulate them after the event. The danger with becoming consciously aware, in the heat of the moment, is "a loss of fluency and even, in extremes of self-consciousness, in paralysis" (Claxton, 2000, p. 35).

When a situation is familiar, automatic responses are triggered, responses that are adequate for that situation. When this happens the structures of both systems (organism and environment) continue to change together through a process of structural coupling,

but in such a way that the cognitive domain of the organism in its environment does not expand and the behaviours of the organism, from the perspective of an observer, do not change. The behaviour of the organism is adequate, it is good enough for the situation at hand, and there is no need for adaptation. It is when a behaviour is not adequate that adaptation must take place. As a practitioner (such as a teacher, or a teacher educator) this adaptation can be described as an active process, a deliberate one in the sense that we must recognise when our behaviour is not effective so that we can adapt by choosing to respond differently.

For enactivists, responding differently is linked to making distinctions. The finer the distinctions that our structure allows us to make, the wider the range of potential responses we might have in any given moment. Therefore, becoming expert involves a process of expanding our cognitive domains, that is, the range and differentiation of perturbations that we are able to encounter, each triggering an expanding set of possible responses. Over time, new responses become automated so that further distinctions can be made in an ongoing process of becoming. Experts are triggered by those perturbations that would otherwise go unnoticed by a novice or even a proficient performer. Connected to these ideas, Dreyfus and Dreyfus (1991) suggest that experts develop their know how and intuition by becoming sensitive to differences, by developing a more refined set of discriminations based on a wide range of familiar situations. Furthermore, Mason (2002) tells us that we develop expertise through "increasing our sensitivity to notice, through having a growing range of different actions to call upon, and through having those possibilities come to mind in the moment" (p. 197).

Brown and Reid (2006) adopt Damasio's (2000; 2006) somatic marker hypothesis to account for mathematics teachers' ability to make complex decisions in the moment of teaching. Somatic markers are most likely stored in the pre-frontal cortex of the brain but are felt as bodily sensations or changes that guide decision making, beyond that which purely conscious mental activity would be able to do. Over time, markers develop so that decisions are made based on past experiences which guide in the moment behaviour and decision making. According to Brown and Reid (2006), somatic markers inform decision-making at two stages, the first stage being "before we are even aware

that there is a decision to be made" (p. 180) since our negative somatic markers mean that many possibilities get rejected before becoming an option. Somatic markers are cognitive structures that enable us to "act spontaneously without pausing to consciously decide how to act" (p. 180). Brown and Coles (2000) explain that negative somatic markers mean that certain behaviours "do not even come to mind as possibilities for action" (p. 168) yet a positive somatic marker means that a potential behaviour "becomes one of a number available for use" (p. 168). The emergence of new somatic markers, based on a history of structural couplings with the environment, could be what allows for new and different behaviours to become possible and for decisions to be made. Perhaps intuition is therefore the culmination of somatic markers that lead to us making decisions without any conscious awareness that a decision needs to be made.

When there is a need for a conscious decision to be made, somatic markers continue to inform this type of decision-making: "It is in this stage that we might be aware of our own somatic markers, when we notice ourselves deciding something because "It feels right"" (Brown & Reid, 2006, p. 180). Our somatic markers are acquired over time, but can adapt and change with our experiences, based on our existing structures and the feedback received from our interactions with the environment. Since somatic markers inform our behaviour at both cognitive modes or stages (intuitive and rational) they could account for the way that an expert practitioner can "respond spontaneously to *complex* situations without deliberation" (Dreyfus & Dreyfus, 1991, p. 238, emphasis original) through developing a refined set of distinctions or markers based on a wide range of familiar situations. Equally, the somatic markers hypothesis could account for the way an expert practitioner may deliberate about the appropriateness of their intuitions when faced with a familiar yet problematic situation.

In Varela's (1999) essay on ethical expertise, he details three inter-related concepts that are described as central to developing ethical expertise: *extension*; *attention*; and *intelligent awareness* (p. 27). Extension is the process by which knowledge and feelings that arise in a familiar situation, in which a particular action is considered correct, are extended in an appropriate way to other, analogous, but more complex situations, where the correct course of action is less clear. "To extend feelings is both to see that one

situation resembles another and to have these feelings "break through" into the new situation" (p. 28). These "feelings" could be translated as Damasio's (2000; 2006) somatic markers. Thus, extension can be seen as the process by which somatic markers are created and also the way in which these markers are made more likely to be activated in future appropriate situations. The process of extension, however, is neither passive nor presumed and Varela counsels the need for "some form of sustained, disciplined practice" (p. 75) as a way of fostering a responsive and compassionate disposition.

Varela (1999) contends that "the specific capacity of the mind that underlies this process [of extension] is the ability to attend" (p. 28). For Varela, attending requires that we perceive clearly in order to identify "correspondences or affinities" (p. 28) through gaining a description of the situation such that all relevant aspects are included not only those that can be reduced to some form of categorical analysis. In a similar vein, van Manen (1991) counsels that our tacit, intuitive nature as teachers, our "pedagogical perceptiveness", is learned "in subtle ways by attuning ourselves to the concrete particulars of situations" (p. 208), or, as Bateson (2000) has it, by keeping accessible the "pragmatics of particular instances" (p. 142) so that behaviour can be modified for every potential instance.

The process of extension "presumes that people can and will *attend* to what needs to be done using *intelligent awareness*" (Varela, 1999, p. 28, emphasis added). Intelligent awareness takes a "middle way" (p. 31) between the two extremes of spontaneous action and rational calculation:

> [i]ntelligence should guide our actions, but in harmony with the texture of the situation at hand, not in accordance with a set of rules or procedures. […] The truly ethical person can, like any other kind of expert, after acting spontaneously, reconstruct the intelligent awareness that justifies the action. And, like any other kind of expert, the truly ethical person can use such a postiori justification as a stepping-stone for continued learning. Indeed, even the beginner can use this sort of deliberate analysis to acquire sufficient intelligent awareness to bypass deliberateness altogether and become an expert. (Varela, 1999, pp. 31-32)

Brown and Coles (2011) build on Varela's notions of deliberate analysis and intelligent awareness in their framing of teacher development. In direct reference to deliberate analysis, they state:

> This post hoc deliberation provokes 'intelligent awareness' as it allows experts to unpick, if necessary, the reasons an action was taken, and hence open themselves up to alternative possibilities in the future. (Brown & Coles, 2011, p. 862).

Intelligent awareness as a middle path between cognitive modes, is reminiscent of van Manen's (1991) pedagogical perceptiveness or the development of what Mason (2002) calls an "inner witness" (p. 184, a notion revisited in section 8.1.8, page 169) that is intimately linked with the process of becoming self-aware. The process of deliberate analysis is akin to the kind of deliberation that Dreyfus and Dreyfus (1991) refer to as the "buttressing of intuitive understanding". This buttressing is what happens when an expert deliberates about the *appropriateness* of their intuitions and is an alternative to "detached, principle based, deliberation" that is "often incorrectly seen as the only alternative to intuition" (p. 241). van Manen (1991) suggests that it is "pedagogical tact" (p. 149) that integrates, in more intimate ways, these two cognitive modes. The process of deliberate analysis is comparable to van Manen's (1991) "reflective thoughtfulness" (p. 206), since it is reflective thoughtfulness as opposed to "artificial thoughtfulness that is created by the mechanical application of an external technique or skill" (p. 207), that leads us to becoming tactful:

> To be tactful is to be physically mindful of the person toward whom one is oriented; to be tactful is to incarnate one's reflective thoughtfulness in concrete situations. If we were to epistemologize the relation between reflective thoughtfulness and tactfulness, we might say that tact is the embodiment, the body work of thoughtfulness. (van Manen, 1991, p. 206)

I conclude this chapter by offering a model (figure 6.2) of the relationships between the various modes of behaviour and states of being, as I have come to see them. The inner most core of the model represents habitual behaviours that, as detailed in table 6.1 (page 108), are those behaviours that sink below the level of consciousness and are unavailable

for inspection. As you move outwards, from the core, into the layer of intuitions, these behaviours, though experienced as spontaneous and automatic, can be made available for retrospective inspection through the process of deliberate analysis. It is through this process, that a state of alertness can be realised, this is the realm of intelligent awareness, and is a middle way between pure spontaneity and rational deliberation. In this state we are able to attend to what needs to be done, to perceive more, to gain a clearer description of the situation. Figure 6.2 is not supposed to be read as indicating subsets, rather, it should be seen more as a layering, with habitual behaviours at the core, way below the surface of consciousness. One seeing is to imagine the image three dimensionally, with a core, and a surface.

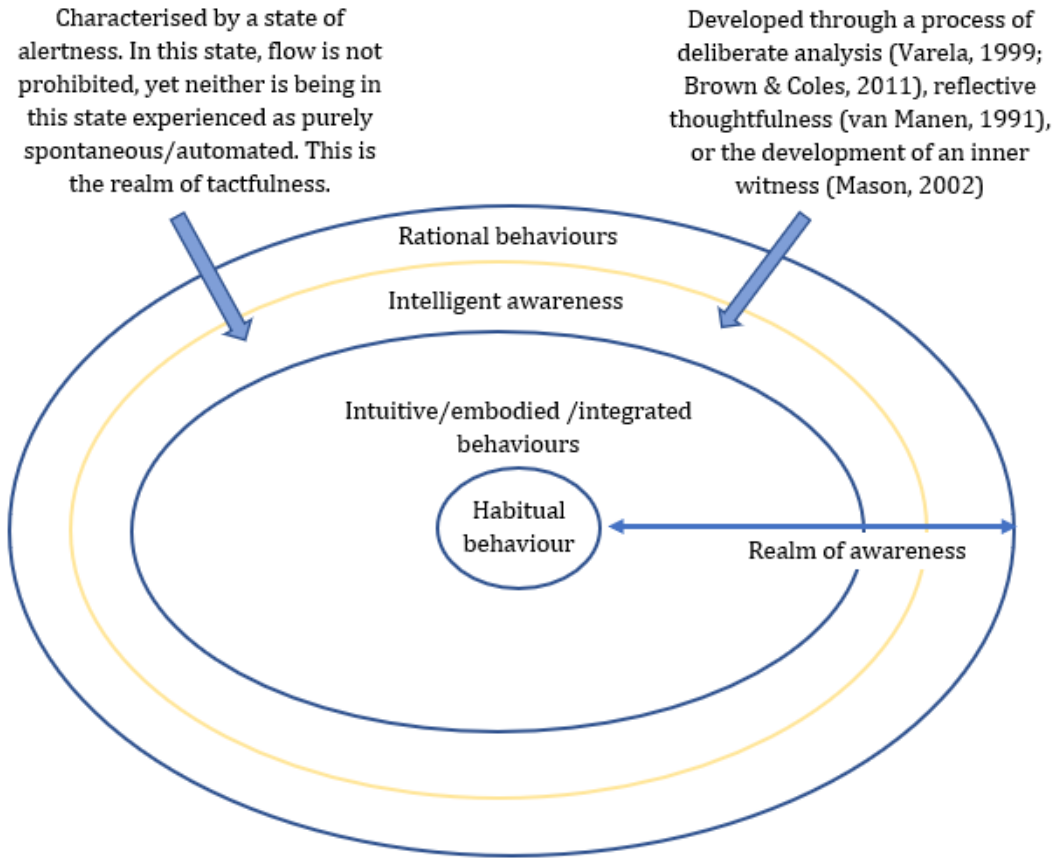


Figure 6.2: Model of related behaviours.

For Varela (1999), intelligent awareness is developed through the process of deliberate analysis, which involves "disciplines that facilitate the letting-go of ego-centered habits and enable compassion to become spontaneous and self-sustaining" (p. 73).

~~~~~
~~~~~

6.4 Towards a narrative-enactivist research methodology

In this chapter, I have explored a range of ideas in relation to modes of behaviour and developing expertise. Through exploring these ideas, I have encountered and discussed several methodological principles that build on those principles already uncovered in the preceding chapters. In the following chapter (chapter seven), I bring those principles together, from the tradition of narrative inquiry and from enactivism as a theory of cognition in order to express a narrative-enactivist methodology for *researching* how I am becoming a mathematics teacher educator, which will ultimately inform an approach to analysing my data (from context two), the product of which is the development of an overarching narrative-enactivist methodology for becoming a mathematics teacher educator (expressed in chapters eight and nine).

~~~~~
~~~~~

Chapter seven

A narrative-enactivist methodology for *researching* my becoming

Chapter seven brings together several of the ideas developed through the previous six chapters in formulating a narrative-enactivist methodology for *researching* how I am becoming a mathematics teacher educator. For that reason, I offer here a review of the thesis so far. The motivation for this study (chapter one) was a personal and professional one, which arose out of my need to learn a new set of practices and a new way of being with mathematics teachers as a new mathematics teacher educator. Over the course of the first few chapters (one, two and three), I considered a range of methodological issues that are pertinent to a study aiming to research how I am becoming a mathematics teacher educator. In considering these methodological issues, I began uncovering a number of methodological principles in relation to a self-based narrative inquiry that then informed my approach to analysing one conversation between myself and Sam, an experienced mathematics teacher (chapter four). Through analysing that conversation, a central issue was revealed (which I expressed as "being limited by my own internal set of criteria") that provoked the need for an explanatory theory (the enactivist theory of

cognition). Through exploring enactivist ideas, I was able to uncover a number of additional methodological principles in relation to how I am becoming a mathematics teacher educator and a re-framing of the research problem (which I expressed as "how can I explore past and current spaces in order to see more and see differently as a mathematics teacher educator?") in more technical terms (chapter five). This new framing prompted me to conceptualise the process of developing expertise as a mathematics teacher educator (chapter six), which included further methodological principles to complement those principles already uncovered in the preceding chapters. Having uncovered a range of methodological principles across a number of chapters (specifically, one, two, three, five and six), in chapter seven I bring those principles together that relate specifically to *researching* how I am becoming a mathematics teacher educator. Chapter seven is thus a narrative-enactivist methodology for *researching* how I am becoming a mathematics teacher educator, which I express as a set of eight key methodological principles. This research methodology informs my systematic approach to analysing a set of audio-recorded conversations (specifically, the audio-recorded conversations from the collaborative group of in-service mathematics teachers, referred to as context two), an approach that I present later in this chapter (section 7.4, page 133).

7.1 Encapsulating the key methodological principles

For the eight key methodological principles (see table 7.1), I present a group of extracts of the originating text from which each principle was derived (along with the original location of those extracts in the right-hand column). I have assigned each of the key methodological principles a label that, for me, represents the overarching theme that connects the extracts of originating text, expressed as a key guiding principle. I have included enough of the originating text for each extract to make sense, but I have used a greyscale to emphasise the most significant parts of each extract that informed my categorisation of each key methodological principle.

Key methodological principle (and associated extracts of text)	**Location**
[KMP1] Engaging in all four stages of narrative inquiry	
Bateson (1979) refers to "*context*" and "*pattern through time*" (p. 23, emphasis original) as that which connects the component parts within a story. According to Bateson, "without context, words and actions have no meaning at all" (p. 24), so, for any story to be meaningful, there is always a context, since it is "the *context* that fixes the meaning" (p. 25, emphasis original).	Section 2.4, page 26
In terms of narrative as a process of inquiry, Clandinin and Connelly (2000) tell us that: "in the construction of narratives of experience, there is a reflexive relationship between living a life story, telling a life story, retelling a life story and reliving a life story" (p. 71). These four stages of narrative inquiry are neatly summarised by Clandinin (2013) as "*living, telling, retelling, and reliving"* (p. 34, emphasis original). I offer the following as my interpretation of these stages that I view as always being in relation to one another, where narrative inquiry is the ongoing process of moving back and forth between and across the four stages: we *live* out our stories, we *tell* the stories we have lived, we inquire into the stories we have told (through *retelling*), (and in doing so) we *relive* our stories.	Section 2.5, page 28
Researching through telling and retelling stories is a generative process, one consequence of which is that "possibilities emerge for reliving in more thoughtful and responsive ways in the future" (Clandinin, Huber, Steeves, & Li, 2011, p. 34). The process of retelling requires that we go beyond regarding stories as fixed objects or entities. It is "the retelling of stories, that allow for growth and change" (Clandinin & Connelly, 2000, p. 71), akin to a process of becoming.	Section 2.5, page 29
[KMP2] Focusing on process (as opposed to outcome)	
I realised quite early on in my research that I was more interested in the process of learning (how) rather than the outcome of learning (what). This interest in researching process was motivated by my sense of *purpose* as a mathematics teacher educator: supporting the process of learning in others (i.e., prospective and in-service mathematics teachers) rather than ensuring those others would reach any particular end state.	Section 2.3, page 23
I needed to develop a methodology that allowed me to demonstrate an ongoing, ever-changing, process rather than a fixed state of being.	Section 2.3, page 24
[KMP3] Attending to the other	
I can imagine, for example, how keeping a diary could become problematic if it were to become a completely isolated activity, that is, if you only ever considered your practice from your own perspective. After all, as Mason (2002) counsels, "studying oneself can become solipsistic and even narcissistic, if gaze is always inward" (p. 174).	Section 1.2, page 7
I resolved to examine *how* I was learning […] by researching the process of my learning in relation to the learning of others (i.e., prospective and in-service mathematics teachers).	Section 2.3, page 24
For Lather, this breach is about diffusing "the power of the author, and thus improv[ing] the 'democracy' of the findings" (Lather, 1991, p. 92). This shift from thinking about the researcher as the sole owner of the research outcomes, to the reader as "actively construct[ing] possible counter-interpretations" (de Freitas, 2007, p. 336), has been a particularly powerful idea, helping me to shape my own approach to research. This active construction by the reader is what de Freitas calls "reading	Section 3.5, page 49

Key methodological principle (and associated extracts of text)	**Location**
otherwise" (p. 336, emphasis original) and she argues that it is through an approach to narrative research, where literary, fictional, or poetic strategies are employed by the researcher, that readers are most likely to engage in the act of reading otherwise.	
"As we identify ourselves with the protagonist of a story, *we* live his or her feelings and actions without having to act ourselves" (van Manen, 1997, p. 70, emphasis original). I want to give my readers the opportunity of gaining insight into certain aspects of my professional life; to explore the meanings that emerge for them as they read and have the freedom and permission to do so. I want the reader to find what resonates for them in my story and to consider what that might mean for their own practice.	Section 3.5, page 49
An important feature of my study is the attention that I pay to the other […] I consider my becoming a mathematics teacher educator always in relation to those who I am working with […] what materialises […] is a methodology for becoming a mathematics teacher educator […] on three distinct (but interrelated) fronts […]: 1) In relation to the process of learning to teach mathematics and mathematics teachers. 2) In relation to *researching* how I am becoming a mathematics teacher educator. 3) In relation to a way of working with mathematics teachers.	Section 3.7, page 55
Much of the theoretical premise of this study is around establishing a discipline that supports the development of self-awareness; a central feature of my becoming a mathematics teacher educator is the development of self-awareness that allows me to attend to others (i.e., prospective, and in-service teachers of mathematics).	Section 3.7, page 54
[KMP4] Utilising multiple perspectives in multiple ways	
I have utilised my own multiple perspectives (through the processes of telling and retelling stories and through performing multiple analyses of the recorded conversations with teachers of mathematics) [...] as well as actively seeking the perspectives of others.	Section 3.3, pages 42-43
By reading my diary entries and by listening to the recorded conversations, I am given access to my past perspectives, not solely from the meanings that are communicated in the words themselves, the accounts given in the diary entries, for instance, but through the active process of memory and imagining that are evoked through the process of reading and listening. I also have access to my past perspectives through my actions in the moment of my conversations with others. These observed changes in perspective feature explicitly in the stories that are told, as an expression of my becoming a mathematics teacher educator. When a range of perspectives (and associated tenses) are utilised, I have endeavoured to be explicit about this in advance.	Section 3.3, page 44
I use paradigmatic-type narrative analysis and narrative-type narrative analysis (Polkinghorne, 1995) in the way that I work with and on audio-recorded conversations with teachers of mathematics (from context one and context two). Accordingly, I draw on "creative analytic practices" (Bochner & Ellis, 2016; Ellis, 2004; Richardson, 1999; Richardson & St. Pierre, 2018) in working with my recorded conversations (from context one and context two). In so doing, I combine the process of telling stories with analytical techniques more associated with traditional data analysis. […]. One lens through which to view the world is with creative arts and another lens is scientific/formal analytical, yet these need not be contradictory ways of seeing as some might have it. In fact, we see more clearly with the two lenses at play, "we see best with both lenses focused and magnified" (Richardson, 1999, p. 666). Creative analytic practices are not alternative or experimental, rather, creative analytic practices are creative *and* analytical.	Section 3.6, page 50

Key methodological principle (and associated extracts of text)	**Location**
[KMP5] Producing a credible account	
According to Richardson and St. Pierre (2018), a *layered* text is a "strategy for putting yourself into your text and putting your text into the literatures and traditions of social science" (p. 834). They suggest that one means of producing a layered text is to write a narrative about an event that has been especially meaningful to you before stepping back to look at the narrative from your "disciplinary perspective" (p. 834), which involves inserting, into the narrative, relevant analytical statements.	Section 3.7, page 53
[i]n my explicit retellings of stories previously told, I do this retelling from a new and different theoretical perspective than any I had available to draw on in the original telling. The building of theoretical ideas throughout this thesis is a key feature of producing this layered text.	Section 3.7, page 53
[KMP6] Facilitating the letting go of ego-centered habits	
According to Etherington, reflexive methodologies are those that remain "close to the hearts and minds of practitioners who value using themselves in all areas of their practices (including research) and who also value transparency in relationships" (Etherington, 2004, p. 16). A key feature of my research has been about uncovering those hidden assumptions and biases that limit what it is possible for me to see in the world of mathematics teaching, a process that is necessarily self-exposing.	Section 3.7, page 54
To see more and differently requires us to open our habitual ways of seeing the world up to question, what I think Davis (2004) meant by "creating the conditions for the emergence of the as-yet unimagined" (p. 184).	Interlude two, page 94
For Varela (1999), intelligent awareness is developed through the process of deliberate analysis, which involves "disciplines that facilitate the letting-go of ego-centered habits and enable compassion to become spontaneous and self-sustaining" (p. 73).	Interlude four, page 123
[KMP7] Enabling new ways of making distinctions	
As human beings, we perceive an object as separate from its background, or in enactivist terms, we *observe* a unity as separate from its medium, by "making an *act of distinction*" (Maturana & Varela, 1998, p. 40, emphasis original). According to Maturana (1988a) the "basic operation [that we perform] in the praxis of living is the operation of distinction" (p. 5). Each and every time we refer to something (implicitly or explicitly) "we are specifying a *criterion of distinction*, which indicates what we are talking about and specifies its properties as being, unity, or object" (Maturana & Varela, 1998, p. 40, emphasis original).	Interlude two, pages 87-88
Furthermore, we only notice what concerns us, which is conditioned by our existing conceptual structures and our cultural context, meaning that many stimuli that could potentially trigger a response go unnoticed, that is, they are not perturbations.	Interlude two, page 92
By expanding the range and differentiation of interactions, those interactions, which a living system actually encounters (i.e., perturbations) that trigger structural changes, will also expand. In return, new ways of acting and interacting in the future are made possible. As human beings, what is possible to notice in the environment is literally enhanced, as is the potential for new and different responses, triggered by future interactions, opening new ways of acting up in a recursive process of learning.	Interlude two, page 94
Learning, therefore, requires a change to our ways of making distinctions, a change in the way we see the world that we bring forth.	Interlude two, pages 94-95

Key methodological principle (and associated extracts of text)	**Location**
For enactivists, responding differently is linked to making distinctions. The finer the distinctions that our structure allows us to make, the wider the range of potential responses we might have in any given moment. Therefore, becoming expert involves a process of expanding our cognitive domains, that is, the range and differentiation of perturbations that we are able to encounter, each triggering an expanding set of possible responses.	Interlude four, page 119
Dreyfus and Dreyfus (1991) suggest that experts develop their know how and intuition by becoming sensitive to differences, by developing a more refined set of discriminations based on a wide range of familiar situations.	Interlude four, page 119
[KMP8] Prioritising the development of intuitive understanding	
Extension is the process by which knowledge and feelings that arise in a familiar situation, in which a particular action is considered correct, are extended in an appropriate way to other, analogous, but more complex situations, where the correct course of action is less clear. "To extend feelings is both to see that one situation resembles another and to have these feelings "break through" into the new situation" (Varela, 1999, p. 28) […] The process of extension, however, is neither passive nor presumed and Varela counsels the need for "some form of sustained, disciplined practice" (Varela, 1999, p. 75) as a way of fostering a responsive and compassionate disposition.	Interlude four, pages 120-121
For Varela, attending requires that we perceive clearly in order to identify "correspondences or affinities" (Varela, 1999, p. 28) through gaining a description of the situation such that all relevant aspects are included, not only those that can be reduced to some form of categorical analysis […]. van Manen (1991) counsels that our tacit, intuitive nature as teachers, our "pedagogical perceptiveness", is learned "in subtle ways by attuning ourselves to the concrete particulars of situations" (p. 208), or, as Bateson (2000) has it, by keeping accessible the "pragmatics of particular instances" (p. 142) so that behaviour can be modified for every potential instance.	Interlude four, page 121
The process of extension "presumes that people can and will *attend* to what needs to be done using *intelligent awareness*" (Varela, 1999, p. 28, emphasis added) […]. "Intelligence should guide our actions, but in harmony with the texture of the situation at hand, not in accordance with a set of rules or procedures […] the truly ethical person can, like any other kind of expert, after acting spontaneously, reconstruct the intelligent awareness that justifies the action. And, like any other kind of expert, the truly ethical person can use such a postiori justification as a stepping-stone for continued learning" (Varela, 1999, pp. 31-32).	Interlude four, page 121
The process of deliberate analysis is akin to the kind of deliberation that Dreyfus & Dreyfus (1991) refer to as the "buttressing of intuitive understanding". This buttressing is what happens when an expert deliberates about the *appropriateness* of their intuitions and is an alternative to "detached, principle based, deliberation" that is "often incorrectly seen as the only alternative to intuition" (p. 241).	Interlude four, page 122

Table 7.1: Eight key methodological principles for researching how I am becoming a mathematics teacher educator.

For ease of reference, I summarise the eight key methodological principles here:

Engaging in all four stages of narrative inquiry	[KMP1]
Focusing on process (as opposed to outcome)	[KMP2]
Attending to the other	[KMP3]
Utilising multiple perspectives in multiple ways	[KMP4]
Producing a credible account	[KMP5]
Facilitating the letting go of ego-centered habits	[KMP6]
Enabling news ways of making distinctions	[KMP7]
Prioritising the development of intuitive understanding	[KMP8]

In section 7.2, I explain how my narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator, which has been expressed through the eight key methodological principles, has informed my systematic approach to analysing a set of recorded conversations with a collaborative group of in-service mathematics teachers (from context two).

7.2 Enacting the key methodological principles

The narrative-enactivist methodology described in this chapter arose out of a need to develop an approach to researching how I am becoming a mathematics teacher educator. Specifically, I needed a systematic approach to analysing a set of audio-recorded conversations in a way that was in keeping with a self-based narrative inquiry and that was informed by the enactivist view of cognition.

The audio-recorded conversations, on which I have based my systematic approach, took place between myself and a collaborative group of in-service mathematics teachers (that I have been referring to as context two, see section 2.2.2, page 22), who were engaged in a project designed to support mathematics teachers in developing the mathematical reasoning of the students in their own classrooms and the classrooms of mathematics teachers from their wider departments. My role in the group was to support a feedback session where the mathematics teachers would talk about what they had been doing in their schools in relation to the work of the reasoning project. There were eight group

feedback sessions in total, from across the two years of the project, seven of which were audio-recorded. These audio-recordings comprise much of the raw data that forms the basis of my systematic approach to analysis [KMP3].

In the remainder of this chapter, I will describe the *process* of enacting the eight methodological principles (i.e., my narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator), which includes a description of the *form* that my analysis has taken (in chapters eight and nine). To make explicit links to the key methodological principles, I have inserted a reference to the specific methodological principle(s) that has informed particular aspects of my systematic approach. For example, when I describe an aspect of the process of data analysis, where I refer to my process of making distinctions, I have inserted the tag [KMP7] within the text. I begin by detailing some of the outlines (which I refer to as boundaries) in relation to my process of data analysis.

7.3 Imposing boundaries

It has not been straightforward for me to identify the boundaries in relation to my process of data analysis. As is often the case, in a self-based narrative inquiry, the boundaries are necessarily blurry between the different stages of living the data (through its creation), telling and retelling about the data (through its analysis), and reliving the data (through new and different ways of seeing) [KMP1]. Rather than moving through these stages sequentially, I found each stage was part of a recursive process in that my initial analysis of early recorded conversations fed back into the subsequent creation of new data, which was then analysed and so on.

As a consequence of needing to deal with certain blurred boundaries, I have deliberately chosen to impose some more distinct ones in order to articulate my systematic approach to data analysis. One such distinct boundary is the particular point in time that marked the beginning of the process I am about to describe (although this boundary is imposed for pragmatic reasons, rather than an actual boundary since I acknowledge that my history of experiences up to that point in time will have determined the way in which the

process unfolded). Thus, my description begins from a point in time, the moment I began *transcribing* the first collaborative group feedback session.

I also needed to impose boundaries on my data set. My analysis (chapters eight and nine) draws directly from four out of the seven audio-recorded feedback sessions (specifically, the first, second, third and seventh). I focused my analysis most intensively and exhaustively on the first two audio-recorded feedback sessions since these gave me most direct access to my actions (and non-actions) as a *beginning* mathematics teacher educator. My earliest conversations with the group of mathematics teachers provided me with a firm grounding from which to express my becoming, over the course of the subsequent years (see the two-phase approach detailed in section 7.4, page 134) [KMP2]. I wanted any categorical analysis to have been generated from the earliest point in time, when I had fewest effective behaviours, since this allowed me to demonstrate change over time. I used the third and seventh recorded feedback sessions in different and less intensive ways than the first two feedback sessions. I describe these different but complementary techniques later in this chapter (see section 7.5, page 141).

As Richardson (1999) reminds us, "the process and the product [of analysis] are deeply intertwined; both are privileged" (p. 661). Thus, in this chapter, I present both the *process* of data analysis and the *product* or *form* that the analysis has taken. I present the process (section 7.4, page 133) and the form (section 7.5, page 141) in separate sections, again imposing a boundary for pragmatic reasons, yet this separation is done with a recognition that "[t]he product cannot be separated from the producer or the mode of production or the method of knowing" (Richardson, 1999, p. 661). I begin with describing my process of analysis.

7.4 My systematic approach to analysing recorded conversations

Given the nature of my study, as one that is focussed on my process of becoming a mathematics teacher educator [KMP2], my data analysis was enacted during two distinct phases, that I refer to as phase one and phase two, briefly described here before I elaborate on both phases more thoroughly:

Phase one (from February 2017 to September 2017):

Transcribing the first two audio-recorded conversations whilst making reflective notes alongside the transcript (the full transcript for feedback sessions one and two along with "phase one comments" can be found in appendices 3a & 3b, pages 300 & 329, respectively).

Phase two (from August 2020 to March 2021):

An initial re-listening to the audio-recorded conversations, whilst making reflective notes alongside the transcript and existing phase one comments. Then, recursively analysing both audio-recorded conversations and existing transcripts, combining categorical analysis with the process of telling stories. In this phase, I also used the third and seventh recorded conversations (the full transcript for feedback sessions one and two along with "phase two comments" can be found in appendices 3a & 3b, pages 300 & 329, respectively. Partial transcripts for feedback sessions three and seven can be found in appendices 3f & 3g, pages 406 & 410, respectively).

This two-phase process allowed me to show my own multiple and changing perspectives on the same data, a form of telling and retelling the story of those group conversations in relation to me becoming a mathematics teacher educator [KMP1].

7.4.1 Phase one of data analysis (transcribing and initial comments)

In phase one, I focussed my analysis purely on the first two audio-recorded feedback sessions. As a new mathematics teacher educator at that time, I did not know what types of observable behaviours there were to observe in the group conversations. I was unable to make many meaningful distinctions at that point, so I chose to use bold type to highlight any parts of the transcript where the words being spoken stood out to me during the initial process of transcribing (see section 8.1.2 for more details). I also made reflective notes alongside sections of transcript whenever something occurred to me, as demonstrated in the example in table 7.2 (for transcription conventions see page xii). As is the case for the full transcript, table 7.2 contains a series of columns consisting (from

left to right): the timestamp from the audio-recording; the name of the speaker (pseudonym); the transcribed text; and the phase one comments.

Time	Name	Transcript	Phase one comments
41:20	Simon	But then again though, this is what we were saying, we did it for year eight didn't we, and we split off into groups. I think we found trying to plan **a question that links other stuff into that topic for low ability was something we found that particularly difficult**. I don't know whether that was just because we weren't really thinking, or it was just the nature of ratio or whatever, but we've just found that that was harder. **I don't know what other people have found but we're just trying to avoid not revisiting for a year basically and just trying to build in lots of different things. Maths is a connected thing rather than we're gonna do ratio then we're gonna do algebra, then we're gonna do something else. They're all the same, you're teaching everything all the time, that's what we're trying to build in.**	We could have spent five minutes here working on this issue as a group. This seems to be a big idea – what mastery looks like in one school – you're teaching everything all of the time – what might this look like?

Table 7.2: A section of transcript from first feedback session including phase one comment.

I made fewer comments on the transcript of the first recorded feedback session compared to the second. Between the first and second feedback sessions, I used an extract involving Sam from the first feedback session at a national mathematics education conference (a story I tell in section 8.1.3 of the analysis, see page 155), where several more experienced mathematics teacher educators were present. The ideas from those who participated in my conference session (summarised in appendix 3c) fed into subsequent comments [KMP4] made as part of the transcription process during phase one, as well as being evident in some of my responses during the second feedback session itself (for an example of a response that was informed by the ideas of others, see section 8.2.6, page 195: "What do you mean by normally?").

7.4.2 Phase two of data analysis (a recursive process)

In phase two, I returned to the full set of audio-recordings. I listened to them all, in chronological order. I made notes and listened again. Having decided to focus my analysis most intensively on the first two audio-recorded feedback sessions, I set to work on these two conversations. I took to going on long walks, listening to the conversations and dictating notes as I went. The walking seemed to get me into a space where I could listen uninterrupted by the usual patterns of behaviour (my own and others) that are triggered

from sitting at my desk [KMP6]. After each walk, I would transfer my comments (referred to as "phase two comments" in tables 7.3 and 7.4, and in the full transcript found in appendices 3a & 3b, pages 300 & 326, respectively) onto the transcript. Presenting phase one comments alongside phase two comments within the transcript is one way of demonstrating change of perspective, as part of my process of becoming a mathematics teacher educator.

As well as commenting in direct relation to particular moments within the conversation, I also began keeping notes of a more general nature, which included connections I was making across the different parts of the conversation and with my broader experiences and ideas. Making these notes (specific and more general) marked the beginning of a process of combining categorical analysis (of the two earliest recorded feedback sessions) with a process of telling stories [KMP4], a process that is further elaborated on in section 7.4.3 (see page 136). Once the process of commenting (phase two) was complete, I listened again to the recorded conversations, this time, with the transcripts and comments in front of me, to begin a systematic process of labelling.

7.4.3 The process of labelling

Tables 7.3 and 7.4 are examples taken from the full transcript of the first two recorded feedback sessions (see appendices 3a & 3b, pages 300 & 326, respectively). Both tables contain the same columns as table 7.2, as well as two additional columns, "phase two comments", and "label" (the colour coding links the label with the specific part of the transcript that the label is associated with). In this section, I describe the process by which these labels (and others) materialised. The full list of labels can be found in appendix 3d.

Time	Name	Transcript	Phase one comments	Phase two comments	Label
10:11	Alex	Yeah, so in the future it is them thinking about how they can use it in their lessons as a tool. I'm sure people think about questioning but sometimes people don't think that much in advance as part of their planning process…	I could have said, "How do you know?" or "What gives you that impression?"	As well as developing their own practice, the members of this group are supporting others in their departments – so the challenge here is how to get them to reflect on this without making assumptions, **speaking for others.** Encouraging evidence etc.	*Speaking for others*

Table 7.3: Example one of a section of worked transcript from second feedback session.

Time	Name	Transcript	Phase one comments	Phase two comments	Label
58:28	Beth	The only thing that I'm kind of thinking there is, is it very procedural? Are they following a method and what happens when you want to ask a question? Does it follow the same pattern? Is that where your questioning needs to come in because you haven't checked for understanding of the concept, you've checked for replication of the process.	I find this comment so insightful and interesting, and I suppose in reflecting on this I come to see that there are many points being made, many experiences to share in the room. Why do I need to share my own? So, am I back to asking what, if anything, do I share?	"Can conceptual understanding be developed through patterns?" "Is process only ever replication?"	*Distinctions*

Table 7.4: Example two of a section of worked transcript from second feedback session.

Generating labels was an iterative process that was initially informed by the comment-making process (particularly phase two commenting, but not exclusively), since comments often referred to a particular phenomenon in the data along with a label for that phenomenon (for an example of this, see table 7.3, *speaking for others* in bold type). Sometimes a label marked the absence of a phenomenon, rather than the presence. By absence I am referring to a missed opportunity, an opportunity to have acted differently in the moment in the group conversation. For instance, one label that emerged, *doing actively*, was used to indicate moments during the feedback session where, as a group, I saw an opportunity to pause the teacher and work on some mathematics, instead of leaving the teacher to continue with a description. Hence, some labels indicate an observed phenomenon and others indicate a potential intervention. In all cases, the labels are at a meta-level, so rather than a label being a description of the *content* of a comment, the label is a description for the *type* of comment being made by the speaker, or the *type* of intervention that could have taken place, or the *type* of phenomena being

observed. Identifying the type of comment being made, or type of phenomenon being observed and responding to that, is something I am learning as a mathematics teacher educator (see chapter nine, page 247).

Once a few labels had become established (i.e., assigned to more than one instance), there was a need to review them individually and as a set, to pay attention to what made the labels distinct from one another, as well as what connected them. Since learning requires a change to our ways of making distinctions [KMP7], I decided to take the labels, working on them in a way that was designed to increase the likelihood of me recognising the same phenomenon in the future, both in subsequent data analysis and, more importantly, in future conversations with teachers of mathematics. Table 7.5 (which is identical to tables 8.2 and 9.1, each a key to the column headings in tables 7.6, 8.3, 8.4, 8.5, 8.6 and 9.2) and table 7.6 illustrate the process of refining my distinctions and enhancing my potential to identify "correspondences and affinities" (Varela, 1999, p. 28) through gaining a clear description of the situation [KMP8]. For each phenomenon (and associated label), I considered and formulated multiple aspects. For instance, what the phenomenon was distinct from (table 7.6, column two), and what the observed behaviour could look like (table 7.6, column three), either in the moment, or over time. I considered this part of my approach to analysing the data as significant in terms of becoming sensitive to differences (Dreyfus & Dreyfus, 1991). By considering each phenomenon in the way described, using the column headings in table 7.6, I was able to return to the transcripts and locate further examples of the phenomena, developing a more refined set of discriminations based on a wide range of familiar situations (Dreyfus & Dreyfus, 1991) [KMP8]. Working on the transcripts and on the emerging labels was a recursive process, with each new distinction feeding into the subsequent identification of new or more refined distinctions (and associated labels):

Label	for the phenomenon.
Distinction drawn	as distinct from…
Descriptions	of observed behaviours
Potential strategies	* In the moment. ** Anticipatory (setting up).
Functions	of potential strategies.
Notes	on useful additional points.

Table 7.5: Key to column headings (for tables 7.6, 8.3, 8.4, 8.5, 8.6 and 9.2).

Label	Distinction drawn	Descriptions	Potential strategies	Functions	Notes
Slide (from account-of to account-for)	Staying with the detail of an observable description.	Mode of speaking changes from detailed description to evaluation/judgement/ expressed values. Ambiguous term/ phrase/label used (e.g., split). Reason given for making a choice as opposed to describing what happened.	* Prompting a staying in the detail (e.g., by clarifying a subjective term). * Asking a clarifying question. * Reflecting back the mode of speaking.	- Clarifying ambiguous terms/labels (a particular label may mean different things to different people.) - Gaining access to an image of a teacher's classroom. - Delving deeper into the detail.	Sliding is not necessarily problematic, e.g., when a teacher moves from the description of an event to identifying, from that detail, a related issue.

Table 7.6: Example of working on a label in a systematic way.

The process of extension requires sustained and disciplined practice as a way of enabling actions that have been deliberated on to arise in appropriate ways within other, analogous, but more complex situations (Varela, 1999) [KMP8]. Thus, I considered two further aspects of each phenomenon, specifically, *potential strategies* (table 7.6, column four), that I could imagine myself using, and the *functions* (table 7.6, column five) of those strategies. Establishing *functions*, in the context of interrogating each phenomenon in the way described, was a particularly powerful mechanism that supported me in identifying analogous situations within the data (by asking myself "what is the function here?", and "what purpose is served?") and to consider whether a potential strategy may or may not be appropriate. Establishing an explicit sense of purpose (or set of *functions*), against each phenomenon, allowed me to deliberate on the appropriateness of my past actions (as well on the appropriateness of potential future actions). Moreover, developing an explicit sense of purpose also expands the possibility for me to act differently in the future.

Once a few labels had been generated, I began pulling together sections of transcript that had been assigned the same label, and physically gathering them together. This process of gathering multiple instances of the same phenomena, was a way of identifying "correspondences or affinities" and to "see that one situation resembles another" (Varela, 1999, p. 28), [KMP8]. Once I had started gathering similar instances together each in a separate table (see appendix 3e, page 379), I began by formulating hypothetical

utterances (see examples in tables 7.7 and 7.8) as a way of using my imagination, to "extend [my] feelings […] and to have these feelings "break through" into […] new situation[s]" (Varela, 1999, p. 28) [KMP8]. This process of gathering and extending was a way of enhancing the possibility of future recognition as well as the possibility of new and different ways of acting in the future as a mathematics teacher educator. Bold type is again used to highlight those parts of the transcript where the words being spoken stood out to me during the initial process of transcribing (see section 8.1.2 for more details):

Slide (from account-of to account-for)		**Hypothetical utterances**
Session 1 (00:03:53)		
Sam	I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? **I picked that structure because of all of the structures that Paul showed me, that was the one that made my head absolutely bend round. I thought, well if it makes my head bend, let's see how good my year tens really are. It completely split my class massively.** The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure.	"Is challenge something you always value in your teaching?" "What do you mean by split?" "How do you know, what was happening?"

Table 7.7: Example of a hypothetical utterance from the first feedback session.

Drift		**Hypothetical utterances**
Session 2 (00:32:43)		
Beth	Yeah, I did something quite similar, it was year eleven, not key stage three but, we were talking about the multiple-choice questions at the start of the paper. I circled the right answer and said, "why is this correct?", so they didn't need to think about that, they knew which one was correct. Then I gave them the question again three more times with the other three answers circled and said, "now you need to tell me what the question could have been if this was the right answer". It was really interesting because they did things, I didn't expect them to do. It was a ratio one, it was what fraction of the drink is orange? I thought they would just change the type of drink each time, but they also started reordering the ratios as well, which for me, showed a better understanding. I was a bit nervous, is it the right thing to do? Just give them the right answer and hope? But actually, it worked really well because it really did help me to understand what they understood.	"A number of you have contributed to this discussion. I wonder if we can get back to the original issue? Or if you could each articulate the issue you have in mind in the examples you have given."

Table 7.8: Example of a hypothetical utterance from the second feedback session.

At this point, I began writing the stories that have now become chapter eight: stories that were emerging throughout my process of dwelling in the recorded conversations; and stories that informed what was possible for me to see within the data. As Mason (2010) confirms:

> Externalizing and labeling salient moments in retrospect is part of bringing to expression, of story telling. Labels act as axes or foci around which experiences can gather […] They can come to be associated with distinctions that might be informative, actions that might be relevant, and stances that might be fruitful. (Mason, 2010, p. 41)

I found that through telling stories about lived experiences, I began to make more sense of where the labels had arisen from and how they might connect to others. As more labels emerged, certain labels merged (because they served the same *function* or prompted the same *strategies*) and other labels split out into separate labels that served different *functions* and called on different *strategies*. Thus, the definitive set of labels that I have come to use in this study emerged out of the interplay between the process of categorial analysis and the process of living, telling, retelling and reliving stories [KMP1], all of which were informed by my broader teaching experiences, dialogue with others and my reading of related literature [KMP5].

7.5 The final form of my analysis

Chapters eight and nine are the product of my systematic process of analysing the recorded conversations from context two, as outlined already in this chapter. In keeping with my narrative approach, chapter eight is presented as a story, organised into four separate *episodes*:

> [I]n coming to terms with my past I can only do so from the present, through the act of interpretation; I seek to "read" the events of my life as episodes in an evolving narrative, the parts shaping the whole and the whole shaping the parts, in an undivided movement of the creation of meaning. (Freeman, 1998, p. 42)

The four episodes that comprise chapter eight represent four of the five groupings of phenomena established during my process of data analysis (the fifth grouping is dealt with in chapter nine. The reason for this separation is explained at the beginning of chapter nine, see page 247). You can view each episode as a separate story, each story representing a significant aspect of my becoming a mathematics teacher educator. You

can view the set of episodes as coming together to tell a longer story. The titles for each of the four episodes (*Using dissonance; Staying with the detail; Finding conviction;* and *Making it real*), as well as the title of chapter nine (*Going meta*), are in essence labels for a broader set of categorisations. This broader set of categorisations emerged out of my process of analysis as categorisations for the *groupings* of those labels (at the level of phenomena) that I recognised as serving similar functions, of which there came to be five. Each of these broader categorisations captures a significant aspect of the process of my becoming a mathematics teacher educator, a process that continues beyond what is possible to represent as five broad categories. Since each of the four episodes, along with chapter nine, reflect a significant aspect of the continuing process of my becoming a mathematics teacher educator, they are in a sense *methodological dimensions* [KMP2].

The process of creating chapter eight involved numerous instances of restructuring so that the collection of episodes would tell a coherent and meaningful story that captures my evolving perspective in relation to working with teachers of mathematics. In selecting lived experiences beyond those directly related to context two, I needed to make decisions about what to tell, what not to tell, and in what sequence. Not only does the story capture my evolving process of data analysis, but it is also organised in a way that reflects a series of significant events in my continuing journey of becoming a mathematics teacher educator.

Each episode, in chapter eight, makes explicit use of selected extracts from the recorded conversations to demonstrate particular phenomena, including a small number of extracts from the third feedback session (in episode four) and the seventh feedback session (in chapter nine). I use extracts from the latter two feedback sessions as part of telling a story of my own changes in practice, as opposed to the way described in relation to the first two feedback sessions. On occasion, where appropriate, I return to earlier stories, from previous chapters, and offer a retelling (a reseeing of events) based on a renewed perspective [KMP1]. I use theory to produce a credible account [KMP5] by describing and explaining certain aspects of the story and to bring about new ways of seeing. Importantly, I have come to realise that, together, chapter eight and chapter nine form my narrative-enactivist methodology for becoming a mathematics teacher

educator, informed by the narrative-enactivist methodology for *researching* my becoming that has been the focus of this current chapter. Throughout chapter eight, I actively draw on creative analytical practices [KMP4] as a way of combining my categorical analysis from working on the first two recorded conversations, with the process of telling stories.

The titles and the sequencing of the episodes reflect how, at the time of writing, I was making sense of the group conversations in relation to my broader experiences as a mathematics teacher educator, having benefitted from regularly reflecting on my practice alongside my colleagues and supervisors. It is not possible to disentangle my becoming from the becoming of others. Though I am not the first to utter the words that have become many of my labels (both for the phenomena and for the episodes), they have come to represent key dimensions of my becoming a mathematics teacher educator. Each episode is a journey through the various distinctions that have emerged in my becoming a mathematics teacher educator, grounded in the conversations between myself and the group of mathematics teachers, representative of the process of living, telling, retelling, and reliving stories [KMP1]. On their own, the distinctions that I have made lack the depth of meaning that they take on in the context of the fuller story that is behind the emergence of these distinctions.

In this chapter, I have outlined a narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator (expressed as eight key methodological principles), which has informed my systematic approach to data analysis, based on a set of audio-recorded conversations. The product of enacting this systematic approach to analysing data resulted in the creation of chapter eight and chapter nine. I begin chapter eight by setting up the subsequent four episodes that together tell part of my continuing story of becoming a mathematics teacher educator.

~~~~~
~~~~~

Chapter eight

A narrative-enactivist methodology for becoming a mathematics teacher educator

This chapter tells a story of my becoming a mathematics teacher educator over the course of the past five years (and beyond). The chapter in its entirety is both the product and the process of enacting the systematic approach to data analysis set out in chapter seven. I present the chapter as a set of four episodes (as explained in section 7.5, page 141) where each episode represents a different methodological dimension in relation to my becoming a mathematics teacher educator:

Since this chapter consists of four episodes, it is significantly longer than any of the other chapters. I view the whole chapter as a longer story of my becoming, with each episode telling a story of its own as well as being part of the longer story. Each episode can

therefore be read in isolation or within the context of the whole chapter. Before episode one begins, it is necessary to outline some important features of each episode, specifically, *the content and form of each episode*; *the use of dialogue from the feedback sessions*; and how each episode contributes to *a narrative-enactivist methodology for becoming on three distinct levels*. Thus, I dedicate a subsection to each of these aspects, and have marked the beginnings of these sub-sections using italics.

The content and form of each episode: At the beginning of each episode, I provide some background on the title of that episode, although an enhanced sense of the meaning behind each of the titles will only come from reading the episodes in full. Each episode then tells a story that connects the different phenomena as identified and labelled during the process of analysing the recorded feedback sessions. Table 8.1 shows the list of labels for the emergent phenomena, organised by the four episodes that make up chapter eight. The labels sometimes form distinct sections within an episode. Sometimes multiple labels gather to tell a particular part of the story. Like the titles for the episodes themselves, the meaning behind each of the labels will become apparent from reading the stories. Throughout this chapter I italicise every instance of a label.

Episode one: ***Using dissonance***	**Episode two:** ***Staying with the detail***	**Episode three:** ***Finding conviction***	**Episode four:** ***Making it real***
Dissonance	*Detailed description*	*Should/ought/had to*	*Asking why?*
	Slide (from account-of to account-for)	*Speaking for others*	*Unresolved*
	Doing mathematics	*Inner/outer*	*Commitment*
	It/that/this	*Teaching issue*	*Importance of context*
	Doing actively	*Teaching strategies*	*Drift*
	Slide (from mathematics to mathematics teaching)	*Inclusion*	*Research opportunity*

Table 8.1: Summary of labels for phenomena (by episode).

Within each episode, I integrate ideas from literature as a way of creating a layered text that enables a process of retelling stories. Some of the labels used within my analysis were informed by existing descriptions within the literature, as part of the recursive process of data analysis described in the previous chapter. Other labels will have originated from my own history of experiences as a mathematics teacher educator, which explains why those labels arose from interrogating the data and not others. Thus, the labels do not represent something existing within the data, rather, they represent what I could see in the data. What I could see in the data was determined by my structure. On occasion, I draw on stories beyond those in relation to the feedback sessions to express a richer and more holistic sense of how I am becoming a mathematics teacher educator. It is important to point out that episode one may read differently to the other three episodes. Episode one is based on one label alone (*dissonance*). Rather than a label for the type of phenomenon that I observed within the data, or the type of intervention that could have taken place during the feedback session, *dissonance* emerged as a description for the way I was experiencing certain words and phrases as I listened to the recorded feedback session (see section 8.1.2, page 154). The distinction between episodes one and the other three episodes will become clearer through reading chapter eight.

The use of dialogue from the feedback sessions: Within each episode, I integrate parts of the dialogue from the first two feedback sessions with the collaborative group of mathematics teachers (context two, see section 2.2.2, page 22), sometimes to exemplify a particular issue or moment in the story; at other times as part of the story itself. Extracts of dialogue from the feedback sessions are formatted in the same way as the dialogue between Sam and I in chapter four (see section 4.1, page 57). In episode one (section 8.1.4, page 156), like in chapter four, I augment a section of the first feedback session where Sam is offering some reflections, with a stream of inner consciousness, showing multiple perspectives (my own and others'). I augment this one extract (and no others in the remainder of chapter eight) since that section of transcript was one that I worked on in a distinctive way (described fully in sections 8.1.2, page 154, and 8.1.3, page 155). With other extracts of dialogue, I communicate my changing perspectives as I narrate the story from my *now* (at the time of writing) perspective, making sure I am explicit about the

perspective being offered. To access my *then* perspective, I have consulted my phase one comments within the full worked transcripts (see appendices 3a & 3b, pages 300 and 329, respectively), as well as my diary entries made around the time of the first two feedback sessions. Throughout the process of data analysis and in creating chapter eight, my focus has been mainly on generating new ways of seeing, as opposed to uncovering old ways. Having said that, one purpose of augmenting Sam's extract in episode one, is to offer some early instances of introspective analysis (see section 8.1.4, page 156), as a way of uncovering hidden assumptions and biases.

I chose to include examples of dialogue from the first two feedback sessions that I felt best demonstrated the phenomenon or phenomena being explored, through the story being told, but many more examples of each phenomenon were identified during the process of data analysis (see appendix 3e, page 379). Extracts do not necessarily appear in chronological order. Before each extract I indicate the originating feedback session and the time stamp which corresponds to the time stamp in the full transcript (see appendices 3a & 3b, pages 300 and 329, respectively). See notes on how to read this dissertation (page xii) for explanation of transcript notation. In episode four, I also integrate parts of dialogue from the third feedback session, where I reflect, during the group conversations on changes in my practice as a mathematics teacher educator.

A narrative-enactivist methodology for becoming a mathematics teacher educator on three distinct levels: Although my narrative-enactivist methodology for *researching* how I am becoming a mathematics teacher educator was articulated, in chapter seven, as eight key methodological principles, this research methodology continues developing through the process of analysing data and as *part of* an emerging narrative-enactivist methodology for becoming a mathematics teacher educator that now exists as chapter eight (and nine). This developing methodology for researching my becoming is reflected throughout each of the four episodes and, in chapter nine, as *one of* three *methodological levels* of becoming a mathematics teacher educator, specifically the second one in the list below. The other two methodological levels relate to the *process of learning to teach*

mathematics and mathematics teachers (level one), and *a way of working with mathematics teachers* (level three). All three levels are summarised here:

1) In relation to the process of learning to teach mathematics and mathematics teachers.
2) In relation to *researching* how I am becoming a mathematics teacher educator.
3) In relation to a way of working with mathematics teachers.

Given the nature of my primary source of data (recorded feedback sessions with a group of mathematics teachers), methodological level three may have been the most foreseeable methodological level to have emerged. However, given the reflexive nature of my research (which involves continuously reviewing the process of research itself) and the recursive process of data analysis (that went beyond looking solely at the recorded conversations), level one and level two emerged as equally significant. In fact, all three methodological levels have been present throughout my thesis, from the very beginning, in the stories I have told and the theories I have developed. It is at this point that I am choosing to make the distinctions explicit, as I aim to articulate my narrative-enactivist methodology for becoming a mathematics teacher educator. It is therefore possible to look at each methodological level in isolation. For instance, if you are particularly interested in conceptualising how learning takes place (for mathematics teachers and mathematics teacher educators) you may choose to pay more attention to level one. If you are interested in research methodologies for researching practice, you may choose to pay more attention to level two. If you are interested in developing a way of working with mathematics teachers, in supporting them to work on their own practice, you may choose to pay attention to level three. The three methodological levels come together as my (all-encompassing) narrative-enactivist methodology for becoming a mathematics teacher educator.

To make distinctions clearer between these three methodological levels (whilst acknowledging the affinities, relationships, and overlaps across them), I present, at the

end of each episode of chapter eight and in chapter nine, a distillation of the particular methodological dimension in relation to each of the three methodological levels, along with an articulation of the overarching guiding principles for the methodological dimension in focus. These overarching guiding principles express my overall sense of purpose, encapsulating the range of functions (see tables 8.3, 8.4, 8.5, 8.6 and 9.2) that have emerged for me through the process of researching how I am becoming a mathematics teacher educator. In chapter nine, I present an overall summary of the five methodological dimensions, in relation to each of the three methodological levels (see table 9.2, page 250). Figure 8.1 shows the various methodological aspects that have arisen throughout this thesis and the relationships between them (for an enlarged version of this diagram, see figure 9.1, page 272):

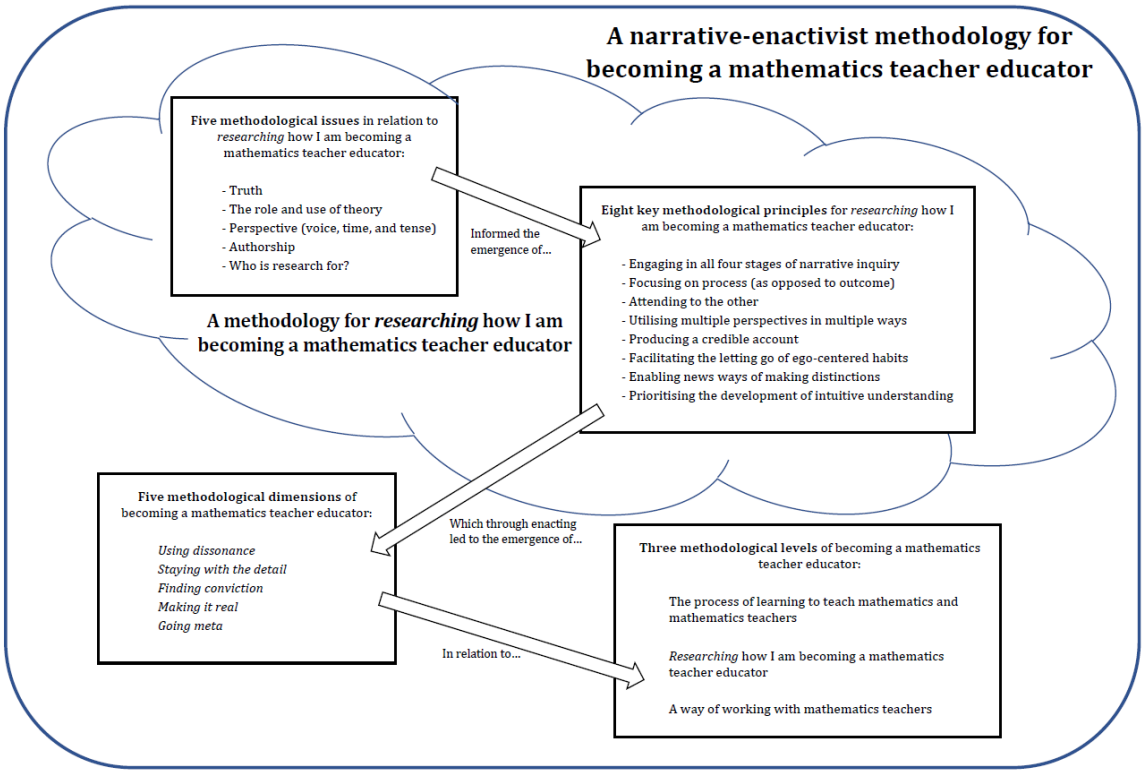


Figure 8.1: Relational diagram showing key components of a narrative-enactivist methodology for becoming a mathematics teacher educator.

~~~~~
~~~~~

Episode One

8.1 *Using dissonance*

The title of this episode also features in the title of one of Laurinda Brown's existing works, *Using dissonance: Finding the grit in the oyster* (Brown & Dobson, 1996, p. 212). I can only assume that the label *using dissonance* stuck with me from when I first encountered the chapter as a new mathematics teacher educator or maybe from a conversation with Laurinda. Having re-discovered Laurinda's writing more recently, I have used it directly within this episode (see section 8.1.7, page 167), making the decision to keep the title, to acknowledge and emphasise the situatedness of my becoming and the importance and role of context and shared practice. It is impossible to say whether the meaning that I attach to a label is the same as somebody else's. I hope that what I can communicate in this chapter is the process involved in my own developing sense of conviction in relation to this label, and others, that, in the early stages of my becoming a mathematics teacher educator, were practically void of meaning.

In this episode, I explore the one label that emerged through the process of analysing the recorded feedback sessions, in relation to *using dissonance.* Table 8.2 (also in appendix 3d, page 369) shows a summary from having worked on the label *dissonance* in a

systematic way. Table 8.2 is the key to the column headings in table 8.3 (as well as tables 8.4, 8.5 and 8.6):

Label	for the phenomenon.
Distinction drawn	as distinct from…
Descriptions	of observed behaviours
Potential strategies	* In the moment. ** Anticipatory (setting up).
Functions	of potential strategies.
Notes	on useful additional points.

Table 8.2: Key to column headings (for tables 7.6, 8.3, 8.4, 8.5, 8.6 and 9.2).

Label: ***Dissonance***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Consonance.	An experience of disturbance or disruption to an otherwise smooth flow.	* In noticing a moment of dissonance in the self, using this to trigger a prompt or question. * Prompting others in retrospectively re-entering moments of dissonance as a way of identifying issues in relation to practice. * Encouraging others to locate their own moments of dissonance in the future.	- Provoking a more thoughtful response (as opposed to reacting emotionally or judgementally). - Exposing hidden assumptions and biases. - Locating potential issues within practice that can be worked on.	An awareness of dissonance in the self is one aspect. Using the dissonance of others is another aspect.

Table 8.3: Summary of label in relation to *using dissonance*.

The story of episode one begins from the very beginning of the first feedback session (involving myself, the project lead, Paul and nine local participating mathematics teachers). The terms “project lead”, “Japanese lesson study”, “structured activities/ structures”, “gap task”, “key stage 3” are all explained in the glossary (page 291). Examples of structured activities presented in the first workshop are in appendix 3h (page 412).

8.1.1 The first feedback session

I was feeling nervous. I knew they were a friendly bunch since I had already met the majority of them at the first workshop, a little over a month before this one. I remember

being pleased that Sam had joined the group, I felt I knew her pretty well since I had been visiting her in school for a few months by then. Having somebody I had already worked with felt reassuring. It was not the teachers I was nervous of though. I was nervous because I did not feel prepared for what I was about to do, and I could not help feeling like I was out of my depth.

At the first workshop, Paul, the project lead, introduced us all to a range of structured activities designed to provoke mathematical reasoning that could be used in mathematics classrooms with groups of children, along with the research question: "How does regular use of one or more of these structured activities impact on students' precision when reasoning mathematically?" The teachers who had joined the workgroup, in pairs from participating local secondary schools, were also introduced to the principles and processes derived from a Japanese lesson study approach to professional development, which they were asked to put into action on return to their schools, in their pairs and also with their wider departments. In short, the first gap task involved participating teachers sharing the variety of structured activities introduced to them during the initial workshop with members of their own departments. Having introduced the range of structures, teachers from participating schools were encouraged to get members of their departments working in pairs to plan a lesson, aimed at key stage 3 students, that utilised one or more structures. Each member of the pair would deliver the lesson to their chosen key stage 3 group whilst the other member of the pair would observe and take observation notes. The teachers were asked to enact the principles of lesson study, which included a post-teaching discussion, involving both the teacher and the observer, focussed on analysing evidence from the lesson itself as a way of realising the detail of any specific adaptations to the original lesson plan. The suggested final step was for the observing teacher to then deliver the adapted lesson to their own key stage 3 group, again observed by the other member of the pair and again followed by a post-teaching discussion. I was lucky enough to visit two pairs in their schools to observe their lessons and to take part in the post-teaching discussions, something I had thoroughly enjoyed and that had allowed me to get to know the participating teachers and to see them in action in their classrooms.

It was my task during the second workshop to support the group of teachers in having a meaningful feedback session based on their range of experiences from doing the gap task. I had imagined that running a feedback session was about supporting a productive conversation between the participating teachers so that they would learn from their own experiences and from the experiences of others in the group, but I knew there was more to it than just getting them to tell each other what they had done, I just was not sure what exactly that was. I took some comfort from feeling I knew a thing or two about supporting children's mathematical reasoning from my recent time in the classroom. I imagined I might have something to offer where that was concerned, but I felt daunted by the responsibility. The impending conversation felt high stakes. Although I felt I was reasonably experienced in supporting mathematical discussion between students in the classroom, I knew I was not attuned to listening to the voices of such a diverse set of teachers and schools. I wanted to know what exactly I could be listening for. At that time, I wanted to know the rules for supporting this kind of conversation. I have since realised that expertise is not about knowing rules (see discussion on expertise in interlude four, page 118).

We helped ourselves to drinks and sat down around the large table that I positioned just before the teachers had arrived. I asked them to make sure they were sitting next to their school partner and I took my own seat at the table, making sure I was sitting as a member of the group rather than at any sort of physical distance. I turned the audio-recording device on and placed it in the middle of the table (see ethical considerations in sections 2.2.1 & 2.2.2, and full ethics applications in appendix four). I had made a few notes the day before, in preparation for this moment, they were supposed to be my opening words. I remember looking down at the notes, briefly, but it did not quite feel right to read so instead I began to speak:

> Tracy: Welcome everybody, for the next hour or so we will be sharing our experiences of doing the gap task. It is important each person or pair has time to share, to get into some of the detail of what happened in the lessons as well as the process of working with one another and with your wider departments. You will, for example, need to explain

> to one another which structure you used and how. If you can bring to mind any particular moments that have stuck with you from the lesson or lessons then you can share those and you may have notes to refer to as well from any post-teaching discussions you were able to have. As listeners, try not to ask questions to begin with, whilst the person speaking is having their time, but do make a note of them so they don't get lost. You will get the opportunity to ask them. I don't mind who starts but whoever wants to start, just begin by telling us what happened. We can start anywhere.

The conversation continued from this point. Sam was the first of the teachers to offer her reflections, reflections that I return to shortly in this episode (see section 8.1.4, page 156). Firstly, it is important for me to set up how I use "creative analytical practices" (Bochner & Ellis, 2016; Ellis, 2004; Richardson, 1999; Richardson & St. Pierre, 2018 (see section 3.6, page 50)) in the way that I analyse Sam's reflections. To do this I need to elaborate on what happened immediately after the first feedback session.

8.1.2 Salient moments

Having audio recorded the first feedback session with the group of mathematics teachers, I was keen to return to the conversation as soon as I could. I set to work, transcribing the first few turns in the conversation (involving myself and Sam). It was not long before I started noticing changes in myself in relation to certain words and phrases being spoken. As I listened to Sam reflecting on her experiences, I became aware of the occasional disturbance to an otherwise smooth process of listening and transcribing. I ignored these disturbances initially, but after a short while, as my awareness of these disturbances continued, I decided to highlight the words and phrases that triggered any disruption to my otherwise smooth flow (indicated as bold type both in Sam's extract below and in the full transcript, see appendix 3a, page 300). I seemed to be marking these salient moments by virtue of some change in my perceptual apparatus.

According to Mason (2002), if a moment is salient, it is likely that the observer "has some interpretation which gave the incident some significance" (p. 117). This suggests that

although it is the comment that is labelled as salient, the salience itself belonged to me, rather than being a feature of the comment itself. The salience, which according to Mason could be signalling some hidden assumption or bias, was manifesting itself as a disturbance to my otherwise smooth functioning. Having transcribed the first few turns, I decided to focus on the salient moments from Sam's extract. I attempted to interrogate each moment to try and uncover what was triggering a disturbance, in each instance, to surface what it was *in me* that meant I was marking certain moments as salient and not others. I represent this early analysis of each salient moment (indicated using bold type) within Sam's extract (section 8.1.4, page 156), as an inner stream of consciousness (in purple italics), using a similar approach to that used in chapter four (section 4.1, page 57) with an early conversation as part of context one (section 2.2.1, page 19). I use present tense for my inner voice to emphasise the analysis being from my *then* perspective. My then perspective was written during phase two (section 7.4.2, page 135) analysis, so, to access my then perspective, I used the original notes that I made at the time of transcribing (i.e., phase one, section 7.4.1, page 134).

8.1.3 The potential for using multiple perspectives

I was troubled by the realisation that many of my reactions to the various teacher's utterances were potentially rooted in misguided intuitions and interpretations. I was motivated "to guard [myself] against prior commitment, (including its extreme form, prejudice)" (Mason, 2002, p. 248), by practising being in question and seeking resonance within an expanding community. I was curious to find out whether experienced others might respond in the same way, if they themselves were to listen to Sam speaking. I recognised a need to "open up new possibilities by introducing multiple perspectives on the same piece of data" (Helliwell, 2017a, p. 2).

Fortunately, the Spring British Society for Research into Learning Mathematics (BSRLM, see glossary, page 291), day conference was being held weeks after the first feedback session. Given my focus of interest, it felt like the perfect opportunity to share with others my experiences of working on Sam's extract including the process of transcribing the

conversation and highlighting particular moments. With Sam's permission, I presented her initial turn at the conference to a group of mathematics education researchers and mathematics teacher educators. I wanted to know what, if anything, was salient for them in what they heard Sam say; if I had missed any opportunities to intervene at any point. The conference session was an opportunity for me to learn how I might have acted differently during that feedback session, and in future feedback sessions, with the group of mathematics teachers (which could potentially get extended to a range of different scenarios, such as de-brief conversations). Having shared some of the context of the project and of the feedback session, I provided the conference participants with a transcript of the short extract and asked them to record any significant moments and/or potential ways of intervening if they had been involved in the conversation for themselves. I was interested in similarities and differences across the group of participants and so I invited a discussion (on which I captured my own reflective notes, see summary in appendix 3c, page 368) about this before sharing my own highlighted moments from the three-minute extract. In Sam's extract (section 8.1.4), I represent the perspectives of the conference participants as a retrospective analysis of Sam's reflections, interspersed throughout the augmented extract. I use past tense to emphasise this layer of analysis as being (slightly) more recent than my introspective analysis that I offer an as an inner stream of consciousness (in purple italics).

8.1.4 The first feedback session continues

In this particular meeting, Sam was the sole representative from her school. The extract below follows directly from my prompt above, and Sam is recounting her experience of planning and using one of the suggested structured activities with her class of year 10 students (students aged 14-15 years). The structured activity she mentions is one that consists of presenting a set of calculations alongside a stimulus, asking, "What were the questions that resulted in each of the calculations?", or some variation on that idea (see appendix 3h, page 412). Note, the phrase "top set" is explained in the glossary (see page 291) along with "early entry", "non-specialist", "subject knowledge enhancement course" and "starter".

Sam: I have a top set year ten and we had just done sine rule, cosine rule, and area of a triangle without knowing the perpendicular height. I went for the structure, here are some calculations that a student might have performed based on this stimulus [Sam holds up a diagram of a labelled triangle (see figure 8.2, taken from my field notes) and a separate list of associated calculations, such as $15^2 + 24^2 - 2(15)(24)\cos 32^o$ and $\sin^{-1}\left[\frac{24\sin 32^o}{15^2+24^2-2(15)(24)\cos 32^o}\right]$].

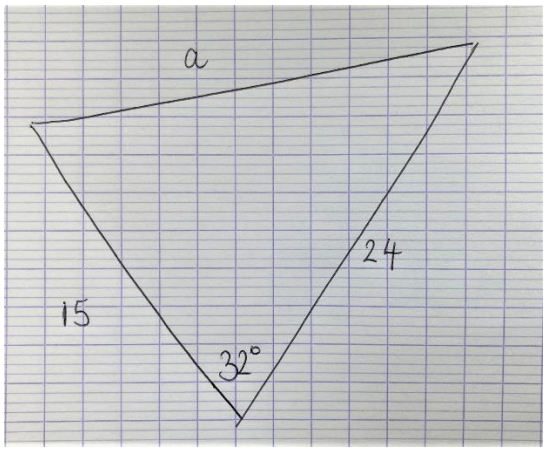


Figure 8.2: Sketch stimulus used by Sam.

I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? I picked that structure because of all of the structures that Paul showed me, that was the one that made my head absolutely bend round. I thought, well if it makes my head bend, let's see how good my year tens really are. **It completely split my class massively.** The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure.

I get the impression Sam is labelling some of her year 10 class as not capable. I realise this is an evaluation on my part though, based on little information. I expect my judgements say more about my own history of associations rather than anything that Sam is saying.

There were some suggestions from the conference participants around challenging assumptions without casting judgement, for example, in relation to ability, or getting stuck, and Sam's apparent dichotomy implied by the use of the word "split", which was taken by some to suggest a division between those who can and those who cannot.

> Sam: I don't think I had made a good enough job of making sure they understood the structure. On reflection, **I think I should have done something much simpler first.** I should have gone for something much simpler so they understood the idea before throwing in some quite horrific maths on them.
>
> *What's wrong with being stuck? I have always valued challenging students, yet who is to say what Sam meant by "simpler"? She did say the task made her own head bend round. I assumed she valued challenging her students too.*
>
> I actually started with the same triangle and asked them to come up with a minimum of three possible examination questions that would need the sine rule, or whatever, to solve. That was fine. That was good, but there was a massive jump between that and the structure using the eight calculations. In terms of the whole department, we have a lot of non-specialists.

Some of the conference participants marked this last sentence as a shift in topic, from Sam talking about her personal experience of teaching to her experience of working with her department more broadly. There was a sense from one participant that these shifts in patterns of talk are possible to observe in the moment of the conversation and that they might indicate a good time to pause, to seek some sort of resolution, before moving on. Thus, any shift in a pattern of talk could mark something worth drawing attention to as a mathematics teacher educator.

> Sam: I mean, I'm a non-specialist, I'm a Physicist, I only did a subject knowledge enhancement course two years ago, but my maths is ok.

I think a lot of my department would really struggle with opening themselves up mathematically to that.

It makes me feel a little uncomfortable when I feel that somebody may be limiting the potential of others. I have come to be mindful of making generalisations about groups of people, especially ones that ascribe limitations. Sam's comment, however, may well have been based on the views of those very teachers. I wonder what "opening themselves up mathematically" means to Sam? It could mean something very different to the way I am interpreting it.

One prompt offered during the conference session, which I found particularly helpful, was in relation to having an awareness that we can only ever speak for ourselves (which informed a label I use in episode three: *speaking for others*). So, Sam's comment, "I think a lot of my department would really struggle with opening themselves up mathematically to that", could have potentially triggered an intervention such as, "start again, how do you know? What is your evidence?". This intervention could have prompted a re-seeing in Sam of the situation being described, or, some more information about what had led Sam to make that comment.

Sam: So, I think we probably have to teach the structure with something slightly simpler and then encourage them to stay a little bit out of their comfort zone but not to the extent where they are completely freaked out. It took me ages to plan it. The first time I did it, I did it actually with Paul in the room, do you remember? I'd done it wrong, because I then thought I'd come back to it the next day and I thought I'd try it for myself and there were two of the calculations where the actual the basic maths didn't make any sense, so it was interesting. **It was quite hairy at times.**

I certainly recognise that feeling when things start to go wrong in a classroom. Who exactly did it get hairy for though?

The first task took about five minutes. The calculations task took about ten so it was quite a meaty starter that could have gone into a

> whole lesson if I'd felt like it I think. I think I should have allowed more time. I think I should have actually unpicked each one of the calculations, but as it was the class were forming that kind of horrible situation where some got it, and others really didn't get it and were really cheesed off that they weren't getting it. So, **I had to put some emergency repairs in place.**
>
> *Why such drastic measures? Emergency repairs imply there was some serious damage being done. I do not correlate being confused or stuck with doing damage, but perhaps there is more to the situation than I am getting from Sam's story.*

Some of the conference participants picked up on what was described as a focus on what did *not* work, and suggested a re-focus on what did work and building on that. Other participants noticed Sam's use of "should" and "had to" which gave the impression that Sam assumed there were no alternative courses of action than those she had taken or that she would choose to take if she were to do something differently in the future (e.g., "I should have gone for something much simpler", "I should have allowed more time", "I should have unpicked each one of the calculations", "I had to put some emergency repairs in place"). One suggestion was to reflect this use of language back to Sam. Another was to see the first "I should" as Sam articulating an awareness of doing something differently, and that becoming a mathematics teacher educator was partly about noticing when this happens. As a mathematics teacher educator, it might be possible therefore to flag something negative that forces a change. For instance, "I had to put some emergency repairs in place", is not seen as wrong, but, through questioning, could lead to, "I don't like it when the kids get stuck". Once a statement like this has been shared, one possibility would be to open up to the rest of the group: "So what do you all do when kids get stuck?", potentially expanding the space of what is possible, for the teachers and for their students. The conversation continued from there (for the full transcript of the conversation, see appendix 3a, page 300). Sam was asked a few questions, which she responded to before the next pair began to reflect on their own experiences in relation to the gap task (section 8.1.1).

8.1.5 A new label

That session at the BSRLM conference, and the resultant conference paper (Helliwell, 2017a), marked a significant moment for me in my becoming a mathematics teacher educator, where I shifted my focus from looking directly at the development of the participating mathematics teachers, to focussing on how I am becoming a mathematics teacher educator through working with teachers of mathematics. Without realising it at the time, the session and the paper were an initial step towards developing a narrative-enactivist methodology for researching my becoming including a systematic approach to analysing the early recorded feedback sessions with the group of mathematics teachers (see section 7.4, page 133). I began to understand that I could see more in those recorded conversations than I was able to see in isolation, that I could work on myself and the data simultaneously so that what stood out, what I noticed, could be more and different. I was motivated to stay alert to any future disturbances, in the moment of any interaction (with that particular group of mathematics teachers, or indeed more widely with the group of prospective mathematics teachers that I was teaching in a range of teacher education scenarios), disturbances that could potentially trigger a range of effective (mathematics teacher educator) behaviours. These disturbances I have now come to view in relation to the notion of *dissonance.*

~~~~~
~~~~~

Interlude five: On dissonance

Mason (2010) tells us that "[b]iologically, a dynamic equilibrium is sustained until there is a significant but not overwhelming disturbance" (p. 24). He explains that no learning takes place "until there is some disruption to the working out of internalized actions (habits, automaticities), until some dissonance is experienced" (p. 24). It is this disturbance, dissonance, disruption or interruption, to our otherwise smooth flow, that triggers us to become deliberate in our actions. According to Festinger (1957), cognitive dissonance occurs when a person participates in an action that goes against one or more of their beliefs, values, or ideas. Dissonance can be momentary, often experienced as a "psychological discomfort" (Festinger, 1957, p. 2) that can trigger us into action in an attempt at reducing dissonance and returning to consonance (or equilibrium).

In some cases, however, dissonance may not be experienced only momentarily, instead it may persist. For instance, I might tell myself that I should get some exercise, how good it is for me to go out for a walk or a run, yet I stay sitting down at my desk for most of the day. I might reason to myself that I need to spend what precious time I have working rather than getting exercise, or alternatively, I might avoid the thoughts entirely. I may put on my running clothes and training shoes as soon as I get out of bed in the morning and tell myself that I will work more productively having exercised because I know that both of these actions will make it more likely that I get myself up and out of the door. My actions, whether they consist of reasoning to myself, putting on different clothes, or ultimately going for a run, are all, according to Festinger, an attempt at reducing the dissonance that has been created, a dissonance that will persist for me whenever I am not in the regular routine of getting some exercise. Festinger's (1957) theory of cognitive dissonance is based on his hypothesis that dissonance (and its associated discomfort) motivates us to try to achieve consonance either by reducing dissonance (by changing our behaviour or through a process of mental reasoning) or by trying to avoid situations that may cause or increase dissonance. In chapter six, I told a story from when I was a newly qualified mathematics teacher (see section 6.2, page 102) where I would never address the whole class for more than a few moments. By inquiring into this story, I

reported on my behaviour with that group as causing me "a certain amount of discomfort" (page 106). A retelling of this story could frame the situation as one where I was experiencing persistent dissonance, yet I seemed unable, at that time, to modify my actions in order to achieve consonance. One possibility is that I managed to reduce the dissonance to a tolerable level by convincing myself, perhaps without realising, that the way I was behaving was in fact the most effective option for those individuals.

A useful and related idea from Whitehead (2000) comes in the form of his phrase "living contradiction", which he uses to express the experience of "holding together two mutually exclusive opposite values" (p. 93). Whitehead claims that he experiences himself as a living contradiction when he recognises that he holds a belief or value, yet denies it in his practice (2000, p. 93). Within mathematics education, the reported misalignment (between mathematics teachers' beliefs and behaviours) is often framed as the distinction between a mathematics teacher's espoused model of teaching and learning mathematics and their enacted model of teaching and learning mathematics (Ernest, 1989). Some mathematics education research reports on *consistencies* between espoused beliefs and enacted practices (e.g., Stipek et al., 2001), but more commonly, research findings report on *inconsistencies* (e.g., Beswick 2004; Cooney, 1985; Voss et al., 2013) or that findings are *mixed* (e.g., Da Ponte & Chapman, 2006).

From an enactivist perspective, beliefs are not a meaningful concept and neither do they *direct* our behaviour, so phrases such as *living contradiction* do not really make sense, even though as a description of an experience it is one that I resonate with strongly. The fundamental issue I am exploring here is the relationship between (explicit) intention and action. As Mason (2003) questions:

> From an enactivist perspective, the distinction between belief and behaviour is artificial: acting is knowing and knowing is acting (Maturana & Varela, [1998]). Your observed behaviour is what you know to do, and what you know to do is enacted. I haven't yet resolved for myself how this sits with my own experience of an inner life often in conflict with outer manifestations: I intend not to do something, but find myself doing it; I set myself to do something yet I forget or shrink from doing it. (Mason, 2003, p. 288)

One possible explanation is that our *espoused* beliefs, i.e., what we *say* we believe (either privately or publicly), are actually narratives that we create. In order to account for our actions, we bring our intentions to expression, to communicate our ideals. Yet in some cases these explicit intentions have not (yet) become embodied behaviours, and we need to work on ourselves, for example, through enacting the key methodological principles presented in chapter seven (section 7.1, page 126), to make this happen.

One reading of the enactivist view of cognition is that we *know* something once it has become an embodied behaviour, i.e., what we do. For Wittgenstein (1975), the notion of certainty is neither propositional nor conceptual, but instead corresponds to a "sureness" (p. 511) that has characteristics of an automatism or instinct. According to Wittgenstein, our basic certainties are not propositional but ways of acting that once formulated *appear* like propositional beliefs. What we formulate, or bring to expression, could thus be something else, some ideal maybe or what others might call a belief. Even our own formulations only have the potential to become *actualised*, i.e., embodied behaviours that belong to the realm of intuition, in the way they do for an expert. Our propositions and formulations exist as narratives. They are stories that thread together and give meaning to numerous fragments of experience. We tell ourselves these stories, we live them, in order to make sense of our lives and to align what we do with what we know. We retell stories on seeing them differently. Overtime, these reseeings and retellings are what I imagine is meant by having wisdom.

Enactivism does not proclaim to be a theory of everything. According to Simmt and Kieren (2015), "[e]nactivism as a methodological frame for mathematics education research is a form of research that is occasionally and multiversally incomplete" (p. 316) by which they are suggesting the need for complimentary theories or methodologies in order to say more about the phenomenon under investigation. Festinger's (1957) theory of cognitive dissonance is one example of a complimentary theory, Damasio's (2000; 2006) somatic marker hypothesis is another. Festinger's (1957) theory has been described as more notional than explanatory, as are many ideas that describe everyday experiences and phenomena, but I find it extremely useful, nonetheless. Damasio's (2000; 2006) somatic marker hypothesis (see interlude four, page 118), as used by

Brown and Reid (2006), seems to offer a theory that is compatible with Festinger's notion of *momentary* dissonance. Brown and Reid (2006) use Damasio's (2000; 2006) somatic marker hypothesis to account "for the ability of most people to make decisions quickly and continually in the course of their lives" (p. 179). In analysing one teacher's mathematical activity, the authors describe the teacher's decision making in terms of a conflict between a positive and a negative somatic marker (as opposed to a conflict between an action and a set of beliefs). Specifically, they suggest that negative somatic markers can make us feel uncomfortable as we move to act and that it is a negative somatic marker that creates (momentary) dissonance.

A recognition that dissonance comes from within, is evident in Maturana's (2002) words: "nothing external to a living system can specify what happens in it; all that an observer sees as external to a living system can only trigger in it structural changes that are determined in it" (p. 12). An enactivist interpretation of the notion of dissonance would be that dissonance is a response that is triggered when an individual interacts with its environment. Thus, the interaction *triggers* a response that is experienced as dissonance, yet the dissonance itself is determined by the existing structure of the individual. A different individual, interacting with the same environment may not experience any dissonance at all.

~~~~~
~~~~~

8.1.6 Persistent dissonance as a motivation for change

Festinger's (1957) theory suggests that, over time, we act in ways that lead to a reduction in persistent dissonance, in order to return to a state of consonance. Persistent dissonance can thus be a motivator for change, provoking a change in behaviours. Unlike my story as a newly qualified teacher, where I experienced persistent dissonance yet no change in behaviour, the following story is instead one of *using* dissonance to bring about changes in behaviour.

I had been teaching mathematics for around ten years when I inherited a Year 9 group (students aged 13-14 years), almost all of whom I had not taught previously. I began working with this group in my usual ways, doing what I had done with Year 9 groups in previous years. However, having taught the group regularly for a few weeks it became apparent to me that things in that classroom did not *feel* right, I was experiencing minor, but persistent, dissonance. The students were working hard, there were no significant tensions created by challenging behaviour and they all appeared to be learning. However, I was struggling to connect with this group in the way that I had connected with others. The problem was, my embodied behaviours, my established ways of being in a classroom, were not triggering the same responses from this group that I was expecting from them. I would ask questions that I had asked a number of times before and used the same familiar tasks as I had used for numerous years, but the students were not responding in the way that I was wanting them to. This was troubling me.

After quite a few weeks of teaching this Year 9 group, I decided I needed to start paying closer attention to the detail of what was happening in the moment of the lessons. The first thing I remember noticing was my experience of silence as being uncomfortable. The silences in themselves were not the issue, since I had become comfortable with silence, but there was something about the quality of those silences that was making me feel uncomfortable. In these moments, it felt not only as if the students lacked the confidence to speak, or that they had not yet formed a response, but it was as if they lacked the confidence to *think*. I realise, of course, that I had no access to their inner thoughts or feelings, but based on my hunch, I began implementing strategies that could support the students in speaking to one another and thus thinking. For instance, when I asked

questions, I would ask the students to individually write down their response to the question (not necessarily in their exercise books and not necessarily a full solution) and a question of their own that they might want to ask. They were then asked to speak with the person next to them or with the people on their table about their responses. Only then would I ask some individuals to share with the whole class and this would include everybody hearing the question that they wanted to ask. After a relatively short period of time, the students began talking to one another without my prompting them to. Even more importantly to me, they began to ask their own questions. With these new behaviours now in place, I returned to a state of consonance and I was able to resume my usual state of flow in the classroom.

8.1.7 Using dissonance during the process of retrospective analysis

In Laurinda's book chapter, *Using dissonance* (Brown & Dobson, 1996), she describes a way of working with a collaborative group of students, all studying for a master's in mathematics education. It is a way of working that I was myself exposed to as one of the prospective mathematics teachers that Laurinda taught (although I cannot say that I had any idea at that time of the significance of this way of working). I have attempted to maintain this way of working with groups of prospective mathematics teachers I now teach on the same course. Rather than presenting this way of working with mathematics teachers as described in Laurinda's chapter, I offer my own version, as I have come to see it enacted with the group of prospective mathematics teachers that comprise my tutor group (usually between 10 and 15 individuals) each year at the university.

One tradition that has been maintained on the course for prospective mathematics teachers is our Friday morning tutor group sessions. These Friday morning sessions take place every week for the first six or so weeks each year, often following a period of a few days where the prospective mathematics teachers have been in school on teaching placement. At the beginning of a tutor group session, I usually say something like "think back over the last few days in school and try to locate a moment of discomfort, something that has stayed with you up to now, potentially something happened where you were

unsure what to do." One of the bases for this initial stage is that "[t]eacher learning often seems to begin with a recognition of something not working" (Brown & Coles, 2020, p. 88). Furthermore, "[i]ncidents which stay in memory are usually ones in which we have considerable emotional and intellectual commitment. They are 'important' to us for some reason" (Mason, 2002, p. 39). After a few moments, when it feels as though the group have had enough time, I ask for somebody to share their moment as briefly as they can without any evaluation, explanation, or justification, only a description of the moment itself, with the minimum context needed for the description to make sense. The other group members are then asked to share any moments that were triggered for them in hearing that opening incident. These triggered moments may or may not be the same as the first incident they had brought to mind. More important is that the incidents are connected in that they are "sparked off either through resonance ('that reminds me of …', or 'that's similar too …'), or dissonance, a strong feeling of difference" (Brown & Dobson, 1996, p. 214). Once a few connected moments have been shared, we then work as a group to identify the issue or theme that runs through all of the incidents, so that we can move to consider possible courses of action if the issue were to arise again in the future:

> Provoked articulation through the use of dissonance is a powerful tool for uncovering implicit theories and beliefs, and thereby creating the platform from which the individual can move to a new position of conscious decision making. (Brown & Dobson, 1996, p. 215)

As a new mathematics teacher educator, I was keen to learn this practice of working with groups. I had a clear sense of the structure of the tutor group sessions and could generally follow the steps as described above, yet I had no real sense of the significance of what I was doing. As a result, I would feel nervous in anticipation of the sessions and unsettled throughout the sessions themselves. Becoming a mathematics teacher educator, for me, involves developing conviction about my actions. I have known to trust in the traditions and practices of close others, having been a recipient of those practices, experiencing the depth of learning that is possible. Yet, at the same time, I realise the importance of engaging in a constant process of making practices my own. The process of researching my practices has played a considerable role in doing this. The process of retrospectively

re-entering moments of dissonance has been confirmed to me, when I am becoming a mathematics teacher educator, as an invaluable activity to get teachers of mathematics to engage in, as a way to learn from their experiences. I find myself, as a mathematics teacher educator, engaging in exactly the same activity to learn from my own experiences. To be moved in the moment to act differently, however, involves something closer to a sustained and disciplined approach such as that informed by my narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator (as set out in chapter seven).

8.1.8 Harnessing an in-the-moment awareness of dissonance

One function of using dissonance is as an indicator of something to be worked on retrospectively. Another function is to develop an awareness of dissonance in the moment of teaching or working alongside teachers of mathematics as a way of triggering a more thoughtful response. An evocative image used by Mason (e.g., 2002, p. 184; 2011, p. 42; 2017, p. 3) is that of two birds, one busy eating, the other looking on. One interpretation Mason offers of this image is that the two birds represent different forms of attention. The bird eating, caught up in the moment of doing, represents the form of attention that gets caught up in the moment, automatically or possibly even, habitually. The bird looking on, however, represents an "inner witness" (2002, p. 184), a form of attention that allows us to be awake in the moment. According to Mason (2002), the awakening of an inner witness comes about through engaging with the *Discipline of Noticing*, which Mason (2002) summarises as four interconnected actions (Mason, 2002, p. 95). The essence of noticing "is being awake in the moment to possibilities" (Mason, 2002, p. 144), a process that is intimately linked with becoming self-aware, that is, awakening the form of attention needed to act differently in the moment. In this view, becoming a mathematics teacher educator requires a form of self-awareness that is attuned to moments of dissonance, by being awake in the moment so that an increasing set of possibilities for action become available.

Staying alert to dissonance, whilst attending to others in the moment of an interaction, is the work of "deliberate analysis" (Varela, 1999, p. 32). As Brown and Coles (2012) describe:

> We need to be alert to such moments [of dissonance] in order to reconstruct (in the moment) the awareness that is clued by the discomfort. This is the working of deliberate analysis. In this mode of being we are not carried away by spontaneous action, nor distancing ourselves by adopting a stance of rational calculation. We are engaged with the other and their concerns, yet alert to disruption of the process of coming to see more through such attention to the other. (Brown & Coles, 2012, p. 230)

For me, the eating bird symbolises our most pervasive mode of being, our immediate and spontaneous coping (Dreyfus & Dreyfus, 1991; Varela, 1999) that has developed from our history of all our lived experiences. The second bird, what Mason refers to as self-awareness, stays alert to moments of dissonance so that we can recognise moments of choice. When the two birds act together, accessing both forms of attention, this is the mode of "deliberate analysis" (Brown & Coles, 2012, p. 230; Varela, 1999, p. 32), which allows us to "reconstruct the intelligent awareness that justifies the action" (Varela, 1999, p. 32), without experiencing "a loss of fluency and even, in extremes of self-consciousness, in paralysis" (Claxton, 2000, p. 35). Dissonance can thus be harnessed as a force for change in two ways. Firstly, through retrospectively identifying moments of dissonance and deliberating about the appropriateness of our actions. Secondly, through staying alert to moments of dissonance as they occur through being in a mode of deliberate analysis, a form of self-awareness that allows us to attend to the other in more responsive ways.

In the final part of this episode, I summarise *using dissonance* as a methodological dimension in relation to three methodological levels of becoming a mathematics teacher educator. Then, to conclude this episode, I share my overarching guiding principles (explained on page 149) for *using dissonance* as I have come to see them.

Using dissonance **in relation to the process of learning to teach mathematics and mathematics teachers:**

Retrospectively locating moments of dissonance and identifying the issue triggering the discomfort. Considering, alone and with others, the appropriateness of the behaviours that triggered the dissonance.

Prompting more thoughtful responses in the moment of teaching.

Monitoring the consequences of actions by paying attention to further moments of dissonance (or consonance).

Using dissonance **in relation to** ***researching*** **how I am becoming a mathematics teacher educator**

In the process of listening and transcribing, staying alert to moments of dissonance and recording those moments for subsequent analysis.

As part of a process of data analysis, retrospectively integrating moments of dissonance in relation to the data being analysed, being transparent about hidden assumptions and biases being uncovered through this process.

Utilising multiple perspectives and sharing own analyses by seeking resonance and dissonance in an expanding community (to bring into question any awareness clued by moments of dissonance).

Telling and retelling stories as a way of re-entering the awareness that is clued by dissonance.

Using dissonance **in relation to a way of working with mathematics teachers**

Staying alert to instances of dissonance, in the moment, provoking a more thoughtful response, rather than reacting emotionally or judgementally.

Prompting others in retrospectively re-entering moments of dissonance as a way of identifying issues in relation to practice.

Encouraging others to locate their own moments of dissonance in the future.

Overarching guiding principles for *using dissonance*

Developing an open mind. Trapping judgements and a false sense of accomplishment. Learning about oneself and others. Triggering a range of considered and thoughtful responses in the moment of interactions.

~~~~~
~~~~~

Episode Two

8.2 *Staying with the detail*

In episode one, I discussed ideas around links between what we set ourselves to do and what we actually do; between what we say we value and what we do in practice; and the resulting experience of dissonance. I explored the potential for harnessing dissonance, a methodological dimension of becoming a mathematics teacher educator, both retrospectively and during in-the- moment interactions with mathematics teachers as one way of triggering more considered and thoughtful responses or interventions. As with episode one, the title of this episode is also one that features in the title of an existing piece of work. This time the label *staying with the detail* appears in the titles of two existing works by Laurinda Brown and Alf Coles. Firstly, in the title of a book chapter by Laurinda and Alf, *Staying with the detail: The use of story as a pedagogical tool within teacher education* (Brown & Coles, 2013) and then a later article by Alf entitled *Facilitating the use of video with teachers of mathematics: Learning from staying with the detail* (Coles, 2019). As a phrase, *staying with the detail* is very much part of our everyday discourse as a small group of mathematics teacher educators at the university. Again, as with episode one, I chose to keep the title since it symbolises such an important dimension of how I am becoming a mathematics teacher educator. Thus, the label itself

is not my own, but the process of making it meaningful is my own and I want to acknowledge the influence of my more experienced colleagues, demonstrating the situatedness of my learning (see interlude two, page 84), that is, the relationship between my learning and the culturally constructed social and material settings in which my learning takes place. The story of episode two is one of me finding the meaning behind the label that I have come to refer to as the second methodological dimension in my becoming a mathematics teacher educator.

In this episode, I explore the labels that emerged through the process of analysing the recorded feedback sessions that I grouped under the methodological dimension, *staying with the detail*. A summary of the labels that have emerged in relation to the story of episode two are summarised in table 8.4 (also in appendix 3d, page 369), starting with the label *detailed description*. A key to the column headings in table 8.4 can be found in table 8.2 (page 151).

Label: *Detailed description*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Talking in more general/ abstract terms.	Teacher is describing an account of an event.	* Commenting on when teachers are giving a detailed account, and when they are not, prompting them to describe in detail. * Encouraging a teacher to give a detailed description of what happened in the moment of a lesson, potentially a moment of momentary dissonance. ** Setting up way of talking about experiences at beginning of session.	- Establishing conversation norms in relation to reflecting on past experiences. - Preventing evaluative modes of speaking. - Trapping judgements. - Supporting the development of new insights and awarenesses. - Developing new basic-level categories.	

Label: ***Slide (from account-of to account-for)***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Staying with the detail of an observable description.	Mode of speaking changes from detailed description to evaluation/judgement /expressed values. Ambiguous term/ phrase/label used (e.g., split). Reason given for making a choice as opposed to describing what happened.	* Prompting a staying in the detail (e.g., by clarifying a subjective term). * Asking a clarifying question. * Reflecting back the mode of speaking.	- Clarifying ambiguous terms/labels (a particular label may mean different things to different people.) - Gaining access to an image of a teacher's classroom. - Delving deeper into the detail.	Sliding is not necessarily problematic, e.g., when a teacher moves from the description of an event to identifying, from that detail, a related issue.
Label: ***Doing mathematics***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Describing the mathematics being done.	Teachers are engaging in doing mathematics together.	* Prompting a mode of doing (as opposed to describing).	- Developing mathematical insights.	
Label: ***It/that/this***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
The thing/idea/ phenomenon that *it/that/ this* represents.	Ambiguous use of the word *it, that* or, *this*.	* Asking for clarification.	- Avoiding speaking at cross-purposes.	The object/idea/ phenomenon that *it/that/this* represents.
Label: ***Doing actively***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Describing the teaching scenario.	Teacher running an activity (with the group as opposed to describing activity).	* Prompting a mode of doing/enacting (as opposed to describing how something was/could be enacted).	- Giving direct access to an activity, to experience it more closely.	This label emerged from my recognition of the need to extend the doing mathematics label, *doing actively* does not need to be doing mathematics but it represents the difference between doing and describing doing. Linked to doing mathematics but can apply to teaching mathematics or working with teachers, all of which can be done actively as opposed to talking about the doing.

Label: ***Slide (from mathematics to mathematics teaching)***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Describing an account of a teaching scenario or a professional development scenario.	The moment doing mathematics becomes talking about teaching mathematics.	* Resisting the slide, by prompting a staying in the mathematics. * Embracing the slide, by relating back to practice (after a period of doing mathematics).	- Resisting common responses to some are of mathematics. - Encouraging new ways of seeing mathematics. - Encouraging new practices.	Sliding is not necessarily problematic, e.g., when a teacher moves from doing mathematics to considering what that might mean for the teaching of mathematics and their practice.

Table 8.4: Summary of labels for phenomena in relation to *staying with the detail*.

As mentioned at the beginning of chapter eight (page 145), the emergent labels sometimes form distinct sections within the episode, and sometimes multiple labels gather to tell a particular part of the story. Episode two begins with a retelling of part of a story already told, in the form of a diary entry from 1st March 2016 (see section 1.3, page 9), of a lesson de-brief conversation with prospective mathematics teacher Hayley.

8.2.1 *Detailed descriptions*

When Hayley first started speaking about the lesson she had just taught, she spoke in general terms about her overall sense of teaching the group, as opposed to talking about moments from the lesson. She seemed to be speaking in an evaluative mode, expressing an interpretation of her (negative) experience of working with the group. It was as if she was talking about how she *felt* about the groups rather than about what happened in the teaching of the group. At that time, I did not recognise this mode of speaking, or distinguish it from other modes. I just knew that I was experiencing that part of the conversation as difficult, that it did not feel productive. In that moment, I did not know what I could do to change the situation.

Jaworski (1990) refers to an evaluative mode of speaking in relation to working with groups of mathematics teachers in using video (of mathematics lessons) as a tool for mathematics teachers' professional development. Following the joint viewing of an extract of video, a discussion takes place between the mathematics teachers and the mathematics teacher educator (Jaworski). She explains that a key purpose of using video

with groups of mathematics teachers is "to provide a shared experience which can form a starting point for discussion of teaching" (p. 61). A problem arises in these discussions when the starting point is based on the teachers' feelings and reactions that arise from watching the video, feelings and reactions that manifest themselves in an evaluative mode of talk:

> The problem with comments such as these is that they invest all of their energy in interpretation and judgement of the acts and intentions of the particular teacher of the video. Such comments, particularly if from the first speaker to open the discussion, can condition what follows and result in little of value for the teachers concerned. (Jaworski, 1990, p. 63)

According to Jaworski, beginning with an evaluative mode of talk can be a barrier to a constructive discussion between the mathematics teachers. Coles (2013), also in relation to using video for professional development, explains that "[i]f teachers begin in an evaluative mode of discourse, it appears it is hard to shift discussion to any other mode" (p. 167). Both Jaworski (1990) and Coles (2013) discuss principles for working with groups of mathematics teachers using video. A common feature from both authors is the practice of reconstructing what was seen in the videoed lesson by the members of the group, for those members to reach a consensus or agreement on what happened during the extract of video, as far as possible, without entering into interpretation. This initial reconstruction phase takes place before shifting into a different, more evaluative mode of discussion, that can lead to the "identification of important issues and the possibility of working on these issues in the classroom" (Jaworski, 1990, p. 63).

The process involved in teachers reconstructing what they have just observed in a video is a little different to the process of teachers reconstructing lessons they have taught themselves or observed at some point in the past. Inevitably, without a record of the lesson, the process of reconstructing the lesson is done using the active processes of memory and imagination (see section 3.1, page 36). What gets reconstructed is potentially therefore a retelling of a story that may well have been told multiple times before, for instance, through a process of self-reflection on the lesson, or with other

mathematics teachers who may have observed the lesson. Each new telling can bring about a new seeing of the events along with the various evaluations and justifications that come with that kind of lesson analysis. It is possible, however, to work at reconstructing the lesson from memory as I observed several times during the feedback sessions with the collaborative group of mathematics teachers. In section 8.2.2, I give an example of a detailed description leading to an insight.

8.2.2 *Detailed descriptions* leading to an insight

It is during the first feedback session, quite early in the conversation. Alex is describing a moment from his lesson on pie charts. I asked him to clarify what question he had asked the students in his class to motivate a discussion, to which he replied, "It was how would you find each of these sections, what would be the easiest starting point?", which he said holding up the image and text from figure 8.3 before giving the following account [session 1, 00:15:08] of what happened at precisely that point in his lesson:

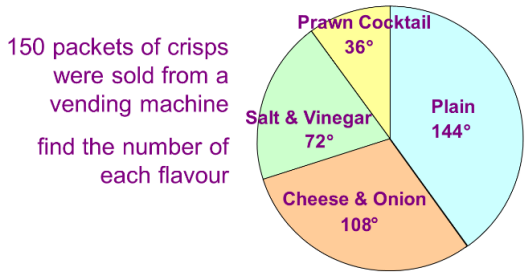


Figure 8.3: Alex and Ellen's first pie chart task (Steward, 2020).

Alex: So, then a kid said, "well I know that thirty-six over three hundred and sixty is the equivalent of a tenth". Then someone else interjected and said, "well then, if that means a tenth, this [pointing at the prawn cocktail slice in figure 8.3] will be fifteen". "OK perfect, we'll write this on the board". "Which will be the next easiest to find?" Then someone else said, "it's got to be seventy-two, because that's double the amount". Then someone else said, "well if you've got seventy-two, we know what one four four is, so take that away". Then someone said, "well you can take it away from the rest of it to find what the hundred and eight was", and someone else said, "why would you do that? Why don't you do thirty-six plus seventy-two

makes one hundred and eight?" Then someone else said, "well you could also do one hundred and forty-four minus thirty-six". So, the numbers are really important.

Sam: So, the choice of numbers is really important in terms of the reasoning.

Returning to the recorded conversations during phase two (section 7.4.2, page 135) of my analysis and relistening to Alex giving the description in the extract above, I was struck by two things. Firstly, I was struck by the level of detail Alex gave, and, secondly, I was struck by what listening to that detail did for me, in terms of giving me access to his classroom. In fact, I felt like I almost had direct access to what was happening during the moment described. At the university, when we interview candidates for the course for prospective mathematics teachers, we ask them to describe a mathematics lesson they have observed in as much detail as they can. When the interview candidate does this effectively and the description is detailed, it evokes a clear image of the classroom being described. Many of the candidates struggle to get to a level of detail required to evoke an image of the classroom being described and need a lot of support to steer them away from evaluation and judgement. Giving detailed descriptions of events are not necessarily natural or straightforward things to do. Sometimes candidates can be quite surprised by how much detail they noticed (or did not).

Given the group's focus on developing mathematical reasoning, it may not be particularly surprising that Alex's account of a moment from his lesson was a reconstruction of the mathematical reasoning performed by some of the students in his class. However, Alex's mode of talking (in this detailed way) was not a distinction I was able to make at the time of the conversation. If it had been, I think it would have been useful to point out to the others how Alex's reconstruction of the reasoning led to an insight (another distinction I was unable to make at the time) in relation to what it was about the task ("the numbers are really important") that enabled this process of reasoning to happen. Sam's rearticulation of the insight, as she saw it, was confirmation of their developing consensual understanding of the insight.

~~~~~

Interlude six: On basic-level categories

In researching a way of working with teachers of mathematics in collaborative groups, Brown and Coles (2020) draw on enactivist theory and particularly the enactivist theorist Rosch (Varela, Thompson & Rosch, 1991) who conceptualised how humans categorise the world as fitting into three broad levels of increasing abstraction: a subordinate level (or detailed level); a basic-level; and a superordinate level (or abstract level) (Brown, 2004; Brown, 2006; Brown & Coles, 2000; Brown & Coles, 2020; Coles, 2013; Lakoff, 1987; Rosch et al., 1976; Rosch, 1999). According to Rosch (1999), the basic-level of categorisation is the "default level at which categories [are] interpreted" (p. 74), they are the categories that have "perceptual, linguistic, and developmental priority" (p. 74). Most of the time we speak, we do so in terms of basic-level categories (which is why it can feel unnatural when we ask interview candidates to speak in a more detailed manner), they are "the most abstract [categories] where the same kinds of action are used for all members of that category" (Brown & Coles, 2020, p. 88). Basic-level categories do not exist in and of themselves, rather, we experience the world through our basic-level categories. One mechanism to bring about the process of creating new basic-level categories is "through a descent into the detail of experience" (p. 88), where existing basic-level categories can be unpicked to reveal a more detailed description of the world (at the level of specific observable behaviours, the subordinate level), where finer distinctions can be discerned. Through a process of labelling, these finer distinctions can become new basic-level categories and, as a result, we experience the world differently; we are more likely to recognise these distinctions again in the future (in the moment of teaching, say).

Let me try to ground these ideas in relation to the extract above. From Alex's description of a moment from his classroom, at the detailed level, a new basic-level category emerged for Alex (and a potentially new one for Sam) that was labelled as "the numbers are really important". Since we experience the world through our basic-level categories, they "are linked to immediate action" (Brown & Coles, 2020, p. 88) and are thus key to functioning effectively as a teacher (or indeed as a teacher educator). As a result of "the numbers are

really important" becoming a basic-level category, Alex (and potentially Sam) are likely to notice the choice of numbers in any potential task, if the function of the task is developing their students' mathematical reasoning.

I offer a parallel example now, this time in relation to the process of enacting my narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator. At the point in the process where I began noticing certain phenomena within the recorded conversations, the labels that emerged were labels for my basic-level categories. For instance, *detail* emerged early on as a label for one of my basic-level categories, and one that I began attaching to several instances of phenomenon that I recognised as *detail*. Through a recursive process of listening to the conversations, working on the data, reading, telling stories and then relistening to the conversations (a process of *staying with the detail*), I began to discern finer distinctions within my existing basic-level category *detail*, such as *detailed description* or *slide* (see table 8.4, page 174), which became new basic-level categories, each with one or more associated functions (see table 8.4). For example, one function of *detailed description* was *supporting the development of new insights and awarenesses* (also a basic-level category) that further enabled me to recognise more instances of the phenomena within the data. Hence, if as a mathematics teacher educator, one function is to support teachers of mathematics in developing new insights, then a potential strategy (see table 8.4) might be *encouraging a teacher to give a detailed description of what happened in the moment of a lesson, potentially a moment of momentary dissonance* (see interlude five).

~~~~~
~~~~~

8.2.3 Retelling the story of a de-brief conversation

Returning briefly to where this episode began, the early de-brief conversation with prospective mathematics teacher Hayley (section 8.2.1), I offer a new insight of my own. This new insight relates to my use of verbatim lesson notes during that de-brief conversation, which, in the moment of the conversation, I turned to without a sense of deliberateness, seemingly as a result of having no other alternatives available:

> *My strategy was to turn to my notes from the lesson and read them out, hoping that she might see a positive in what she had been saying […] Through talking, Hayley was able to unpick her negativity and talk about what she did that she would like to have done differently and that the issue of the students' poor behaviour was actually an issue concerning a lack of motivation.*
> (Diary entry, 1st March 2016, section 1.3, page 9)

Hayley's opening comments were both evaluative and negative, yet, in the moment, I was unable to recognise this problematic mode of talking. I only recognised a sense of not knowing what to do. I was motivated to act, however, to reduce the dissonance that I was experiencing. In the absence of another alternative, I turned to the verbatim notes that I had taken during the lesson. As a new mathematics teacher educator, I had observed my colleagues using verbatim lesson notes, but I was unaware of the function of doing so, beyond creating a record of the lesson that the prospective teacher could refer to later on. In using the verbatim lesson notes and pointing to specific moments from Hayley's lesson, a space seemed to open up where Hayley's negative evaluation could be explored. By getting into some of the detail of the lesson, we could work together to come closer to a mutual understanding of the situation. Both Hayley and I were able to see the situation differently, informed by one another's perspectives. Part of my process of becoming a mathematics teacher educator involves developing a sense of conviction in relation to certain practices that I have inherited. A second experience that concerns verbatim notes, also introduced in chapter one, was initially captured as a second early diary entry, this time concerning a group interview task:

> During the interview, I take notes, I try to listen to what is said and capture that on my page. I find this difficult as I can't write quickly enough, and my urge is to watch the body language in this performance. I think I will miss out if I don't watch but notes are what we do.
> (Diary entry, 18th April 2016, section 1.3, page 12)

The diary extract captures a moment of dissonance that is triggered by the conflict between my inner "urge" to watch the group of interview candidates working on the mathematics problem, and the established practice of creating verbatim notes. Everything in me wanted to observe the situation unfold, to study the group dynamics and the ways in which the individuals were interacting, with the task and with one another, rather than putting my energy into making notes. Since that time, I have learned that the *process* of recording verbatim notes does not prohibit my capacity to observe, on the contrary, the process seems to force me to attend to the detail of what is happening, away from casting judgements. Attending to the detail as an observer is a mechanism that supports me in using dissonance to mark certain moments during an observation (of a lesson or an interview situation) that can be interrogated further at some point during the subsequent reflective conversation.

Articulating a *detailed description* is not a guarantee that a new insight will be developed or that a new way of seeing will transpire in relation to teaching mathematics but *staying with the detail* of an experience is certainly one means by which new distinctions (or new basic-level categories) can emerge. When it comes to teaching mathematics, sometimes a *detailed description* of an event or scenario does not give sufficient access to the experience or phenomenon being described. In the next section, I present an extract from the first feedback session that includes a detailed description, yet, I have come to see this moment as a missed opportunity to shift into the mode of *doing actively*.

8.2.4 On the value of doing as opposed to describing

As part of the project working with the collaborative group of mathematics teachers, I offered to take part in the lesson study process (section 8.1.1, page 151) by joining pairs of participating teachers as the third member of their lesson study triad (see glossary, page 291). One of the pairs of mathematics teachers whom I went to visit in school was Simon and Mia. Simon and Mia had co-planned a lesson using the same structure that Sam had used, which was to provide the stimulus and a list of calculations (see figure 8.4, taken from my field notes), and ask, "what does each calculation represent?"

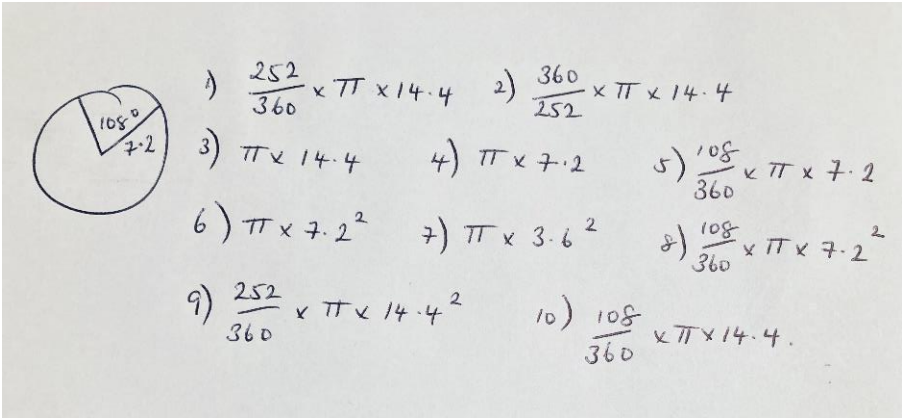


Figure 8.4: Sketch of the stimulus and corresponding calculations used by Mia and Simon.

A couple of days before the first feedback session, I went into their school to observe each of them teach the lesson they had co-planned. The following extract from the first feedback session [session 1, 00:09:07] begins with Simon's account of part of the lesson he taught.

> Simon: We did arcs and sector areas, so I just put a circle up, can you see that there [Simon points to an image including the stimulus and calculations shown in figure 8.4]. I gave them these different calculations and said, "right, what do each of these formulas represent on the diagram?" Some of the calculations didn't actually represent anything in relation to the diagram and in those cases, they had to either sketch or at least try and explain what that formula could be showing. So, it was a similar structure to yours [looking at Sam]. Mia and I approached it quite similarly, didn't we?

Mia: Pretty much the same.

Simon: Yeah, just pairs, talk about it, write something down, discuss with the people around you. That was the most powerful thing, the discussion that came out of it was really, really good. There were a few nice questions like, "if it is three hundred and sixty over two hundred and fifty-two, could that be used to do anything useful?" "What's the point in that?" We just tried to keep the numbers very, very similar. Some of the students were saying, "well maybe it's just a smaller circle". Someone raised the point, "oh there's pi times three point six squared, would that be the area of a smaller circle?" So, I said "well, would it be half the area of one that's got a radius of seven point two?" It just stemmed off from there. But I think it was good putting some in there that were not relevant to the diagram that was quite powerful too.

On initial inspection, I recognised the extract above as an example of a *detailed description*, similar in ways to the one given by Alex (section 8.2.2). Simon does move occasionally into a more evaluative mode of speaking (e.g., "that was the most powerful thing, the discussion that came out of it was really, really good", and "I think it was good putting some in there that were not relevant to the diagram" both of which feel like insightful evaluations). Mainly he describes the detail of what he did and observed. Having observed the lesson myself, I realise the way that I will have experienced what Simon was saying, in the moment of the group feedback session, would have been partly determined by what I saw myself during the lesson and what we discussed as a triad after both lessons were taught. In relistening, over three years later, I hear Simon talking quickly and I can appreciate how difficult it may have been for other members of the group to follow Simon's description. I found myself, as part of the process of analysis, needing to refer back to my field notes (taken whilst observing Simon teach) and having a go myself at the task Simon set for his students. Only then did I feel that I could fully engage again in Simon's description. Sometimes, in learning to teach mathematics, description is not enough. There is something significant about the process of *doing*

actively, as opposed to being told about the doing, that results in a different way of seeing. In the moment of the feedback session, I could have slowed Simon down; I could have suggested he enact that moment of the lesson, rather than describe it, to give us a few minutes to try the task for ourselves. I am convinced this would have given more of the group access to what I had been given access as a classroom observer, to the setting up of the activity, to the design of the task, and to the related conversation from the students.

Similarly, I could have asked Joe (see following extract [session 2, 00:30:06]) to switch modes part way through his description of a lesson that he had taught. Just prior to this point, Joe had introduced to the group the idea of talking less (both as the teacher and for his students) in relation to promoting mathematical reasoning. He suggested that it might be possible to model to the students by showing them (as opposed to explaining) by breaking a concept down into manageable steps:

> Joe: I went back to areas of rectangles and triangles. This time I used a series of diagrams to show the link between the area of a rectangle and the area of a triangle. I went through rectangles first, just put a side value in of say, ten, then I kept changing the bottom length to show that it was multiplying by ten to give the area. Then I changed it around and gave them some to do, did it quickly. Then I did one where I split the rectangle in half. I showed them a few examples, got the idea of halving, changed the way I halved the rectangle, then changed the halving to make a triangle. From there they could make the link that a triangle is always half of a rectangle. I gave them some more examples, so not always halving corner to corner and changing the slant height but keeping the perpendicular height the same, so the area didn't change. I genuinely did not say anything to them apart from at some points to say, "OK, what have I just done?"

Maybe if I had prompted Joe to enact the lesson with us, we could have tried designing a different set of images or examples as a collaborative group. I could have said, "would somebody be able to suggest a different topic or concept that we could try creating a set

of images or examples for?". We could have done some fruitful work on that. *Doing actively* by enacting a moment of teaching mathematics, as opposed to describing the detail of that moment, allows those participating in the doing to experience the teaching of the mathematics as well as the doing of the mathematics. Sometimes there seems to be a need to focus purely on *doing mathematics* (without the additional layer of teaching mathematics). *Doing mathematics* for me is a form of *doing actively* that I explore in section 8.2.5.

8.2.5 *Doing mathematics* together

Joe's idea of talking less (see previous extract) triggered related stories from other members of the group, each in some way related to less talk. The conversation made itself back to Joe [session 2, 00:39:45], who then offered an example of a task that he had used in the following lesson. "NRICH" is explained in the glossary (see page 291):

> Joe: I did an NRICH task, it's the one where they give you two rectangles. One of the side lengths is ten and you have to figure out what the missing length is for the perimeter and area to be equal. [...] I was anticipating, they were going to take forever, they would say, "oh what do I do?" But with a little bit of guidance; "how about you try a different side?" "Does it have to be a whole number?" "OK you're very close what should you do now?" Just little question prompts like that.

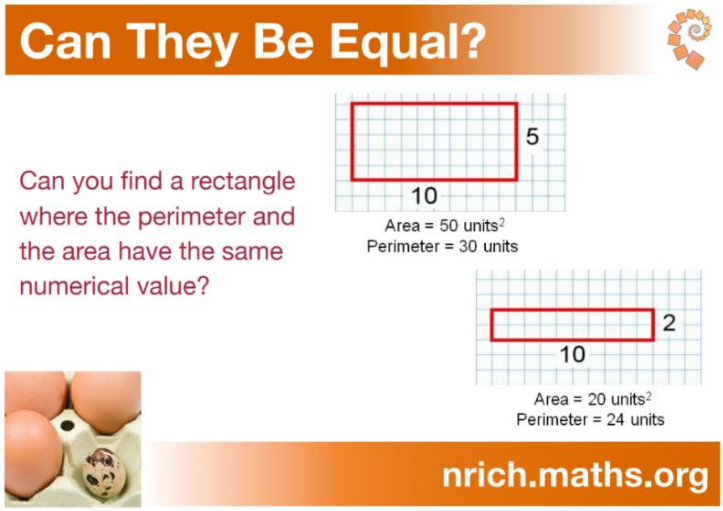


Figure 8.5: Task used by Joe (University of Cambridge, 2021).

I am familiar with the task that Joe introduced, having used a version of it multiple times in my own teaching of mathematics. I wonder if my (assumed) familiarity can be unhelpful, seeing what I am hearing as what I have done myself, leaving me feeling less curious. I think that being in conversation with mathematics teachers, supporting their learning, requires curiosity; to listen like everything being said is true. Listening like everything being said is true (or listening openly), does not mean having to agree with everything that is being said, rather, it means using dissonance to trap a false sense of accomplishment and to trigger a response of wanting to know more. I could have been curious in the moment where Joe introduced the task. I am curious now to know what we would have done as a group, if we had paused and spent five or ten minutes working on the mathematics for ourselves.

I was myself taught by mathematics teacher educators who established doing mathematics as a primary source of learning to teach mathematics. In one respect, doing mathematics as a precursor to teaching mathematics feels almost too obvious a point to be dwelling on. However, doing mathematics as a precursor to teaching mathematics can manifest itself in many ways. Throughout the PGCE course (see glossary, page 291), but particularly in the early stages, when the prospective teachers are starting to plan mathematics lessons, we strongly encourage them to spend time dwelling in the mathematics before they start committing to a planned sequence of activities and a lesson plan. For many of the prospective teachers, this dwelling will begin with familiarising themselves with the content of the intended curriculum (e.g., finding out the key learning outcomes for a particular topic). They often look in textbooks or on websites and remind themselves of the mathematics content and attempt some exercises. They may search for existing online resources and talk to one another about their ideas. As tutors, we suggest other activities, such as exploring related representations and models and trying them out with different examples. We might, for their very first lessons, suggest an open question that they could investigate to see where it takes them mathematically. Doing mathematics, in whichever way, is essential to the process of planning to teach. Doing mathematics as planning to teach mathematics is something I find myself prompting as I support prospective teachers over the course of the year, yet

previously I have struggled to articulate the purpose behind this activity, even though, intuitively, and through my own experience of doing so, I feel it is essential.

Mason and Davis (2013) use the phrase "a way of being *with* mathematics knowledge" (p. 194, emphasis original), that resonates strongly for me in relation to doing mathematics as preparing to teach mathematics. For Mason and Davis, being with mathematics knowledge:

> enables a teacher to structure learning situations, interpret student actions mindfully, and respond flexibly, in ways that enable learners to extend understandings and expand the range of their interpretative possibilities through access to powerful connections and appropriate practice. (Mason & Davis, 2013, pp. 194-195)

It is "being *with*" that fits closely with my own experiences of working on mathematics as a mathematics teacher and now as a teacher educator, in a way that I never experienced as a student of mathematics. For me, being with mathematics as preparing to teach mathematics, is a form of *staying with the detail*. Just as a detailed description of a classroom event can provoke new insights and ways of seeing past happenings, being with mathematics can nurture the mathematical awarenesses needed in the moment of teaching mathematics to respond mindfully and flexibly. Being with mathematics *together* as a group of mathematics teachers is a powerful way to access more than is possible to see by doing mathematics alone. In the moment of the group feedback session, I missed opportunities to encourage being with mathematics together. Joe's mention of that familiar task could have triggered me into getting the group doing and being with mathematics together.

There are also warnings to heed when doing mathematics together as teachers. Something to be aware of is the possibility of frustration if those insights that are developed alongside colleagues, are not observed in the classroom when engaging students with the same task. The frustration can lead to the common phenomenon of "switching into telling and explaining when things are not going to plan, especially for teachers in the beginning stages of trying to 'work differently'" (Mason & Davis, 2013, p. 183). Developing ways of working differently was on the agenda for our collaborative

group. I know my own frustrations have often triggered my desire to tell, for the students I teach to develop the same insights as I would have done in working on the mathematics for myself. My "teacher lusts" (Boole in Tahta, 1972, p. 11, see interlude one, page 69) have, on many occasions, taken over. It is quite liberating to have come to the realisation that only I can come to my own mathematical insights. Doing mathematics on my own, and with others, developing new mathematical insights, is not about preparing to change others, or to force them to see things the way I see them. Doing mathematics as planning to teach mathematics is about being ready to support students (and now teachers) of mathematics to see more and differently, not seeing what I see in the way that I see it.

I now recognise a few possible modes of being as part of a collaborative group. There are different ways of talking about experiences (description mode, evaluative mode) then there are different ways of doing actively (enacting part of a teaching episode, doing mathematics). Becoming a mathematics teacher educator, working with teachers of mathematics, involves a process of attuning myself to these different modes; being sensitive to when a mode changes (which I have labelled as a *slide*, see table 8.4, page 174) or when it might be fruitful to prompt a change. In the next section, I explore these *slide*s in more detail.

8.2.6 On becoming attuned to *slide*s

It seems that in everyday conversation, which for me includes conversations about mathematics teaching, it is natural to move between description, evaluation, and reason without any awareness (or any need for an awareness) of the change between those different modes of speaking. For instance, when Alex is reflecting, with his colleague Ellen, on his use of two tasks on pie charts early on during our first reflective discussion as a group (the next section of dialogue took place immediately prior to the extract presented in section 8.2.2 [session 1, 00:12:36]). "Median" is explained in the glossary (see page 291):

Alex: We did a matching exercise. So, there was one pie chart where all the slices had the same area. [Turning to Ellen] what was the second one we all thought they were gonna go for, but they didn't?

Ellen: The one when they were all different.

Alex: They were all different. This one [Alex points to the third pie chart on the resource shown in figure 8.6 that he is displaying on his laptop screen]. Then there is one where the frequency is one, two, three, four, five and they didn't look at that at all.

Figure 8.6: Alex and Ellen's matching pie charts resource (Steward, 2020).

They saw this one here [Alex points to the fourth pie chart on the resource shown in figure 8.6] where it's got the largest sector and then looked for the largest number from the tables. When we were going around asking them why they were choosing, the discussions were really rich. It helped that the resource was well designed. It only took me a little while to find it; it was a Median resource. So, we used this [pointing to the resource again] and I thought it was really rich. They had that and they had to match each of the letters with a number. I think we sort of explained the next task briefly [showing a different resource to the group based on figure 8.7] and then they went away and really went for it.

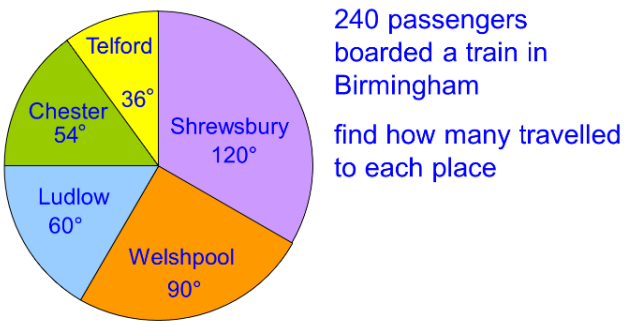


Figure 8.7: Alex and Ellen's second Pie chart task (Steward, 2020).

> Because they hadn't done pie charts […] we went through the workings and they had the discussion; "do we need to find the last one or do we just take it away from the full amount?" That was a really rich discussion.

Alex moves seamlessly between describing what happened, what he thought would have happened instead, his reasons for his decision making and his evaluation of the teaching/learning scenario. For me, this example of talk from Alex, demonstrates, quite typically, the way the teachers (including myself) often spoke during the feedback sessions. At the time of the conversations, I had no real sense of the different modes of talk, let alone how I might pick up on when the mode is changing.

Coles (2019) describes being attuned to the "the distinction between observation and interpretation" (p. 1) in relation to teachers talking about video-recorded lessons. A core enactivist principle is that "everything said is said by an observer" (Maturana, 1988a, p. 5), which means that when we explain our experiences, we always do so in relation to other experiences. Thus, our observations are in fact always also interpretations. We do not have access to a reality that is independent of our actions and our own being in the world, instead, as observers, each of us "builds up [our] own distinctive world according to [our] own distinctive structure" (Capra, 1996, p. 262). Using the language again from Rosch (Varela, Thompson & Rosch, 1991), a more precise distinction might be between *descriptions of observations* (a type of interpretation, but at Rosch et al's (1976) "subordinate" (p. 385) level (or detailed level), at the level of actions) and *descriptions of abstractions* (at Rosch's "basic level" (p. 383), which include evaluations, generalisations,

judgements, and reasons and Rosch's "superordinate" (p. 385) level, which is more abstract again, and includes philosophies).

Mason (2002) offers a useful distinction for supporting the seeing of two different modes of speaking. The labels Mason uses make visible the different functions for the two modes as opposed to describing what these modes are, that is, the labels (account-for and account-of) describe what they do:

> To *account-for* something is to offer interpretation, explanation, value-judgement, justification, or criticism. To give an *account-of* is to describe or define something in terms that others who were present (or who might have been present) can recognise. (Mason, 2002, p. 41, emphasis original)

Through staying with the detail of the feedback sessions, my process of data analysis has supported me in sensitising myself to notice when a *slide* might be occurring from an *account-of* to an *account-for*. Mason (2002) suggests that one mechanism for noticing such a *slide* (change in mode) is to "ask yourself whether what is being described is behaviour, whether it is negotiably visible or audible to others who share a similar culture to your own" (p. 42).

As already discussed during this episode (see interlude six, page 180), the process of labelling (associated with accounting-for) can support future recognition of new basic-level categories, which is why it can be a powerful mechanism for learning to teach mathematics, or mathematics teachers. Certain uses of labelling, however, should be avoided. Staying with the detail by honing accounts-of (as opposed to accounting-for) means avoiding generality and labels, which "can block access to alternative paths, alternative interpretations, and so ultimately to alternative acts" (Mason, 2002, p.51). For instance, as teachers, we need to avoid labelling students with particular (potentially negative) characteristics, since those individuals or groups are more likely to grow to fulfil our labels, if this is the way we learn to experience those particular individuals. Thus, working to separate out our accounts-of from our accounting-for, can reduce the number of unhelpful labels as well as generating new and effective ones. Let me offer another example from the first feedback session (this extract also appeared as part of a

longer extract in episode one, section 8.1.4, page 157, [session 1, 00:03:53]). I have emphasised what I consider to be the significant phrase using bold type:

> Sam: I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? I picked that structure because of all of the structures that Paul showed me, that was the one that made my head absolutely bend round. I thought, well if it makes my head bend, let's see how good my year tens really are. **It completely split my class massively**. The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure.

What constitutes the notion of "split", for Sam, may well be different for different people. If, in the moment of the feedback session, I had recognised "it completely split my class" as a problematic *slide*, one potential response might have been as simple as to ask Sam, "what do you mean by split?" Working on an account-of the notion of "split" could potentially support others in recognising similar phenomena from their own mathematics classrooms, leading to a collection of useful strategies. Working on the notion of "split" may also have avoided subsequent use of the same label ("split") by other mathematics teachers in the group (the example below follows twelve minutes after Sam's initial use of the word [session 1, 00:16:56]), without consensual meaning having been established (as far as consensual meaning is possible). The phrases "top set" and "mixed ability" are explained in the glossary (see page 291). Again, I have emphasised the significant phrases using bold type:

> Ellen: There were a few students in Alex's class when I was observing who didn't engage quickly **even though it was a top set class.** I had to get them going, and they were saying "we don't get it", but they were fine. **The split was much, much, wider in my class**, **because obviously it was mixed ability** and there were some children there

who don't know their four times table. But they all had a go, none of them just sat there, did they?

Assuming there is a consensual meaning across members of the group, in relation to notions such as "split", is likely to lead to talking at cross-purposes. My "normally", for instance (see extract below from the second feedback session [session 2, 00:29:44]), is likely to be different to somebody else's. Though we might assume we mean the same thing, it is not useful to make this assumption. Having worked myself on the notion of a *slide (from account-of to account-for*), after the first feedback session I was more attuned to problematic labels:

Joe: I taught that **normally** last term, it didn't go down very well.

Tracy: What do you mean by **normally**?

[laughter]

Joe: "Here's how to do the area of a triangle". "Here's how to do the area of a rectangle, now do some yourself". Pretty much, give them a procedure, give them a formula, you know the eighty percent of the time lesson when you're stuffed for time at the end of term.

Slide (from account-of to account-for) is one label that I have developed for a type a *slide* that is observable during the feedback sessions. Another type of *slide* is observable when the mode of *doing mathematics* moves to a mode of talking about the doing or the teaching of mathematics (for which I have used the label *slide (from mathematics to mathematics teaching)*). The next section of dialogue comes from the latter stages of the second feedback session. Immediately prior to the next section of dialogue [session 2, 01:03:57] Beth had just drawn two images on a whiteboard (recreated in figure 8.8), explaining that as part of her professional development role across a set of local schools, she had been working on the topic of fractions with some of the local primary school teachers (students aged 5-11 years).

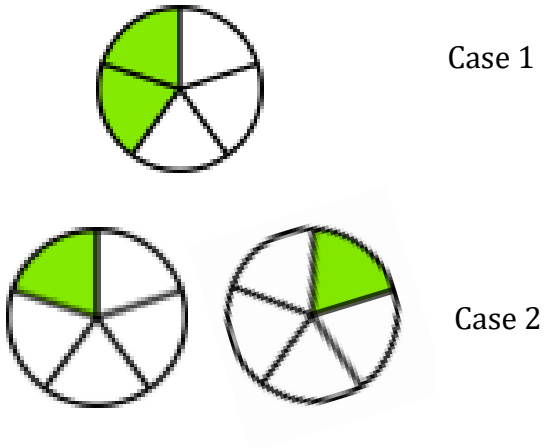


Figure 8.8: Two fraction representations.

Beth: I just asked the teachers to look at these two representations [pointing to a whiteboard where she had sketched the representations recreated in figure 8.8]. So, it's a pizza with five parts, two shaded, and then two pizzas cut into fifths essentially, five parts. I asked them, "what is the same and what is different?" and then I asked them "what does the numerator represent in each case, and what does the denominator represent?" I thought that everyone would be quite clear that here [pointing to case 2] it's two pizzas cut into five parts and here [pointing to case 1] it's one pizza cut into five parts with two shaded. It was amazing how controversial it was. One idea was it [case 2] must be two tenths because there are ten parts there [pointing to case 2], rather than it being two fifths of one pizza and what the whole is. It was really, really interesting. I think for me it was interesting because it showed how fixed our ideas of fractions are and how we might unintentionally limit what children think.

8.2.7 On resisting *slides*

Having shared the images she had used during her professional development session (figure 8.8), along with the specific questions that she asked the group of primary teachers, her mode of speaking then moved smoothly (i.e., there was a *slide*) to talking about what had happened as well as her thoughts on those events ("It was amazing how

controversial it was"). I remember being keen that Beth allow us each some time in the group to form our own ideas in relation to the two representations [session 2, 01:04:43].

Tracy: Well, do you just want to give us a minute to think about this. There might be other people who…

Beth: Yeah, so the first question I asked was "what's the same, what's different?" What I wished I'd asked was "what do you think the kids would say?" and then let people think about how they would address that [I gesture to prompt Beth back to the doing]. So, any responses?

Tracy: Well in both cases you've got two sections highlighted.

It was the first instance of me stopping the flow of the conversation to focus the group on something specific in this way. It was not at all clear to me that we shared a common sense of what Beth's images were representing in terms of fractions. I was also not surprised by the idea that case 2 could be interpreted as representing two tenths. I wanted to give everybody an opportunity to think for themselves about what they saw when they looked at both scenarios (case 1 and case 2).

Mia: It looks a bit like a fifth plus a fifth is two tenths, which is a big misconception isn't it.

Beth: Because there is ten parts in total so your whole is two, yep, any other responses?

Paul: I was thinking about what you would define the vinculum, the fraction line, as.

Beth: Yeah, so that was the next question, what does the numerator represent in each case and what does the denominator represent, I think this is where we came quite unstuck, which was good, it wasn't a bad thing, it was interesting, so…

Tracy: Well, do you want to give us a minute to just think about the answers to those questions. What does the numerator represent in each case and what about the denominator?

It fell quiet for a few moments until a new, but related, conversation begins to take hold between three members of the group. I bring it to a close:

Tracy: Try and stick with this for a minute.

There is a noticeable strength to my voice, which provoked some laughter from some members of the group. The laughter felt like polite recognition from those group members who were not involved in the related conversation, perhaps that they needed more time. I really wanted the group to think about the mathematics before moving to a discussion about what students (or teachers) might say or do or how to work with teachers in professional development scenarios. I wanted to hear the different ways that people were seeing what Beth had presented them with:

Beth: So, what does the numerator represent in each case? Does it represent the same thing each time?

Miguel: Two bits of pizza.

Beth: Both times?

Miguel: Yep.

Sam: Yep, I would say so, to me the bit that goes on top is the bit we are interested in.

Beth: OK, so what about the denominator? If this is always two bits of pizza, what does the denominator represent?

There is a pause and then what sounds like growing recognition from noises made by a few members of the group.

Sam: Arrr, and therein lies the conflict.

Miguel: How big the top bits are, they tell you, it's like you've got two bits, and the five bits is how big it is out of one, I suppose.

Beth: OK, so in this case [pointing at case 2], you think the two are these two? [Beth points to the two shaded sectors, one from each of the circles].

Tracy: I think the two is the two circles [I have sketched figure 8.9 (image taken from my notes)].

Beth: So here [pointing at case 2] is it that I've got two pizzas, and I split it into one, two, three, four, five parts. So it's two things divided into five parts, as opposed to, two out of five parts.

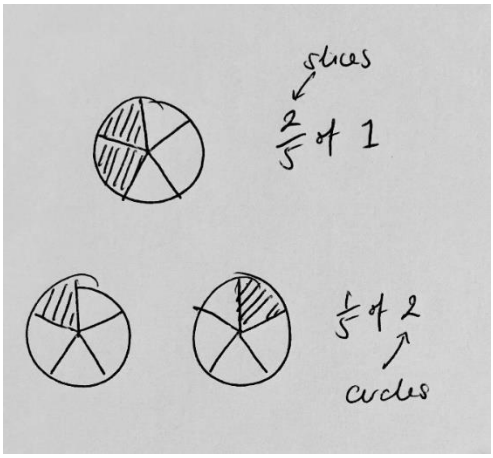


Figure 8.9: Sketch of two fraction representations.

Miguel: Well, I would say that it's a fractional choice. We've got a whole, we've got one whole, one pizza, and you divide by 2, then you shade it and you've got a half. But when you have two fifths then it gets a bit trickier doesn't it. I think, you've got two, as in the two whole ones, but then when you say split it into five, you need to assume the kids know you mean split each one into five.

Miguel has made a *slide* here from *doing mathematics* to talking about *teaching mathematics* ("when you say split it into five, you need to assume the kids know you mean split each one into five"). At which point the conversation continued in the same vein of talking about the mathematics in relation to its teaching. Though there were not many

turns in the conversation before this *slide* took place, to me it feels significant that we allowed that time for doing mathematics before referring to what took place during the professional development event and the implications of the two representations for the teaching of fractions:

> Beth: I could have coloured that one in instead [pointing to a non-coloured sector, adjacent to the coloured sector in the first circle of case 2] and it may have generated some different responses. Showing they were one part but colouring in the different ones to the ones we were highlighting.
>
> Paul: You could have two and then split those into groups of five, so you could have had the pizzas then giving you ten quadrants. Or you could share those into two groups of five which would also be a representation of the same thing.
>
> Beth: Yeah. It made me really reflect on how fixated I have been on this representation [case 1] when I've been teaching. I don't know if you have ever done the chocolate bar task, when you place one on a table, two on another and three on another and then you ask the students to choose where to sit.
>
> Simon: When they stand outside and then walk in.
>
> Beth: When I have done the chocolate bar task, the students have really struggled with was the idea that three things divided by five people is the same as three fifths, and that actually, the numerator there is the three things you're sharing not the three bits you're getting. If that makes sense.

Doing mathematics is common practice in mathematics teacher education scenarios, yet, in my experience, some teachers of mathematics still seem to be compelled to *slide* away from doing mathematics towards talking about teaching mathematics, as if, as mathematics teachers, we have learned to see and therefore talk about mathematics with a kind of mathematics teacher lens. As mathematics teachers, it is an arguably necessary

skill to be able to see through the eyes of our students, after all, learning to teach involves learning to see things that students of mathematics are likely to see, to pre-empt difficulties students will face in learning mathematics, and to design tasks that are worthwhile and fit for purpose. At the same time, though, this mathematics teacher lens, as I am calling it, could be potentially limiting. There is something important about setting aside some time for allowing mathematical insights to develop, for a while at least, setting aside concerns about what a student or group of students may see, do, struggle with, or conclude. Teachers, like everybody else, are prone to habitual interpretations and anything we can do to provoke new and different ways of seeing is likely to bring rise to new awarenesses and possibilities.

A *slide* from doing mathematics to talking about teaching mathematics seems to be a natural teaching behaviour in the same way that a *slide* from giving an account-of to accounting-for is a natural human behaviour, one that requires a certain amount of discipline to resist. In this particular part of the feedback session, I remember having a strong intuition about keeping the group together, focussed on the mathematics being offered, and the questions being asked. I wanted the group members to engage in the task, to answer the questions at the level of the mathematics, and to resist the natural *slide*, for a while at least, before moving to comment on related teaching and learning issues. What was behind my intuition? My own experience of learning to resist the *slide* perhaps. For me, learning to teach was very much about separating out, to an extent, the process of working on mathematics (by myself and with others) and the related implications for classroom teaching. Having said that, I think I always work on mathematics with my teacherly lens intact, perhaps once it is there, you cannot remove it.

One of my tasks, as a mathematics teacher educator, is to recognise when a *slide* is occurring, and to respond accordingly. In some situations, an effective behaviour might be to resist the *slide*, in others it might to highlight or even encourage the *slide*. The following episode begins with some examples of effective *slide*s (in terms of mathematics teacher learning), sometimes the process of *staying with the detail* can lead to a *teaching issue* (see table 8.5, page 206) being articulated, a meaningful label for a phenomenon

that a teacher recognises from their own experiences as something they would like to work on in their practice. In the final part of this episode, I summarise *staying with the detail* as a methodological dimension in relation to three methodological levels of becoming a mathematics teacher educator. Then, to conclude this episode, I share my overarching guiding principles (explained on page 149) for *staying with the detail* as I have come to see them.

Staying with the detail **in relation to the process of learning to teach mathematics and mathematics teachers:**

Separating out accounts-of from accounting-for, avoiding evaluative ways of speaking and allowing new insights and awarenesses to arise from the detail.

Trapping judgements and avoiding unhelpful labels that characterise groups of, and individual students/teachers.

Engaging in *doing* (mathematics, mathematics teaching) as opposed to *describing* (mathematics, mathematics teaching), before moving to explicitly consider implications on practice.

Staying with the detail **in relation to** ***researching*** **how I am becoming a mathematics teacher educator**

Dwelling in the detail of data through the recursive process of listening, transcribing, re-listening, accessing field notes, doing mathematics, reading, telling, and re-telling stories as a way of developing new basic-level categories. Labelling these categories to increase future recognition.

Working on labels for categories of data in multiple ways (e.g., considering a description of the observable behaviour, associated potential strategies and related functions of those strategies).

Staying with the detail **in relation to a way of working with mathematics teachers**

Becoming attuned to the different modes of talk and prompting for clarity when ambiguous labels/terms/phrases are used.

Noticing when a slide might be taking place and responding accordingly by resisting the slide or encouraging it.

Identifying opportunities for doing mathematics or doing actively before considering implications on practice.

Overarching guiding principles for *staying with the detail*

Developing (new and different kinds of) insights. Letting go of assumptions and preconceptions. Being open to newness and difference. Releasing self from judgement and becoming curious. Listening like everything being said is true (or listening openly).

~~~~~
~~~~~

Episode Three

8.3 *Finding conviction*

I have written previously about *finding conviction* in relation to becoming a mathematics teacher educator (e.g., Helliwell, 2017a; Helliwell & Brown, 2020; Brown, Brown, Coles & Helliwell, 2019). I keep returning to *finding conviction* as a description that fits with my experiences over the last four or five years. I suppose it was inevitable that *finding conviction* would emerge again as a central notion in researching how I am becoming a mathematics teacher educator. In the previous episode, I explored how *staying with the detail* has become a meaningful methodological dimension in my becoming a mathematics teacher educator. One purpose of dwelling in the detail of an experience is to support new and different ways of seeing, beyond our initial readings of a situation. As structure-determined systems, what is possible for us to see as humans is determined by our structures (and associated history of experiences); the way we respond to a stimulus from the environment is determined by our structures, rather than the stimulus itself. Being structure determined does not mean, however, that we have no choice in how to respond in any given situation, even in relation to those responses that we experience as automatic (see chapters 5 and 6 for full discussion). Episode three is, in different ways, about making choices. Firstly, in relation to *nurturing a sense of agency* (see section 8.3.1)

and, secondly, in relation to *fostering a sense of ownership* (see section 8.3.5). Together, agency and ownership have emerged as the essential components of *finding conviction* (one of the five methodological dimensions of becoming a mathematics teacher educator), both in relation to the process of learning to teach (mathematics and mathematics teachers) and in relation to a way of working with teachers of mathematics.

Table 8.5 (also in appendix 3d, page 369) is a summary of the labels that emerged, during the process of analysing the first two recorded feedback sessions, in relation to *finding conviction*. The meaning behind each of the labels in table 8.5 will become more apparent through the story of episode three, as will their relationships with the notions of agency and ownership, which form the basis of the episode. I begin by exploring what *nurturing a sense of agency* has come to mean to me through researching how I am becoming a mathematics teacher educator. A key to the column headings in table 8.5 can be found in table 8.2 (page 151).

Label: *Should/ought/had to*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Could, choice, multiple possibilities.	Using *should* or *had to* in relation to teaching decisions	* Flagging the use of *should* etc and suggesting shift in mode of speaking. * Opening to the wider group.	- Supporting an awareness of choice, a sense of agency.	Suggesting a lack of choice and limits a sense of agency. Potentially flagging an issue of ownership.
Label: *Speaking for others*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Speaking for ourselves only.	Teacher making a claim beyond themselves, for example, about a group of individuals.	* Flagging up when a global claim is being made. ** Setting up "we only speak for ourselves" from the beginning.	- Avoiding unhelpful/limiting assumptions.	Limiting the agency of others (as opposed to nurturing).

Label: *Inner/outer*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Not perceived as external.	Referring to a factor (from an external source) that could be interpreted as influencing their practice, or a perceived characteristic of an individual or group of individuals.	* Reporting back on an observation to force a different awareness. * Prompting to avoid unhelpful labels.	- Accumulating alternative positions. - Provoking new awarenesses. - Challenging a preconception/ assumption. - Supporting a process of making it your own.	Can limit expectations of self and others. Moments where a participating mathematics teacher referred to a factor (from an external source) that could be interpreted as influencing a teacher's practice.
Label: *Teaching issue*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
An observable behaviour. A teaching philosophy.	At basic-level, a meaningful problem encountered in practice. Sometimes being posed as a question, sometimes as a desired outcome.	* Flagging a potential issue. * Working on any potentially ambiguous phrases or terms (e.g., out of comfort zone). * Formulating an issue that others might recognise. * Working with the teacher(s) to formulate an issue that a particular moment or set of moments is speaking to. ** Each person having time to bring to mind a moment of discomfort that might clue an issue.	- Supporting new practices. - Developing a sense of choice. - Supporting the recognition of new categories. - Taking ownership of choices and practice.	Whose issue is it? This category includes teaching issues that do not come from the group members themselves, one aspect of this category is about ownership. Important to agree issues, and that they are consensual (understood in mutual terms).
Label: *Teaching strategies*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Not at the level of behaviour. At basic-level.	Describing a purpose/action (usually in relation to a particular issue, or to achieve something specific).	*Opening to wider group, to gather multiple strategies around a particular issue or idea. * Prompting for more and different potential strategies, and associated actions. ** Sharing the function of gathering potential teaching strategies and actions.	- Developing a sense of agency (i.e., many effective strategies). - Developing new basic-level categories.	Can come from others but relating to an established teaching issue.

Label: *Inclusion*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Absence of participation	Lack of verbal contribution from members of the group	* Inviting comment from those who have not yet spoken. ** Setting up an expectation of having time (limited amount of time, order of speaker established in advance, not waiting for own time to make comments).	- Including all viewpoints - Providing each member with an opportunity to articulate their own issues/perspectives.	Linked to ownership. Being actively involved in forming ideas as opposed to passive.

Table 8.5: Summary of labels for phenomena in relation to *finding conviction*.

8.3.1 *Nurturing a sense of agency*

By sense of agency, I am referring to an independent capacity to make choices. Agency is about initiating and controlling our own actions. A sense of agency can be nurtured through paying attention to the limitations we may be placing on ourselves and on others, both in the way we view and speak about our actions and experiences, and in terms of imagining future possibilities. As a mathematics teacher or as a mathematics teacher educator, *nurturing a sense of agency* also relates to how we learn to respond in the moment of teaching (mathematics or mathematics teachers), which involves developing an awareness of choice. Becoming a mathematics teacher educator involves a process of nurturing my own sense of agency as well as *nurturing a sense of agency* in others (i.e., the mathematics teachers with whom I work).

One way of *nurturing a sense of agency* in conversation with others (during a feedback session, or a de-brief conversation) is to support a shift from the retrospective "I had to…", which suggests (to self and others) a distinct lack of agency, to "I could…" (Mason, 2002, p. 88), which instead communicates a sense of choice and of future possibility. In section 8.3.2, I briefly explore the phenomenon that I have labelled *should*/*ought*/*had to*, a phenomenon that I have observed in conversation with a variety of mathematics teachers, which was brought to my attention during the BSRLM conference (see glossary, page 291) in March 2017 (see section 8.1.3, and appendix 3c, page 368).

8.3.2 From *should* to could

Pimm (1993, p. 28) explains that one of his purposes as a mathematics teacher educator working with teachers of mathematics is for them to move away from asking the question, "what *should* I be doing in my classroom?" towards the more enabling "what *could* I be doing?" Use of the word *should* suggests the existence of a single, most effective response, given any particular situation. I can think of a few situations where there is a definitive course of action, where *should* is completely appropriate. For instance, you *should* report every instance of a safeguarding (see glossary, page 291) issue. Generally, however, in the teaching of mathematics there are no single solutions, no blueprints to follow, classrooms are just too complex for that. Take the following extract from Sam (repeated from episode one, section 8.1.4 [session 1, 00:04:33]) as an example of the phenomenon that I have labelled *should*/*ought*/*had to*. I have used bold type to highlight instances of *should* and *had to*:

> Sam: I don't think I had made a good enough job of making sure they understood the structure. On reflection I think **I should** have done something much simpler first. **I should** have gone for something much simpler so they understood the idea before throwing in some quite horrific maths on them […] So I think **we probably have to** teach the structure with something slightly simpler and then, and then encourage them to stay a little bit out of their comfort zone but not to the extent where they are completely freaked out […] I think **I should** have allowed more time and I think **I should** have actually unpicked each one of the calculations but as it was the class were forming that kind of horrible, I get it, I really don't get it, and I'm really cheesed off that I don't get it. So **I had to** put some emergency repairs in place.

Given Sam was the first teacher to offer a retrospective account, an intervention at this point in the session could have established some conversation norms. For instance, at each moment where one alternative was offered (e.g., "I **should** have done something much simpler first", or, "I **should** have allowed more time"), simply prompting a "what

else?" may have been enough to provoke further alternatives, or "there are no shoulds" could have shifted Sam's mode of speaking. I consider both of these potential interventions as related to *nurturing a sense of agency*. Interestingly, Sam is one of the only participating teachers to suggest alternative ways of acting in the moment of teaching during her reflections. Despite the fact that she uses *should* and *had to*, early on (Sam is the first participating teacher to speak during the first feedback session) she models a way of using her time with the group, to identify potential issues and suggest possible different approaches (i.e., *teaching issues* and *teaching strategies*, see section 8.3.4, page 211). Other teachers in the group tended to use their time differently in the first two feedback sessions, mainly reporting back on successes. Reporting on successes is a valuable use of time that can provide other members of the group with new ideas, and potential new basic-level categories. In some cases, reporting on successes resulted in new insights being developed, or at least shared. A new insight generally followed a *detailed description* (for an example, see section 8.2.2, page 178). What might have benefitted more of the teachers would have been to ask them to share any uncomfortable moments (i.e., moments of *dissonance*), which may have provoked a mode of speaking closer to Sam's in the above extract. In sections 8.3.3 and 8.3.4, I explore a process for supporting teacher learning through conversation with others, to enable a mode of speaking that is closer to Sam's, a process of establishing *teaching issues* and associated *teaching strategies*.

8.3.3 Purposes as basic-level categories

In episode one, I explored the relationship between intention and action, and how, through becoming attuned to moments of *dissonance*, it can become possible to respond in a more thoughtful way, by *using dissonance* to signal a moment of choice. In episode two, I briefly introduced Rosch's (e.g., Rosch et al., 1976) theory for how humans categorise the world, focussing particularly on what Rosch refers to as "basic level categories" (Rosch et al., 1976, p. 385) as those that are linked directly to action. According to Varela et al., (1991), basic-level categorisation is the point at which "cognition and environment become simultaneously enacted" (p. 177), where there is a

coordination between form and function, and where distinctions are perceived as "affording certain kinds of interactions" (p. 177). This coordination of form and function could explain why so many of our behaviours are experienced as automatic, i.e., what we perceive is inseparable from the associated action. For example, *sitting* on a *chair*; *drawing a diagram* when *encountering* a *geometry problem*; or *assisting a child* when *a child is stuck*. There may be a range of associated actions, for any basic-level category; I might, for instance, find myself *closing my eyes and visualising* a geometry problem, rather than drawing a diagram.

The example, *a child is stuck*, is a particular type of basic-level category that relates directly to teachers, that Brown (Brown, 2004, p. 3; Brown, 2006, p. 1451; Brown & Coles, 2000, p. 169; Brown & Reid, 2006, p. 181; Reid, Brown & Helliwell, 2017, p.440) refers to as "purposes":

> [Purposes] are guiding principles that structure student teachers' learning from their own experience. Purposes are significant for researchers, teachers and teacher educators in that they provide a level of description that allows an individual to see whether they are acting effectively or not and can lead to changing her/his behaviours. (Reid, Brown & Helliwell, 2017, p. 434)

My process of researching has, in essence, been a process of developing my purposes in relation to becoming a mathematics teacher educator. Each label that has emerged from the process of data analysis, both at the episode-level (e.g., *finding conviction*) and at a more micro-level (e.g., *teaching issue*), are labels for some of my purposes. In section 8.3.4, I explore two such purposes that I have labelled *teaching issue* and *teaching strategies.*

8.3.4 *Teaching issues* and *teaching strategies*

As a mathematics teacher, I am presented with a moment of choice every time I notice a child has their hand up. I might choose to assist them directly, in fact, assisting a child who is stuck might be my usual course of action. I could, however, decide not to assist them and consider doing something else instead. This might sound obvious, yet, in the

moment of teaching, when there is so much going on, our usual automated responses are likely to dominate. We tend to do what we have tended to do (see chapter six for a related discussion). Through *staying with the detail* of a moment (or series of moments) of *dissonance*, involving children putting up their hands, I can develop new basic-level categories through establishing *teaching issues*. For instance, a *teaching issue* might emerge from the process of *staying with the detail* that is something like "how to deal with a student who is consistently putting their hand up and asking for help". Having established this as a *teaching issue,* it can then be possible to collect a range of potential *teaching strategies* and associated actions that could be used next time a teacher encounters the same issue. One potential teaching strategy might be *encouraging independence* and associated actions could be saying to the child, "just have a think about where you might be able to start, and, once you have come up with something, let me know" or having a class discussion, with a prompt "what do we do when we get stuck?" If *encouraging independence* becomes a new basic-level category for a teacher (one of the teacher's purposes), then the next time they encounter a child who is stuck, they may have a range of possible actions to draw upon. In this moment of choice, a new practice may be observed, triggered in relation to *encouraging independence.* Different children performing the same action (e.g., putting their hand up) may trigger different responses, depending on existing basic-level categories. Over time, teaching a class, it is possible that new basic-level categories emerge in relation to different children (we might call this differentiation), thus responding differently to different children putting their hands up. Having an awareness of choice, in the moment of teaching mathematics, or in the moment of working with mathematics teachers, is the result of nurturing a sense of agency.

Teaching issues can arise through *staying with the detail* of a teaching experience. The process of labelling can be a fruitful way to develop new basic-level categories in relation to teaching mathematics, which can lead to new and different ways of seeing in the mathematics classroom. Once a new *teaching issue* has been established (which includes assigning it with a label), associated potential *teaching strategies* can then be gathered that may in turn become new basic-level categories (or "purposes"). The process of gathering potential *teaching strategies* and associated actions, is especially effective

when others are invited to contribute, in order to accumulate a range of possibilities, beyond those that are available to any one individual. Through a process of dwelling in the recorded feedback sessions, I observed several occasions where a participating mathematics teacher articulated a *teaching issue* (*teaching issues* are highlighted in bold type). I recognised, in those moments, missed opportunities to pause the conversation, to support the speaker in establishing one or more potential *teaching strategies*, as a way of bringing new basic-level categories into existence. For instance, near the beginning of the second feedback session [session 2, 00:12:33], when Joe shared something he had been thinking about in his teaching:

> Joe: I would love to explore **how we can get students to express their reasoning in different ways**, so maybe pictorially, things like that. It's a real hurdle at the moment.

A pause at this point, to get Joe to unpick a little more what he would like to achieve and what exactly he was seeing as the "real hurdle", could have led to some potential *teaching strategies* in relation to "how we can get students to express their reasoning in different ways" For some teachers, this *teaching issue* may have led to potential *teaching strategies* that several participants might recognise. Having identified the potential *teaching strategies*, we could then have worked together as a group by accumulating stories and associated actions in relation to those potential strategies.

In a different example, this time from towards the end of the second feedback session [session 2, 00:59:44], Simon begins with a *teaching issue*, "how open do you make things in order to facilitate the students' reasoning, and what is too open?", which he follows with a description of an activity that he set up with one of his classes:

> Simon: I did forty percent of fifty pounds equals fifty percent of forty pounds. I put that statement up, and then I walked around. I had a quote board, so anyone that said anything interesting, I just wrote their name on the board and I used that as a discussion point. Out of that, we had a few things that came out, like, "oh it works for that, but what about if it's thirty percent of sixty pounds, or something

else?" So, they changed the numbers. They asked, "what about if it's odd?" "Does it matter of it's odd?", "Does it matter if it's even?", "What if I double one value and halve the other, does it still work?" **Then it just led off to lots of other really nice questions. But I hadn't deliberately structured a lesson like that before, and just that open, and we kind of lost our way a little bit**.

In listening to Simon's account, I am curious about what happened in the moment where he and his students lost their way, or what happened leading up to that moment. If I had paused the conversation and encouraged Simon to *stay with the detail* of the moment he and his students lost their way, perhaps he would have been able to establish some potential *teaching strategies* (or purposes, such as, *working on student generated questions*) from which we could have then deliberately gathered a range of associated actions (such as, *working together to try to group the questions*; *rephrasing questions as mathematical conjectures; inviting comments on the range of questions; choosing which question for the class to work on;* or *allowing students to choose a question to work on*) that Simon (and others) may have been moved to try in the future.

In re-listening to the following extract from Lucy [session 1, 00:58:07], which occurred near the end of the first feedback session, I identified multiple *teaching issues* (highlighted in bold type) each of which we could have worked on as a group alongside Lucy. The phrase "mixed ability" is explained in the glossary (see page 291):

Lucy: My initial thought was **when do I jump in and how long do I leave them to suffer** […] **What I find challenging is knowing when to step in**. Because it's mixed ability. For the weakest kids I probably leave them for a few minutes, and I usually put a timer on the board, so I know how long they've had thinking about it. I find it really difficult not helping them. I tried to break it down and give hints to those that were stuck, to try to get them to recollect how they would do it. We were talking earlier about different methods of doing one question, **I want the students to give me more than one method, but typically at the end of the lesson, and only when I know**

everybody has understood. Otherwise, I feel that it makes them a lot more confused than they would have been. If you have taught them one way of doing something, and they are still grappling with that method, to have somebody else come and show another way; "I haven't got my head around this way yet, and you are showing me something else". I feel more confident when a student comes to the board to offer a different method once most of them have understood the first method. **They find it hard to explain what they are doing, to put it into words, some of them are not fluent enough to explain**.

A few moments later [session 1, 01:01:57], Lucy began to share what she might have done differently during the lesson she had been describing. In essence, she begins considering alternative actions:

Lucy: If I had more time, I would have made the questions a bit wordier and possibly put a few more hints on paper for the least able kids, because it was mixed ability. To stop me from actually going over, and instead just encouraging them to read the hints.

Having marked Lucy's comments above with the labels *teaching issue* and *teaching strategies* another category also emerged. I used the label *inner/outer* to mark moments where a participating mathematics teacher referred to a factor (from an external source) that could be interpreted as influencing a teacher's practice and *potentially* limiting their sense of agency. For instance, a school-wide initiative (such as "a focus on literacy and numeracy", [session 2, 00:11:20]), or a perceived characteristic of an individual or group of individuals (such as "mixed ability", as referred to in both of Lucy's extracts, as well as by others). Although these examples might be recognised as having originated from an external source, they are examples of existing basic-level categories for the teachers involved. In the first example, a school-wide initiative such as "a focus on literacy and numeracy", Alex and Ellen reported on changes in their practice, i.e., a new basic-level category, in relation to this whole-school initiative. One interpretation of this kind of change in practice is that it has been *imposed*, yet it is not possible to *impose* a change in

practice. Instead, I have come to see this instance as an example of *the importance of context* (see discussion in section 8.4.3, page 234), which need not involve limiting a sense of agency, rather, it can involve a process of making sense of and enacting new practices that have been introduced by somebody other. As discussed in the previous episode (see section 8.2.6, page 193), certain labels need to be avoided since they "can block access […] to alternative acts" (Mason, 2002, p. 51), i.e., limiting a sense of agency. I have heard myself, on several occasions, saying to prospective mathematics teachers, "try not to label the child as badly behaved", or "try not to label the group as low ability", or "try not to tell yourself you cannot get the children to listen". I could have addressed the "mixed ability" label that I now see as limiting Lucy's sense of agency. A label such as "mixed ability" or "low ability" is a label for an imposed social structure, which need not become a determining factor in terms of what Lucy sees as possible in her own classroom. A label such as "mixed ability" can also suggest the lack of a sense of ownership when it is viewed as external, in other words, something that does not belong to us. However, "mixed ability" has become a basic-level category for Lucy; it is what she knows and thus her practice is determined by that. In this instance, I could have attempted *fostering a sense of ownership* by addressing the use of these kinds of labels and working at establishing different, more enabling, basic-level categories. In section 8.3.5, I explore the process of *fostering a sense of ownership* in relation to becoming a mathematics teacher educator.

8.3.5 *Fostering a sense of ownership*

By sense of ownership, I am referring to the *feeling* that my actions, including my thoughts, belong to me. Mathematics teachers can find themselves in situations where values, viewed as external to their own, are seen as being imposed. A striking example of a potential new practice, seen as being imposed, was illustrated in chapter four, where Sam spoke about a situation in her school where she had been told that teachers should spend less than ten minutes talking at the beginning of the lesson (see page 66). When a new practice is seen as being imposed, is can act to limit the sense of ownership that makes teaching such a fulfilling occupation. Through the process of analysing my

conversation with Sam (section 4.1, page 57) I came to see myself, in that conversation, as trying to impose my values, to change Sam's practice in ways that reflected my own desires. Over time, I have come to recognise any attempts at imposing a way of being in the classroom as both unethical and futile. There is a big difference, however, between imposing new practices and introducing new possibilities, which will either be ignored, or will be put into practice (i.e., become new basic-level categories).

The final extract of dialogue in this episode demonstrates to me the importance of feeling a sense of ownership. On the surface, it might seem in this moment of the feedback session as though we were gathering potential *teaching strategies*; in reality, however, since Lucy's *teaching issue* was never quite established, we were maybe playing a bit of a guessing game. One thing I have learned, from *staying with the detail* of this section of dialogue, is just how important it is that a *teaching issue* belongs to the person whose practice is in focus, to support the possibility for the emergence of new basic-level categories (or "purposes", which may lead to new practices in future teaching situations). The initial speaker is again Lucy. This time the extract of dialogue is taken from the second recorded feedback session [session 2, 00:26:11]. Lucy has just given a three-minute account of a lesson she had taught involving ratio.

Lucy: There is one child in my group that doesn't like answering questions, and I don't, she has really low self-esteem and I'm not quite sure. I know we propose no opt out, but I don't know how to get her to answer a question without, sort-of, …

Tracy: So, the issue is that she won't put up her hand to answer a question?

Lucy: If I ask her to answer a question, and she feels that she is going to get it wrong, she'll say no I'm not answering it. Even though the person next to her is quite confident and would give her a hint. She still doesn't want to.

Ellen: Could you avoid only asking one person to answer. If they've got mini whiteboards and they all write an answer down, then she's answering it along with everyone else?

Lucy: Yeah, I guess so yeah, but I think in her head she's just convinced that she's going to get it wrong even though she's got the ability.

Alex: What's wrong with getting things wrong? Just celebrate the fact that people are getting things wrong. That's an opportunity to learn.

Lucy: Yeah, yeah, that's what I say.

Ellen: That's going back to my old, oh what's the program called? You know the one, I used to use it.

Alex: Was it QI[14]?

Ellen: QI that's it! Where the wrong answers come up. I was hoping someone would say that. I used to use that a lot. I've forgotten about that.

Lucy: Oh yeah, absolutely. Like with the fraction misconception, when someone says something like that, I say, "oh thank you, because this is the mistake that people often make", and then I celebrate it. I even make mistakes in front of them, so I try to celebrate mistakes.

Alex: I use Joe's Nan, and they don't even know who Joe is. It's not Joe it's his Nan that gets things wrong[15] [laughter].

Lucy: But, yeah.

Paul: Maybe if it's about self-confidence and it is a serious issue for that student, then if you have a particular question, or you've planned a particular question that you want to ask that student, then you give her lots of time. Say to her, at this time in the lesson, I'm going to ask you this, so have a go on your whiteboard, and we can have a look at it beforehand. If she's worried about being wrong, then that conversation is maybe between you and her, but maybe that would

[14] QI stands for Quite Interesting. It is a popular British comedy panel show where points are awarded for interesting answers and deducted for wrong and obvious answers.
[15] In the first feedback session, Joe told the group that he often refers to his fictitious nan in reference to making mistakes.

> actually help build up her confidence of you being her teacher, and, you know, enable her to speak.

I cringe each time I revisit my contribution (did I really cut her off like that?). Just as Lucy was getting to the articulation of a *teaching issue*, I try to force it, to put it in my own words, to impose my version of the issue. It strikes me, from that point on in the conversation, we never really get to the bottom of what the issue was for Lucy. Potentially, if Lucy had been given the space to establish her own *teaching issue*(s), we could have worked more effectively at supporting her in gathering a set of potential *teaching strategies* that fit with her sense of the issue. As it was, there was a lot of advice being offered, yet somehow, it was never quite in the right place (at least that is my sense based on Lucy's responses). In hindsight, I doubt that Lucy gained anything much from that part of the conversation. To own a *teaching issue*, does not mean being the first to have encountered it. What seems important is that the issue is one that is recognised by the teacher (or teacher educator) so that any *teaching strategies* that arise in relation to that issue, have the potential to become new basic-level categories (or "purposes") which trigger associated actions. Owning a teaching issue means it has arisen in your own practice, it is something you recognise and is thus personally meaningful.

8.3.6 New purposes as a mathematics teacher educator

A phrase, which has become a label for one of my own basic-level categories, is one that I have written about in a previous chapter (see section 4.1.1, page 63). The label I am referring to here is *there is not one way of teaching mathematics.* The label itself is not original, but it is the label I now associate with one of my basic-level categories or purposes. As one of my purposes, *there is not one way of teaching mathematics* manifests itself in different ways and in different contexts. For instance, at a macro-level, *not one way of teaching mathematics* is reflected in the design of the course for prospective mathematics teachers, in that the course combines multiple different models for running a classroom (groups, pairs, individual, whole class, carousel, etc), multiple different modes of working on mathematics (fast, slow, extended, short; pen and paper, digital,

etc), and multiple different activities. At a more micro-level, during PGCE subject sessions (see glossary, page 291), *not one way of teaching mathematics*, is actioned through drawing on the experiences of the group, as opposed to offering my own experiences, and, during de-brief conversations, drawing on the experiences of the school-based mentor (see glossary, page 291). What was somebody else's label, *not one way of teaching mathematics*, became a label for one of my purposes that triggers a range of associated behaviours, depending on the specifics of the situation at hand. Though the label is shared, who is to say whether it carries the same meaning for those of us who use it. Labels for purposes do not come full of meaning, meaning emerges in and through our interactions with the environment.

Sometimes we find ourselves in situations where our actions trigger a sense of *dissonance*. If we mark those moments of *dissonance* and engage in a process of retrospective analysis in relation to what took place in that moment, one possibility is that we develop new basic-level categories, or purposes, that enable us to act differently the next time we recognise a similar situation. The label we assign, to that new basic-level category, can be an existing label that we have been aware of, that we have heard used by others, becoming meaningful when it enables recognition of a new basic-level category (i.e., enabling new behaviours). One behaviour that I adopted early on, in relation to *no one way of teaching mathematics*, was to only speak about the classrooms of others, as opposed to my own. As a new mathematics teacher educator, I found my experiences of other classrooms was limited, yet since then, I have accumulated many stories from classrooms that are not my own, to the point where I no longer know what I would do in my own classroom. Recognising that I may no longer know how I would teach mathematics (as opposed to recognising that I could no longer teach mathematics), brings with it a sense of sadness, yet, at the same time, feels liberating.

Finding conviction as a mathematics teacher and as a mathematics teacher educator involves *nurturing a sense of agency* and *fostering a sense of ownership*. A sense of ownership can be fostered through a process of establishing *teaching issues* that are, for the teacher involved, personally meaningful in that they provide a description of a situation that the teacher recognises, having encountered that situation in their own

practice. A process of gathering, with others, potential *teaching strategies* (potential basic-level categories) and associated actions in relation to a particular *teaching issue*, can lead to the emergence of new potential basic-level categories that, if recognised can consequently trigger new effective behaviours (i.e., become actual basic-level categories or purposes). When a new basic-level category has emerged for the teacher, a range of new behaviours can come to mind in the moment, which nurtures a sense of agency. A sense of agency can also be nurtured through avoiding the use of labels that act to limit what we perceive as possibilities in ourselves and others. As a mathematics teacher educator, I am looking to *nurture a sense of agency* and *foster a sense of ownership* both in myself and in those teachers I work alongside. In the final part of this episode, I summarise *finding conviction* as a methodological dimension in relation to three methodological levels of becoming a mathematics teacher educator. Then, to conclude this episode, I share my overarching guiding principles (explained on page 149) for *finding conviction* as I have come to see them.

Finding conviction **in relation to the process of learning to teach mathematics and mathematics teachers:**

Identifying *teaching issue*s through marking moments of dissonance, and then retrospectively staying with the detail of those moments.

With others, gathering a range of potential *teaching strategies* for each *teaching issue.* Responding thoughtfully in a moment of choice, by bringing to mind a range of potential actions.

Avoiding developing and using labels for generalisations that characterise individuals and groups of individuals, and only speaking for yourself. Avoiding modes of speaking about teaching that are inherently limiting.

Finding conviction **in relation to *researching* how I am becoming a mathematics teacher educator**

Supporting a process of seeing more and differently by seeking multiple interpretations of the same data.

Supporting the emergence of new basic-level categories by integrating theory with the process of data analysis. Returning to the data repeatedly, each time looking for a different reading of the situation.

Allowing labels for observed phenomena to change as new distinctions are made. Staying open to new ways of seeing data that has already been categorised.

Finding conviction **in relation to a way of working with mathematics teachers**

Fostering a sense of ownership, through supporting others in establishing their own *teaching issues* and avoiding the urge to impose one's own *teaching issues* on others.

Nurturing a sense of agency through organising the gathering of potential *teaching strategies* and associated actions, supporting the emergence of new basic-level categories. Flagging modes of speaking that act to limit possibilities.

Overarching guiding principles for *finding conviction*

Nurturing a sense of agency and *fostering a sense of ownership* both in self and in others.

~~~~~
~~~~~

Episode Four

8.4 *Making it real*

In this episode, I explore the fourth methodological dimension of becoming a mathematics teacher educator that I am calling *making it real*. In the previous episodes, I have focussed primarily on the process of formulating ideas (during moments of retrospective analysis, and in conversation with others) that have the potential to be translated into new practices. In episode three, for example, I explored the process of establishing *teaching issues* and associated *teaching strategies*. *Making it real* is about going beyond the initial phase of formulating ideas to considering mechanisms that specifically support the move to enacting new practices (i.e., the process by which *potential* basic-level categories become *actual* basic-level categories) through "expanding the space of the possible" (Davis, 2004, p. 184). Expanding the space of the possible involves developing a repertoire of possible actions that may be triggered in any given situation.

Since we are structure-determined systems, to act in new ways, there must have already been the *potential* to act differently. Learning is thus about expanding the range of new actions from an existing set of possibilities, it is a process of realising new possibilities. In this episode, I have adopted the phrase *real*-ising to signify this process.

Part of *real*-ising is the process of grounding ideas in reality. One way of grounding ideas is through *staying with the detail* (episode two) of teaching experiences, comparing and contrasting those experiences with the experiences of others, seeking resonance and dissonance. It is through paying attention to the "concrete particulars of situations" (van Manen, 1991, p. 208) and keeping accessible the "pragmatics of particular instances" (Bateson, 2000, p. 142) that our actions can be modified for every potential instance, even those actions that we experience as automatic. When conversations about mathematics teaching occur at a more theoretical or abstract level, this mode of speaking can be enlightening, but not necessarily lead to new ways of being in the classroom. To be transformative, ideas need to be made *real*, they need to become new basic-level categories.

In many ways, this study is about the process of uncovering my purposes as a mathematics teacher educator, alongside the related process of *real*-ising (which, as a recursive process is one way of conceptualising my becoming). It is the process of *making it real*, of *real*-ising, that is a focus of this episode, both in relation to the mathematics teachers with whom I work, and in relation to my own practice as a mathematics teacher educator. Table 8.6 (also in appendix 3d, page 369) is a summary of the labels that emerged during the process of analysing the first two recorded feedback sessions, in relation to *making it real*. The meaning behind each of the labels in table 8.6 will become more apparent through the story of episode four. I begin by exploring the process of grounding ideas in practice before moving on to discussing the process of *real*-ising. A key to the column headings in table 8.6 can be found in table 8.2 (page 151).

Label: *Asking why?*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Asking what or how.	Asking for the *cause* of a situation.	* Guiding use of what/how questions as opposed to why. * Focusing on the detail (what). * Seeking similarity across a collection of stories, being explicit about features of specific acts.	- Avoiding the tendency to explain away or justify acts and to classify a situation prematurely. - Informing ways of acting differently in the future (as opposed to hypothesising about the cause of the issue). - Sustaining the energy that might have arisen in the detail.	
Label: *Unresolved*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Continuing until a resolution is found.	Teacher changes topic of conversation before getting to a resolution (sometimes suddenly).	* Pausing the speaker and bringing the conversation back to the previous unresolved issue. * Checking if the issue has been resolved (as far as is possible)	- Seeking resolution. - Realising possibilities.	Linked to *drift.*
Label: *Commitment*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
An unformed idea or strategy.	Teacher visually commits to trying something new or doing more of in their future practice.	* After a phase of sharing ideas, prompting a distillation and explicit sharing of commitments. * Visibly collecting commitments from the group as session proceeds.	- Realising possibilities. - Grounding in practice.	Formulating commitments could be done personally, could be shared, could be collected visibly (either at the end or steadily throughout).
Label: *Importance of context*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Not considering context.	Explicit (or implicit) references made to context.	* Being explicit about the importance of context. * Working with teachers to explore how specific teaching strategies and associated actions could fit with context.	- Realising possibilities. - Grounding in practice.	The importance of school context.

Label: *Drift*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Staying focussed on a particular point (e.g., collecting strategies, detailed descriptions)	Dialogue (as opposed to monologue) where ideas emerge from one participant to the next.	* Asking group members to summarise the main points that stay with them from the conversation (speak or write) and to translate that into practice.	- Grounding ideas. - Putting ideas into practice as opposed to holding them at a distance.	A lack of focus in the conversation, or the subject of the conversation moves from one point to the next, each point is connected, yet it *drifts* along without resolution (related to the label unresolved). Different to a slide (I use slide in relation to monologue) – drift in relation to multiple people in dialogue.
Label: *Research opportunity*				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
No research opportunity.	Teacher articulates a problem that they have not got to the bottom of or asks a question that could form the basis of practitioner research.	* Suggesting a small-scale research project.	- Encouraging practitioner research, critically examining one's practices.	Motivating practitioner research. Looking for opportunities to reframe what a teacher says as a potential research question.

Table 8.6: Summary of labels for phenomena in relation to *making it real*.

8.4.1 Staying grounded

A phenomenon that I have encountered during a few de-brief conversations with prospective mathematics teachers is a tendency for the prospective teacher to talk about the lesson (that they have just taught and that I have observed) in a way that lacks focus. Usually this involves the prospective teacher shifting quickly from one moment or issue to the next in a burst of energy during the initial stages of the debrief-conversation. When that situation presents itself, one response that I have developed is to slow the prospective teacher down, to focus on one moment or issue and to seek resolution (usually through establishing one *teaching issue* at a time, gathering potential *teaching strategies* and associated actions), before moving on to the next moment or issue. Occasionally, however, I have noticed that the act, of slowing the conversation down, can dissipate the energy that comes in those initial moments of the de-brief conversation. The energy is then lost along with the awarenesses that seem to have been held within

it. Getting beyond the first moment or issue can then become more challenging, as if the moment is lost.

An alternative action, as a mathematics teacher educator, is to facilitate a space where this initial energy can be utilised by the prospective teacher, who, whilst in flow, may share a range of issues that have stayed with them from the lesson. In this case, I might aim to capture (in my notes) what is being said, to then support a slower, more focussed phase of the de-brief conversation, after the initial burst of energy. Having a structure to refer to during de-brief conversations (which basically consists of locating a moment from the lesson that caused *dissonance*, establishing the associated *teaching issue* and then gathering related, potential *teaching strategies* and associated actions) has enabled me to establish a set of behaviours that I am able to enact with *conviction*. As I am becoming a mathematics teacher educator, however, I am learning when to stray from such structures, to allow my intuitions to determine my course of action, intuitions that have been buttressed though a process of researching my own practice.

A related phenomenon in the group feedback sessions is one I have come to label as *drift.* I perceive *drift* as a lack of focus during the conversation (for instance, without the attachment of an established teaching issue), a progression of ideas, where each point is connected to the next, but the overall sense of the conversation seems to *drift* without reaching resolution (and thus related to the label *unresolved*). The next section of dialogue is from towards the end of the second feedback session [session 2, 00:55:48] and demonstrates the notion of a *drift*:

> Ellen: I always think that when people, and it is changing the subject slightly, but when kids have to have scribes in exams, that would disadvantage me terribly. Something happens between here [pointing to her head] and here [holding up a pen] that doesn't involve the rest of me, and if I try to verbalise it, it's sometimes when you're doing a question in class and you haven't come across the question before and kids ask you to explain it I have to do it myself first, so that it goes straight from my head to my pen, without saying

anything, and then once I've got the answer I can work backwards verbalising.

Tracy: The saying gets in the way?

Ellen: Yeah absolutely.

Beth: For me, that's where this idea of planning questioning comes from, in that actually we need to do something…

Ellen: But I could not have done the exam using a scribe where the only chance I had to get marks was by speaking.

Simon: It can be harder to speak than to write can't it.

Ellen: Yeah.

Tracy: Yes, it's like something else is going on.

Ellen: And when people have said to me can you tell me, like even my own children they ask me to help them with something. When they used to ask me what to write, I'd say, "oh you have to pass it to me, because I can't tell you what to write". I just have to write it.

Tracy: And for me that is linked to Joe's idea of showing that set of images, it's as if the words can sometimes get in the way. Why do we need words when we have direct access to something? Students don't necessarily need to verbalise things for themselves, because the reasoning is there, within the structure of what you offer them. For me, that is connected to Ellen needing to write for the words to come.

Ellen: Yeah, I just need to write it.

Alex: We've got one teacher in our department who is Charlie Chaplin[16], he hardly ever speaks. He just does all his stuff on the board, because, same thing, he says that if you're having to listen to the conversation, you're not actually thinking, you're just listening. So, he'll do lots and

[16] Charlie Chaplin was an English comic actor, filmmaker, and composer who was best remembered for his recurring silent movie character.

lots of examples in silence and then when he sets the students off on their work, the kids that don't get it keep looking at the board, and the kids that do continue working. He'll just go through example, example, example, on the board, and then he knows when the kids look away that they've got it.

Tracy: They've got it.

Alex: Yeah, and you don't have to disrupt anyone, everyone's working at their own pace. They can get as many examples as they want.

Simon: It's like if you're happy, ignore me, if not, keep listening until…

Alex: But they don't have to listen because, if you listen, the kids get distracted whereas he just doesn't talk…

Sam: But ignore me doesn't necessarily mean don't listen does it, it can mean ignore what I'm writing.

Alex: Yeah.

Beth: The only thing that I'm kind of thinking there is, is it very procedural? Are they following a method and what happens when you want to ask a question? Does it follow the same pattern? Is that where your questioning needs to come in because you haven't checked for understanding of the concept, you've checked for replication of the process.

Tracy: But this [pointing to Joe's images] doesn't feel like replication of a process.

Beth: No that doesn't, so it depends on what you're doing.

Tracy: It's what it is you're doing?

Beth: Yeah, what's happening.

Alex: Yeah, yeah.

By labelling this section of dialogue as *drift*, I do not mean to suggest it was in any way a poor use of time during the feedback session. In fact, in my experience, dialogue is a

hugely effective way of developing ideas and finding meaning. What the phenomenon of *drift* makes most apparent to me is the way a conversation can potentially be about such different things, to different people. For Ellen, we might have been talking about *embodied knowing*. For Alex, it may have been about *using silence*. For Beth, it could have been a conversation about *conceptual understanding versus replication of a process*. All three (italicised) examples are labels that I have assigned, which means they represent *my* basic-level categories, they are only possible basic-level categories for the teachers themselves. I suspect that the conversation was about something different for each member of the group. From an enactivist perspective, we each bring forth our own world of significance, so it is not surprising that each person will experience the same conversation differently. We each have our own basic-level categories that determine the way we experience the world. One of my basic-level categories (or "purposes") in relation to teaching mathematics is *providing direct access to a mathematical concept* (evident in my third turn in the conversation, "Why do we need words when we have direct access to something?"). In terms of grounding the ideas in each person's reality (i.e., in direct relation to their teaching practices), one strategy would have been to capture the ideas (in the form of notes) as they emerged, to facilitate a more focussed phase of working on some of those ideas in more detail. A second strategy would have been to ask each member of the group to make a note of something they had been struck by during the conversation; what that might mean in terms of their practice, perhaps in the form of something they want to commit to trying for themselves. Of all the participating teachers, Sam was most forward in sharing her commitments. In the second feedback session, she vocalised three commitments within a period of twenty minutes: "Oh I love that. I'm doing that" [session 2, 00:22:04]; "So I don't give them the right answer, but actually, in the future, I think I will" [session 2, 00:33:41]; and "Teaching in silence is a really nice idea. I'm going to give it a go" [session 2, 00:41:07]. Recognising, and then commenting on, a commitment having been shared could prompt a further sharing of commitments from the wider group.

The *potential strategies* mentioned above relate to my growing conviction about the value of grounding ideas in reality (i.e., relating to practice), which might be more

commonly referred to within teacher education as bridging the gap between theory and practice. For me, theory need not purely be the type that can be accessed through literature, or that is shared explicitly as part of teacher education programmes; theory can also be the ideas that emerge through analysing experiences. In this form, theory and practice can emerge together in a recursive process of *making it real.* One way of supporting this recursive process is to make actions real, right there, in the moment. For instance, during a de-brief conversation with a prospective mathematics teacher, making an idea real could involve planning for the following lesson, incorporating a new practice that had come up as part of the discussion.

8.4.2 A commitment to what is observable, as opposed to *asking why*

Another way of grounding ideas in reality is not to go too far beyond what it is possible to observe, paying attention to what and how, as opposed to why. Knowing why something has occurred or why somebody might have behaved in a certain way can certainly be a source of resolution and closure. Asking why can give us access to different perspectives, which in turn might be a source of empathy. As human beings, it is likely that we tell ourselves all sorts of things to justify our behaviour, in our quest to reduce dissonance and return to consonance. *Asking why* can prompt the considering of one's actions, and associated purposes, interrogating observed behaviours. So, what is the problem with asking why? Mason (2002) suggests that asking why (in relation to one's practice) can lead to useful analysis, but on other occasions, it can dissipate the energy that might have arisen through attending to the detail of what has been observed. He offers three forms of *accounting-for*: through asking why, that he calls "*why-ning* (whining)"; "*whys-acreing* (wise-acreing, meaning talking about things when you know very little about them)"; and "*why-sening* (wisening or making wise)" (Mason, 2002, p. 41).

Whining (not meant in the conventional sense of moaning or grumbling), refers to a "tendency to explain away or justify acts" (p. 41), to explain why certain behaviours may have been beyond my control, or why I might have had no choice in a given situation.

Whining is therefore related to a lack of ownership and agency (see episode three). Mason (2002) suggests that whining can manifest itself as seeking reassurance in relation to actions, to ask for confirmation that certain actions were at least good enough, if not optimal, given the situation at hand. Wise-acreing refers to "a tendency to theorise about an incident, to classify it prematurely as an example of a general phenomenon before having delved into details" (Mason, 2002, pp. 41-42). An example I now recognise as wise-acreing, occurred near the very end of the first feedback session [session 1, 01:03:10], soon after Lucy had finished giving her account (see section 8.4.4, page 237) of a lesson she had taught. Lucy had suggested that some of her students might have struggled with a different way of working, from what they had been used to. What followed Lucy's suggestion was a set of hypotheses about the underlying cause of the issue, what might be considered as classifying the issue prematurely as an example of a general phenomenon:

Simon: Do you think that could be about school culture, or ability? We did our lesson with quite high ability classes so maybe they were more willing to try, or maybe they didn't have a fear of being wrong, I always try to have a no fear culture.

Ellen: Maybe because they are more used to variety.

Beth: Or how close to your normal teaching practice that is.

Alex: I think they liked it because it was different, didn't they.

Ellen: I didn't think it was that different.

Beth: Maybe that says it all, that it didn't seem different. It didn't seem different to the kids either.

Tracy: It is interesting Ellen that you taught a mixed year seven group and you didn't have the same barriers that you have described Lucy. It didn't feel like there was something like adding fractions that got in the way of the students accessing that task.

Ellen: Maybe it was because the task was slightly easier, or maybe it's because I'm always saying because, or why? Maybe they are used to, how do you know? that's my usual one, how do you know?

Alex: The kids know that if I can't hear them, it is because they haven't responded with a because.

[laughter]

Sam: That's nice.

Alex: So, after a while the kids will say, "oh that's what I need to say, the reasoning behind it". So, again, maybe it is because we already have that established.

Beth It's interesting to consider if certain structures might match up to certain topics. Maybe the structure was just slightly more aligned with the topic in that instance. So, there were just less barriers for the kids to get through.

As opposed to wise-acreing, "wisening" (Mason, 2002, p. 42), refers to the process of seeking similarity across a collection of stories, being "explicit about positive, negative, and interesting features of specific acts" (p. 42), which can inform ways of acting differently in the future. Rather than hypothesising about the *cause* of the issue, an alternative possibility would have been to delve further into the detail of Lucy's situation. By opening up to the group, stories could then be shared that are triggered by the process of *staying with the detail* of Lucy's account, stories that share similarities and may offer alternative ways of acting. By encouraging this alternative mode of feeding back as a group of mathematics teachers, Lucy may have been more likely to commit to trying something new or different.

8.4.3 *The importance of context*

Committing to trying something new or different is a process of *making it real* that involves a bringing together of a present possibility with a future action. In the process

of accumulating *teaching strategies*, it is likely that some strategies will take hold (potentially becoming new basic-level categories), those we can imagine ourselves enacting, and others will not. What is considered possible, or imaginable, is linked to the *importance of context*.

From an enactivist perspective, the common-sense divisions drawn "among individuals and between "persons" and "contexts" must be abandoned" (Davis & Sumara, 1997, p. 116). Context does not exist in isolation; it is not a fixed background in which activity takes place. The structural coupling of organism and environment means that an individual is not situated *within* a context, rather, the individual is *part of* the context (Davis & Sumara, 1997). For mathematics teachers, one consequence of being *part of* the context (a particular class, within a particular school, which is part of a particular education system, and is located in a particular local community, at a particular point in time, and so on) is that every action is performed in relation to that context. Actions (or interactions) are inseparable from the context (which includes the teacher and the students) in which they arise. Put simply, the same action performed in one context would likely trigger a completely different response in another. The actions of the teacher determine the context in as much as the context determines the actions of the teacher. A particular school context will involve numerous factors, some of which could be interpreted as being external to the teacher (for instance, a system for managing behaviour, a new school initiative for improving literacy levels, or a recent Office for Standards in Education (Ofsted, see glossary, page 291) inspection judgement). However, even though those factors may not directly relate to a particular teacher, the changing context (which is itself an active component), will ultimately shape the actions of the teacher. Thus, context is an important factor in terms of emerging basic-level categories or purposes.

On the PGCE (see glossary, page 291), we often talk about *the importance of context* in relation to the prospective mathematics teachers developing potential *teaching strategies*. When the group (of which there are usually around thirty) come together, at different points in the year, to share stories and ideas in relation to a particular *teaching*

issue, we are careful to point out, as tutors, that certain strategies and associated behaviours will be effective in some contexts and not necessarily others; "It depends on the context of the school" we might say. The differences in context can lead to subtle differences in effective behaviours (that a prospective teacher might have to learn when they move from one school placement to another) so subtle that they can be challenging for the prospective teacher to recognise. In other cases, the difference in context might be more apparent and the prospective teacher may need to act very differently from the way they have been acting in their previous school. In either case, the actions performed by the prospective teacher are always done in relation to the context they find themselves part of, the prospective teacher's job is to learn that new context.

During the feedback session, I identified a few examples where context was either referred to explicitly, or implicitly. For example, in the extract below [session 2, 00:05:10], Alex is sharing some of the context of his mathematics department during the second feedback session as part of his reflections on working with members of his mathematics department at school to develop their use of questioning in the classroom:

> Alex: The context is, we have a couple of new members of staff in the department, and a PGCE student as well. As a result of this, we said, when you come to doing your planning, you don't have to draw it out on an A3 sheet like we did in the last workshop. But do have a think about what paths the lesson could take. What sort of questioning might come up and how you could approach that?

For me, this instance from Alex demonstrates the way that a context can be described in terms of a set of basic-level categories (e.g., "new members of staff" and "a PGCE student") that have been recognised as factors determining a particular approach. Context, that is not separate to the individual, could thus be conceptualised as a continuously evolving set of basic-level categories for an individual in relation to a particular set of practices (e.g., teaching). In another moment (previously introduced in section 8.3.4, page 215, [session 2, 00:11:20]), Alex is reflecting on how he has been making changes in his practice:

> Alex: As a whole school there has been a shift in onus on developing literacy and numeracy, so, we've linked the work on this project to that whole school focus. So, we won't just accept an answer of, say, seventy-two. Why is it seventy-two? Can you expand on that? Trying to get the students to use full sentences. I think that initiative has helped the work we have done during project, because there is a wider school emphasis on expanding answers and developing oracy in the classroom.

Alex describes changes in his practice in relation to basic-level categories that have emerged for him in relation to his work on the reasoning project (where a common purpose has been about developing students mathematical reasoning) as well as the whole-school initiative ("expanding answers and developing oracy").

8.4.4 A process of *real*-ising

Mathematics teachers cannot specify what or how their students learn, they cannot do the learning for their students. They can, however, work on themselves and their practices, creating the conditions for their students to learn for themselves. Similarly, as a mathematics teacher educator, I cannot specify (nor would it be ethical to) what mathematics teachers learn about the teaching and learning of mathematics or what they learn about their own practice. All I can hope to do is to create the conditions for mathematics teachers to learn for themselves, which involves supporting a process of *real*-ising new possibilities. Ideas about teaching and learning mathematics will remain as ideas, as possibilities, until they are realised (put into practice), becoming new basic-level categories or purposes. Through researching how I am becoming a mathematics teacher educator, I have come to recognise several mechanisms that support a process of *real*-ising new possibilities. Mechanisms include immediately committing ideas to action (see section 8.4.1, page 227), conducting small-scale classroom-based research projects (see section 8.4.2, page 232), and using writing as a method of inquiry (see section 2.5, page 29). As part of my narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator (detailed in chapter seven), I recognised the

need to formulate hypothetical actions (see section 7.4.3, page 136), which I have continued to do throughout chapter eight, as a way of using my imagination, to "extend [my] feelings […] and to have these feelings "break through" into […] new situation[s]" (Varela, 1999, p. 28). One way of using the imagination to extend inclinations is through inviting mathematics teachers to write a "'lesson play', in which part of a lesson is presented in dialogue format between a teacher and students" (Zazkis, Liljedahl, & Sinclair, 2009, p. 40). Creating a lesson play is akin to my own process of formulating hypothetical utterances in relation to observed phenomena, as part of my process of analysing the recorded feedback sessions, and throughout this chapter.

8.4.5 My process of *real*-ising

As a mathematics teacher educator, I have become curious about observing change in myself and others. I am interested in how it might be possible to observe the process of *making it real*, in relation to the process of *real*-ising. What is happening in the classrooms of participating teachers, beyond the boundaries of the project, is not accessible to me, at this point, although it would certainly make an interesting follow up study. Without access to the classrooms of the participating mathematics teachers, and without returning to them to ask them about changes they might have made as a result of the mathematical reasoning project (which, in any case, would be an entirely different research study), I decided to return to some of the later feedback sessions, beyond the initial two (my primary source of data). In this final part of episode four, I present a section of dialogue from the third feedback session, where I have identified instances of phenomena (and associated labels) that were absent from my analysis of the first two feedback sessions, in relation to my own actions (i.e., they are new behaviours that I did not observe during the first two feedback sessions). A newly observed practice can be interpreted as an instance of a new basic-level category, or purpose, in action. By the time phase two of my data analysis process began, I had developed a range of new purposes as a mathematics teacher educator, many of which I was recognising in the first two feedback sessions (and labelling accordingly) in relation to missed opportunities. It is likely, however, that many of these purposes were not purposes for me at the time of the

first two feedback sessions, otherwise my actions would have reflected them. When I came to analysing my contributions in the third feedback session, I observed some of these new purposes in action, as opposed to missed opportunities for action. I also identified some purposes, that had not been observed during the first two feedback sessions, thus I needed to assign completely new labels. In terms of process, labels were assigned through listening and re-listening to the third recorded feedback session, transcribing my own contributions to the discussion (see appendix 3f, page 406), and then using existing labels (those in blue, e.g., *doing mathematics*) as well as creating new ones (those in red, e.g., *noticing extremes*) some of which appear in the section of dialogue below. It is not my intention here to demonstrate that my actions, in the moment of those feedback sessions, somehow caused more or less fruitful responses in others. Rather, I intend to present instances of new practices (when compared with the first two feedback sessions) and how I interpret those new practices in relation to the emergence of basic-level categories which likely emerged through engaging in the early stages of my data analysis (phase one analysis, see section 7.4.1, page 134). At this point in the conversation [session 3, 00:15:45], Sam is taking her turn to share what she has been doing in relation to work she has been doing with students on proof. The phrase "starter" is explained in the glossary (see page 291):

Sam: My problem with the year nines was how was I going to make it concrete, how was I going to make proof accessible to them. So, for my starter, I just put that outline on the board [Sam holds up a sketch (see figure 8.10, taken from my field notes)] and asked them to shout out numbers, while I put them in the spaces. They had to think about what I was doing.

Tracy: Well, why don't you do this. Do you want to do this with us? *(Doing mathematics)*

Sam: OK then, can I use the board? [Sam takes a few moments finding space on the flipchart. She draws the image from figure 8.10 for the group to see]. So, I asked them to give me some numbers.

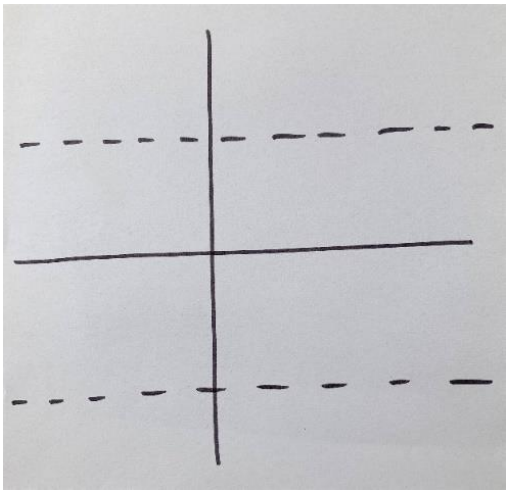


Figure 8.10: A reconstruction of Sam's sketch.

Tracy: Just pretend we're them. *(Doing actively)*

Sam: Yep, OK. So, I'd like you to give me a number. Maybe your favourite number or your house number and I'm going to put it in one of these boxes and I want you to work out what the boxes are. If you think you have got it, you say, no Miss, I think you have put it in the wrong box.

Maria: Six.

Sam: Six, OK I am going to put that there [Sam starts adding numbers onto the image on the flipchart, see figure 8.11].

Joe: Zero.

Sam: Zero, I'm going to put that there for a minute.

Simon: Eight.

Sam: Eight.

Maria: Three.

Tracy: Minus-five.

Sam: Minus-five.

Joe: Pi.

[Laughter]

Sam: It was a bottom set.

Tracy: Can you put pi in there?

Sam: Can I put pi in there? I did actually say whole numbers to them. Where would I put pi. Ummm, no.

Tracy: OK, whole numbers.

Alex: Ten.

Lucy: Seven.

Maria: One.

Sam: I'm gonna put it in there for the minute, I'll come back to it.

Alex: So, top left quadrant, are they below the line, seven and three?

Sam: They are below the dotted line.

Alex: Oh right, OK.

Simon: One hundred and thirteen.

Sam: Thanks! I think it is probably here, but I will need to check that out.

Alex: Minus-six.

Sam: Minus-six.

Maria: I want to know what goes top left.

Joe: Fifteen.

Simon: There you go!

Maria: Yeah OK.

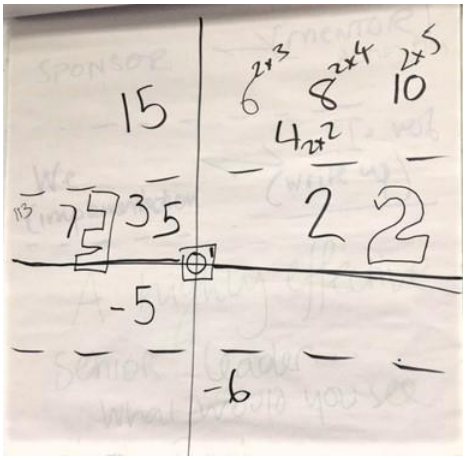


Figure 8.11: Photograph of board created by Sam.

Ellen: So, the y-axis is odd and even.

Sam: Yep.

Paul: Four.

Joe: Two.

Joe: X-axis is positive and negative?

Sam: Yep.

Ellen: Two is the only prime even number.

Tracy: What are the dotted lines?

Ellen: Odd primes and even primes.

Sam: Yeah, so, odd and positive, even and positive, odd and negative, even and negative, and then the primes were in the gap.

At this point, there is a *slide (from mathematics to mathematics teaching)* as Sam's mode of speaking moves to being *about* the activity as opposed to being *in* the activity.

Sam: I just wanted them to think about the numbers and build on something they could do. That brought out, six is two times three, that's 2 times four, that's two times five.

Simon: Oh, that is really clever.

Sam: So, that was useful because I knew we were about to go into the double stuff.

Alex then began talking about a related idea (see appendix 3f, page 406, [session 3, 00:18:55]), a connection that he seemed to be making between Sam's activity and something he had seen elsewhere, a form of accounting-for. In relistening to Alex as part of my process of data analysis, I imagined getting the group to take the ideas that Alex was describing, to engage in the process of *making it real*, either by sharing stories that may have been triggered by listening to Alex, or by working for a while on designing a similar task to Sam's using the ideas that Alex had articulated. In the moment of the conversation, however, I was not yet ready to leave the mathematics behind.

Tracy: Can you have a negative prime number? *(Supporting mathematics learning)*

Alex: You should be able to, it's got two factors.

Joe: But negative three times negative five would give you fifteen.

Tracy: I don't think I've ever encountered negative prime numbers before.

Mia: But, minus-five has got more than two factors, hasn't it? Hasn't it? Minus-one, five, minus-five, and one.

Joe: But three also has the factors negative three and negative one.

Sam: Yes, and we don't talk about those do we.

Becky: We only talk about positive factors.

Alex: We've broken maths.

\+ [Laughter]

Tracy: But I don't think there are, so, OK, I'll leave it. *(Not imposing)*

Sam: That is something to think about.

There is no *correct* way to respond during conversations about mathematics teaching, there are only different responses, with each one triggering a different response. If I am focusing on *making it real*, then certain actions become more readily available than others. As I engage in *doing mathematics*, my way of seeing may be attuned for that activity in relation to a different set of basic-level categories (such as *supporting mathematics learning*). There is likely to be a range of possibilities (ways of seeing/acting) in any given moment. Prompting Sam to get us *doing mathematics* (an existing basic-level category during session three, having worked on previous two feedback sessions during phase one), triggered a new action ("Well why don't you do this. Do you want to do this with us?"). Through *doing mathematics*, a new mode of discussion was enabled, and, in analysing this new mode, new and different labels for categories emerged (in red, both in the section of dialogue above, and in appendix 3f, page 406). Specifically, *supporting mathematics learning* ("Can you have a negative prime number?") or *not imposing* ("But I don't think there are, so, OK, I'll leave it") are both labels for

categories that emerged out of a context of *doing mathematics* together (two labels that I have deliberately left unexplored).

Each time a new practice is observed, possibilities emerge for new ways of seeing. The more you look, the more you will find. It is not the purpose of my study, however, to get to an exhaustive list of labels in relation to a way of working with mathematics teachers across all seven feedback sessions. The focus of this study is on the *process* of becoming as opposed to what I (or others) have become. Change involves seeing more and differently. My becoming, as a mathematics teacher educator, is thus under continuous construction, and this study aims to capture parts of that process. In the final part of this episode, I summarise *making it real* as a methodological dimension in relation to three methodological levels of becoming a mathematics teacher educator. Then, to conclude this episode, I share my overarching guiding principles (explained on page 149) for *making it real* as I have come to see them.

Making it real **in relation to the process of learning to teach mathematics and mathematics teachers:**

Grounding philosophical ideas by considering how those ideas could directly inform practice.

Realising possibilities by: immediately enacting new practices when this is possible; conducting small-scale research projects; making and explicitly sharing commitments; using writing as a mode of inquiry.

Staying with the detail of teaching experiences and comparing and contrasting those experiences with the experiences of others, seeking resonance and dissonance.

Formulating hypothetical actions, and imagining those actions playing out in a range of scenarios.

Making it real **in relation to *researching* how I am becoming a mathematics teacher educator**

Seeking explanatory theories that describe and explain phenomena. Creating a credible account of experiences through integrating theory and practice. Allowing theory and practice to emerge together in a recursive process of analysing practices and making sense of existing theory whilst telling and re-telling stories.

Identifying instances of new behaviours and considering those new behaviours in light of emerging categories.

Making it real **in relation to a way of working with mathematics teachers**

Creating the conditions for mathematics teachers to learn for themselves. Expanding the space of the possible in teachers by developing a repertoire of possible actions that may be triggered in a range of situations.

Using mechanisms that specifically support the move to enacting new practices such as: prompting ideas to be considered in relation to practice; making and sharing commitments; immediately putting ideas into practice; conducting small-scale research projects; using writing as a mode of inquiry; imagining the playing out of hypothetical situations.

Overarching guiding principles for *making it real*

Expanding the space of the possible by realising possibilities. Committing to not going beyond what is observable, observing one's own actions in relation to the actions of others. Constantly and critically reviewing practice.

~~~~~
~~~~~

Chapter nine

Going meta

Chapter eight was the product of enacting my narrative-enactivist methodology for researching how I am becoming a mathematics teacher educator, outlined in chapter seven as eight key methodological principles. The four episodes from chapter eight each represent a different methodological dimension in my becoming a mathematics teacher educator (specifically, *using dissonance; staying with the detail; finding conviction;* and *making it real).* This chapter, chapter nine, represents the fifth methodological dimension: *going meta*. Since part of the function of this final chapter is to provide a brief commentary about the study in its entirety (i.e., a *meta*-commentary), it felt appropriate to position the fifth methodological dimension (which is about *going meta*) differently to the other four, as a layer above (a *meta*-layer), as a kind-of *meta*-dimension (that is, a dimension in and of itself as well as a dimension *about* the other dimensions). The structure of this final chapter is thus three-fold. Firstly, as with the episodes in chapter eight (but somewhat more briefly), I begin with an analysis of the recorded feedback sessions (section 9.1), using the key methodological principles outlined in chapter seven, and using some of the labels that emerged during the process of analysis in relation to *going meta*. Secondly, I engage in a process of looking back over the study (section 9.2)

by summarising the key "find-ing(s)" (Brown, 2015, p. 193) and offering some final remarks about my ongoing process of becoming a mathematics teacher educator, reflecting on the contribution of this thesis, and revisiting my criteria for research discussed in chapter three (section 3.7, page 52). Finally, I look forward (section 9.3) to my continuing process of becoming, particularly in relation to becoming a researcher.

9.1 *Going meta* in becoming a mathematics teacher educator

Since joining the university, as a mathematics teacher educator, the term *meta* has been a regular feature of the conversations between myself and the other tutors on the course, especially in relation to PGCE subject sessions (see glossary, page 291). Sometimes these conversations involve us reflecting together retrospectively on sessions taught, other times, the conversations are focussed on the planning of future sessions. We talk, as tutors, about there being *meta*-tasks during subject sessions, which are made explicit to the prospective teachers. For instance, whilst working with the group of prospective mathematics teachers on a particular mathematics task, an example of a *meta*-task might be for the prospective teachers to notice (and make note of) different *feedback mechanisms* (e.g., using a common space for collecting results; discussing ideas in groups; a process of self-checking; and so on). As a group of tutors, we might also talk about commenting at a *meta*-level. For instance, having collected a range of responses to the prompt "what is algebra?", a comment at a *meta*-level might involve acknowledging the diverse range of responses, or it might involve commenting on particular similarities and/or differences across the range of responses. Commenting at a *meta*-level is commenting *about* what has been said or commenting *about* what is being done, as opposed to responding directly.

Influenced by both Laurinda and Alf, I used the phrase "meta-commenting", as a mathematics teacher, to refer to the kinds of comments I might make in the mathematics classroom, having noticed a particular mathematical behaviour. I would find myself commenting on observing the process of thinking mathematically/being mathematical. For example, if I observed a student putting results into a table, I might comment on that student's behaviour, using the label "being organised", or if I observed a student asking a

mathematical question, I might be moved to share that behaviour with the entire class, labelling it as "asking your own mathematical questions", with a purpose of establishing more of that behaviour within the group. Becoming a mathematics teacher involved me learning to recognise the *process* of being mathematical and commenting accordingly and learning to attend to the *type* of comment or behaviour observed and, again, commenting in relation to that. Commenting at a *meta*-level as a mathematics teacher became something I did automatically (which is not to suggest all my comments were at a *meta*-level, because they certainly were not). When I moved to the university, I found myself searching for a parallel to the meta-commenting on process/type that I had developed in the classroom. Unlike in a mathematics classroom, however, I was not yet attuned to seeing process/type, when working with groups of (prospective or in-service) mathematics teachers on teaching mathematics. I looked for frameworks for verbal meta-commenting (Helliwell, 2017b; Helliwell, 2018), and, with Laurinda and Alf, wrote about working as mathematics teacher educators at the meta-level (Brown, Helliwell & Coles, 2018). Yet I still struggled, in the moment of working with groups of mathematics teachers, on what I might comment upon. To an extent, when I let go of searching for how I might see process in others and began focussing on my own process of becoming a mathematics teacher educator, I began developing a sense of the *types* of comments and behaviours that were possible to comment upon. In fact, only at this point in my study do I now recognise each of my labels as representing opportunities for commenting at a *meta*-level, the labels themselves are at the *meta*-level, since they are labels for categorisations of the *types* of comments and processes observed during feedback sessions. Thus, becoming a mathematics teacher educator involves a process of learning to see *process* and *type* more and differently, a process of seeing at a *meta*-level.

The labels that emerged for me through the process of analysing the feedback sessions in relation to the methodological dimension *going meta*, are presented in table 9.2 below:

Label	for the phenomenon.
Distinction drawn	as distinct from…
Descriptions	of observed behaviours
Potential strategies	* In the moment. ** Anticipatory (setting up).
Functions	of potential strategies.
Notes	on useful additional points.

Table 9.1: Key to column headings (for tables 7.6, 8.3, 8.4, 8.5, 8.6 and 9.2).

Label: ***Distinctions***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Not making distinctions	Teacher explicitly commenting on making distinctions (or potentially on not making distinctions). Implicitly, where a distinction is made but not necessarily acknowledged as such.	* Working on making distinctions. * Opening up to the wider group. * Collecting responses on a visible space (like a whiteboard). * Asking for an example that helps us understand the distinction being made/described. ** Facilitating a common experience (e.g., working on some mathematics together) as a reference point/generative exercise. ** Sharing distinctions from external sources and inviting comment (in combination with previous point).	- Supporting the process of making new distinctions. - Support process of seeing more/ differently.	Seeing (noticing), articulating (describing), labelling. Evidence of teacher learning – articulating a (new) distinction. At content level and at meta level. Teachers using distinctions (e.g., open/closed): Important process of sharing how different people are using these terms. Linked to new insights.
Label: ***Patterns/Connections***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Isolated or detached comments/ ideas/issues.	Explicitly making links across different comments over the course of the conversation.	* Drawing attention to the repeated use of a word/phrase/idea. * Asking what is meant by particular word/phrase across different people's use of the word/phrase. ** Visibly gathering themes (e.g., on a whiteboard), organising them and making connections explicit. ** Adding to existing collations of ideas/issues/ strategies etc over time, as connections are made.	- Checking on understanding of a repeated term (is it being used to mean the same thing?) - Drawing attention to commonalities and persistent ideas/issues. - Checking for resolution (e.g., of a recurring issue).	Patterns can be noticed at content level (e.g., a theme running throughout, or use of a particular word etc). Retrospectively, this might be a process of data analysis, but can happen in the moment of the conversation as well. Patterns/connections also refer to meta-level/process level (e.g., the act of noticing a pattern and explicitly commenting/making links to previous comments). This might be noticing *types* of comments as opposed to noticing re-occurrence of a word/topic.

Label: ***Teacher observes change in behaviour***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Teacher has observed no change in the behaviour of their students.	Teacher gives an example of where they have observed students behaving in a way they have not observed before.	* Highlighting when a teacher describes a new behaviour (explicitly commenting on this).	- Raising awareness of changes in behaviour (increasing likelihood of noticing more changes in the future). - Increasing capacity to notice and thus triggering new ways of responding.	Observable classroom behaviours/situations /scenarios.
Label: ***Teacher going meta***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Remaining at the level of content. Responding directly.	Teacher makes a comment about the conversation or a comment on the process of the conversation. Teacher comments about a situation/experience at the level above the content of the situation/experience.	* Commenting on when a meta-comment has been made (reflecting back).	- Getting to a new awareness about an experience. - Getting to new distinctions.	Evidence of teacher learning. Sometimes a slide is an example of this, an example of when a slide is useful and demonstrates learning. Teachers also going meta within the conversation itself – taking on established collaborative group norms.
Label: ***Existing frameworks***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
Own frameworks.	Using a framework or set of distinctions (e.g., from existing literature or common discourse), in conversation or as a focus of activity.	* Returning to existing framework to add to an emerging set of distinctions from the group. * Publicly collating the categories from the group and then comparing to an existing framework. * Working on making categories visible/ observable in teachers' classrooms by getting to detail of examples. ** Introducing an existing framework to add to an emerging set of distinctions from the group.	- Enhancing existing distinctions. - Seeing more and differently in relation to practices and being able to comment on them. - Providing a common language within a group.	Whose framework? An existing framework could be seen as an external object, so, it is the process of making sense of the framework/what the individual sees in that framework that is significant in terms of learning.

Label: ***Change in practice***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
No change in practice/Status quo.	Teacher explicitly acknowledging a change in their practice or the practice of another teacher. Over time, a teacher's description of a particular practice changes (e.g., from one feedback session to the next). A change is observed in teachers' practice/my practice (as the teacher educator) in relation to previous practices.	* Commenting when a change in practice has been acknowledged. * Probing for more detail about what might have provoked the change. ** Asking teachers to keep a professional/ research diary.	- Raising an explicit awareness of learning (as change in behaviours), increasing the likelihood of noticing future changes.	This category applies to changes in teachers' practices as well as my own.
Label: ***New insight/awareness***				
Distinction drawn	**Descriptions**	**Potential strategies**	**Functions**	**Notes**
No new insight/ awareness.	Teacher explicitly describes having learned from an experience. Following a description, a new summing up occurs in the form of a new insight. Explicitly acknowledging a new awareness.	* Commenting on when a (potentially) new insight is made. * Extending the new insight by prompting for examples.	- Raising awareness of possibility for new insight. -Clarifying the new insight for others. -Creating conditions for further new insights to arise.	Insights/awarenesses could present themselves in many different forms… sometimes following detailed descriptions.

Table 9.2: Summary of labels for phenomena in relation to *going meta*.

I bring these labels together in the following two sections. Firstly, I reflect briefly on the process of making distinctions, observing patterns, and finding connections (section 9.1.1). Secondly, I reflect on the process of learning to observe learning (section 9.1.2).

9.1.1 Distinctions, patterns, and connections

To notice is to make a distinction. In enactivist terms, the "basic operation [that we perform] in the praxis of living is the operation of distinction" (Maturana, 1988a, p. 5). We are always involved in the act of noticing, as we interact in and with our environment. When an interaction *is* noticed (called a perturbation), it triggers a structural change, a

change that is determined by our existing structure. Not all interactions, however, trigger structural changes. Since we only notice what concerns us and what concerns us is conditioned by our existing conceptual structures and our cultural context, many stimuli that could potentially trigger a response go completely unnoticed (i.e., they are not perturbations). To make new distinctions is a process of seeing more and differently, a process of expanding the range and differentiation of perturbations from an existing set of possibilities. Only by seeing more and differently is it possible for new practices to be *real*-ised, this is the process of learning that I have been exploring throughout this thesis, along with ways of enabling this learning in myself and in others.

According to Claxton (2000), it is possible to "pick out and make use of patterns of information in a complex situation (such as a classroom) that are too subtle to be captured in a conscious, articulated account" (p. 36). Thus, even though not everything that we notice is at the level of consciousness, we may still be triggered into action regardless. We are constantly engaging in the process of making distinctions, whether we are aware of this or not. Much of what an expert notices, for instance, is "integrated into their professional functioning so that all they are aware of is a possibility to act, but not necessarily of the distinctions which trigger that act" (Mason, 2002, p. 33). This unconscious noticing, what Mason calls "ordinary-noticing" (2002, p. 33), or more recently "*barely noticing*" (2011, p. 41, emphasis original), operates at the level of perception. To access or reconstruct an incident or an aspect of an incident that was noticed at the level of perception would usually require a reminder from someone or something after the incident took place. Ordinary-noticing is one of three distinct forms of noticing that Mason describes; marking and recording are the other two. Marking is described as "a heightened form of noticing" (p. 33), beyond the level of perception. When we mark something, we become aware of it, at a conscious level, and we are able to re-mark upon it retrospectively, making it available for inspection.

Marking signals something salient about an incident and an incident is salient if we become aware of making a distinction. It follows that marking is a fundamental feature of researching since researching can be described as a systematic process of making new distinctions (it is in this way that researching is equivalent to learning). I resonate

strongly, therefore, with Mason's statement that salience is "almost the essence of research itself" (p. 247).

Put simply, making distinctions is a process of seeing difference within sameness, and making connections is a process of seeing sameness within difference. I return to Bateson's conception of story (discussed previously in section 2.4, page 26) as the "pattern which connects" (p. 13). Bateson (1979) uses the term connectedness not only in relation to the connectedness of events (or "components in the same 'story'" (p. 13)) but also connectedness between people, in that we "all think in terms of stories" (p. 13). During the feedback sessions, teachers told stories of their classrooms and as a group we listened to those stories, often making connections to the stories of our own classrooms. When connections were made across stories, teachers would sometimes be explicit about those connections, for instance, when Simon [session 2, 00:40:20], picked up on a previous point made by Miguel [session 2, 00:39:20], commenting, "You know you said about the not revisiting it for a year thing, that's a really big thing for us too". On several occasions (see instances of *patterns/connections* gathered in appendix 3e, page 379), I commented on connections that I was making in the moment of the conversation, for instance, in the flowing extract ([session 2, 00:57:02], extract also used in section 8.4.1, see page 229), where I make two explicit connections (highlighted in bold type):

> Tracy: And **for me that is linked to Joe's idea of showing that set of images**, it's as if the words can sometimes get in the way. Why do we need words when we have direct access to something? Students don't necessarily need to verbalise things for themselves, because the reasoning is there, within the structure of what you offer them. **For me, that is connected to Ellen needing to write for the words to come**.

On other occasions, the process of making connections was not commented on explicitly, yet connections could be observed in the patterns of talk throughout the dialogue. Thus, another fruitful study, using the same audio-recorded conversations, could involve trying to identify these patterns in talk, and potentially observing how these patterns in talk

form and evolve. In this study, I was less concerned about the *content* of the patterns (as might be the focus of a different kind of analysis, involving thematic coding), rather, I was concerned about the *process of recognising* patterns. The distinction I am making is linked again to *going meta*; I am interested in sensitising myself to when a connection is being made, in addition to the connection itself. Identifying when a connection is being made, is an opportunity to comment at the *meta*-level, with the purpose of making connections visible and inviting further connections to be shared.

9.1.2 Observing learning

Becoming a mathematics teacher educator, involves attuning myself to observing the process of learning. Coles (2018) explores what he observes when he says he observes learning in mathematics teachers. Specifically, he asks, "What might mathematics teacher learning, or knowing, look like *in talk*?" (p. 20, emphasis added) and claims that there are "four ways we might observe teacher learning, via talk" (p. 23). The first two ways (in the list below) are observed through the teacher offering a self-report (either of a change in their behaviour or of a new awareness). The second two ways are observed directly from the talk, yet potentially without the teacher talking being aware of the reporting by the teacher speaking a self-report:

1. A teacher reporting a change in their behaviour (listeners are offered a description of a change in behaviour, in general terms).
2. A teacher reporting on a new awareness (listeners are offered a description of a change and an associated new awareness).
3. Observing a new awareness (listeners observe a new awareness being articulated, in that moment, and what is being distinguished).
4. Observing a change in behaviour (listeners notice a new response to the same stimulus, compared to an earlier instance). (Coles, 2018, pp. 23-24)

According to Coles (2018), a self-report from a teacher of a change in behaviour, "necessitates the teacher being aware of a change they have undergone" (p. 23), which is not necessarily the case when a change in behaviour is observed during the talk itself. Coles argues that observing learning directly, through talk, is stronger evidence (as opposed to stronger quality/depth) of learning, than the evidence from self-reporting. In the transcripts of the first two feedback sessions, I found myself recognising instances of *change in practice* as well as *new insight/awareness* (see appendix 3e, page 379, where all instances of these two phenomena have been gathered). I offer some example extracts from the first two feedback sessions of each of Coles' first three ways of observing teacher learning in talk (in italics below) from the list above, using examples from the feedback sessions. I return to the fourth way of observing teacher learning in talk, in a slightly different way, a little later on.

A teacher reporting a change in their behaviour [session 1, 00:37:27]: In this extract, Miguel is describing a new purpose in his teaching, linked to new behaviours:

> Miguel: The way that I naturally taught, is that I would explain something and then give them the work, and I've tried to move away from that. My attitude used to be, how can I explain this in the clearest possible way so that the students have to think about it as little as possible to understand it and my attitude now has become what is the least I can possibly do to explain it to them, without deliberately making it confusing, so they can make the rest of the leaps themselves.

Miguel goes on to describe some specific ways in which his behaviours have changed.

A teacher reporting on a new awareness [session 2, 01:14:46]: In this extract, Beth is describing an experience of working with a group primary teachers (for students aged 4-11 years) on the process of division, and the point at which she had a new awareness

herself about the process[17]. I have used bold type to point to where a new awareness is marked. For "Dienes blocks" see glossary (page 291):

> Beth: I asked [the primary teachers] what their kids see division as. They said they see it as sharing. Now this is a cautionary tale, because I tried to use Dienes blocks to show long division. In my head, because I'm a secondary teacher, I was sharing, **and then I realised the language you use around long division is grouping**, it's "how many of this go into that", so you're counting up in groups. So, I was desperately trying to model sharing with Dienes blocks, **but realising I wanted groups** and it was all a bit of a mess. Luckily, I learned from that experience and in the end the primary teachers planned out a sequence of lessons where they just spent a couple of lessons representing division both ways and talking about sharing *and* grouping.

Observing a new awareness [session 2, 00:55:10]: In this extract, Mia is reflecting on a new way of seeing a moment from her classroom where one of her students had been "convinced that to get from millimetres to centimetres, you times by ten" (see [session 2, 00:51:18] in appendix 3b, page 329). About four minutes later, potentially as a result of the subsequent conversation, Mia seems to articulate a new awareness:

> Mia: I think it's what you said, because I always think of it as you are going from a smaller unit to a bigger unit, so you need less of them, so that's how we sort of talk about it, but you're right, millimetres to centimetres is getting ten times bigger.

In a study where my primary focus has been on my own learning, as a mathematics teacher educator, I offer an extension of Coles' (2018) framing to include the learning of the mathematics teacher educator (learning to teach mathematics teachers) and the learning of the students (learning mathematics). I offer four *layers of learning* that I am aware of

[17] In Beth's description, she uses the language of sharing and grouping, two images for the process of division (see [session 2, 01:12:50], appendix 3b, page 329, for a conversation between the group about the difference between grouping and sharing).

observing during the feedback sessions, where layer two is the closest to the ways of observing learning by Coles (2018):

1. Observing my own learning (through the labelling of a new basic-level category (or purpose), or through a change in practice in relation to working with the group of mathematics teachers).
2. Observing the learning of individual mathematics teachers (through a new insight/awareness, or through a change in their practice).
3. Observing the learning of mathematics students (through teachers reporting on observing changes in their students' behaviour).
4. Observing the learning of the group of mathematics teachers (through a change in the practices of the group).

Observing at layer three involves a process of observing observing (me observing the teachers observing their students). Observing the learning of others will always be at a *meta*-level since we do not have direct access to the learning of others (observing observing could therefore be seen as being at a *meta-meta*-level). For instance, during the second feedback session, where Joe is reflecting [session 2, 00:34:18] on a particular structured activity (see glossary, page 291, and appendix 3h, page 412) that he has been using with one of his classes, where he comments, "the more I did [of the structured activity] the more they got used to thinking. The easier it became and the better learning we got". The conversation continued [session 2, 00:34:38]:

Tracy: What gave you the sense that the learning was better, what was it they were doing that meant that the learning was better?

Joe: Well, for example, when I was giving the students the biggest, smallest[18] task, they were reasoning more about which values to use in the lesson when I had given them a selection of values to choose from, so they could select in order to yield the result they wanted.

[18] The "biggest, smallest" task Joe is referring to involved students finding values of a and b that maximise and minimise the values of expressions such as a/b, ab, a + b, a – b, 2a, 2a + b, and so on.

The first time I did it, it was very much trial and error. Whereas the second time was very much more about strategy.

Tracy: Oh OK.

Joe: And their different approach was really noticeable.

Tracy: So same structure both lessons, but in the first you let them choose any numbers and in the second, you gave them a selection to choose from?

Joe: Yeah, so how they approached it was different and the speed at which they got to the correct answer and the accuracy of calculation was much stronger in the end. The students were not writing four examples per expression and stumbling on the answer, they were saying "right I think I'm going to get the right answer by following this strategy" and getting there much more quickly.

In this section of dialogue, Joe offers a clear account of observing students' changing behaviours. One comment I might have made, at a *meta*-level, in the moment of the conversation, might have been to acknowledge Joe's observation and to point out a parallel to my own commenting which could be to suggest that when Joe notices changes in behaviour that he associates with being mathematical, that he himself makes that explicit (which he may well have done).

I use this final extract, the only extract from the seventh feedback session (see appendix 3g, page 410), to demonstrate a sense of change in the behaviour of the group (myself and the teachers, point 4 in the list above). In this section of dialogue, starting with Simon's articulation of a *teaching issue* [session 7, 00:20:30], the group seem to, without prompting, launch into offering potential *teaching strategies*. There is a moment where Harry (the project lead (see glossary, page 291) for the second year of the project) interjects with a comment (in the sixth turn below) *about* what the teachers are doing in relation to the work on the project. Sensing the fruitfulness of the conversation (which at that point, I considered to be *unresolved*), I re-direct the attention to Simon's issue which seems to then provoke a *detailed description*, from Leo, about a particular idea developed

in relation to supporting written communication. Vicky prompts for some further clarification and the idea seems to be taken up as a possibility by Simon, in his final comment. For an example of "always, sometimes, never activity" see appendix 3h (page 412):

Simon: You know the talking mathematically thing. From a student perspective, in my lessons, I feel like students are a lot more willing to discuss and consider. For example, during an always, sometimes, never activity, producing a counter-example is now a standard thing. They know if something is sometimes true, to give a reason why, and an example of both. So, I've found that they are really good at being able to discuss, but they are not yet able to communicate their reasoning well on a page. So, I think their reasoning, in terms of verbal reasoning, is really good. In terms of their written communication, it is not necessarily coming through yet and I don't know if that is something I am doing.

Vicky: I've been giving the students sentence starters quite a lot.

Simon: Yeah, I've never even thought to do that.

Joe: There is something maybe about committing it to paper that scares some of them. I give the students mini-whiteboards before they commit to paper.

Vicky: I started giving them sentence starters like "I think… because of…" and "I can show this by…". To start with I would give them different options and they could just cross off the ones they disagree with.

Harry: What I like is that you have really thought about more than the task, sometimes people think the task is the answer to getting students reasoning, but there is more to it than that.

Tracy: Is there anything else about getting written communication more established?

Leo: Yeah, I've got a standard form I use for all of the reasoning activities that I give the classes. So, it is essentially an experiment area, followed by a conclusion area. So, by using it in a standard way, you're encouraging the students to experiment first of all, so even if they don't know really what's going on. So, it's really getting them into that habit; if you're stuck with a question, the first thing you do is experiment, you try something. You can get it wrong. Then that conclusion area is for that sentence, that key thing that I want them to take away. In terms of getting them writing what they are thinking, that has been my way into it, and making it routine. I've got about ten or fifteen of these now, and a list of those I want to make. I have seen a difference there in the classes, in that they know what do to, it's the same as they have been told before. Even if they don't know what is going on, they know that they need to have a go.

Vicky: When have you been doing these? At the start of the lesson? At the end?

Leo: The start of each lesson would be some mixed practice, then we might do some examples and then this would be the finish. Typically, it is about five minutes of them trying things out, maybe writing a conclusion if they feel confident to, and then about five minutes of us talking about it as a class and getting to the conclusion that I want them to get to.

Vicky: So, like a class conclusion.

Leo: Exactly, and then what I will do is wipe off the board, and then the form is double sided, so they can write their own version on the back and then the check is that I ask three different people to read out their own versions, word for word. I am making sure they have clear sentences and that it is coming from them, rather than writing it down from the board.

Simon: That is really nice, I'm going to give that a try.

This extract is by no means supposed to demonstrate cause and effect, but instead to acknowledge an observed change in the behaviour of the group, what Barwell et al., (2020) might refer to as "jointly doing knowing", which means "the collective process of enacting knowing through interaction" (p. 216). To me, this final section of dialogue is a special section to share because it made for comfortable listening (as opposed to the discomfort provoked by listening to many of my responses throughout earlier feedback sessions). This extract, for me, illustrates a way of working as a group of teachers that is closer to an image of a way of working that I have been developing over the past four or five years. Becoming a mathematics teacher educator involves becoming attuned to instances of teachers reporting on the learning of their students learning mathematics, on instances of their own learning and on instances of the group learning together. Instances of observing learning can all be seen as opportunities to comment, as a mathematics teacher educator, at the *meta*-level, on what has been observed, with the purpose of provoking yet more new awarenesses and behaviours.

I bring this section to a close in the same way I have closed the four episodes of chapter eight: by presenting a summary of *going meta* in relation to three methodological levels of becoming a mathematics teacher educator, followed by the overarching guiding principles. After this, I then offer a brief looking back (section 9.2) over the study, doing so by returning to the criteria presented in section 3.7 (see page 52) before briefly looking forward (section 9.3).

***Going meta* in relation to the process of learning to teach mathematics and mathematics teachers:**

Noticing, marking, and recording instances of practice attending to potential labelling/categorising of those practices.

Observing own learning through keeping a professional diary and looking back, paying attention to what might have changed.

Observing the learning of others through attending to the process/type of talk as well as the content.

***Going meta* in relation to *researching* how I am becoming a mathematics teacher educator**

Drawing parallels between the process of researching and the process of teaching and learning to teach (mathematics and mathematics teachers).

Combining telling stories with categorical analysis as a way of seeing more in a set of data and collecting and analysing data over time.

Interrogating labels that emerge for categorisations, telling stories to uncover the meaning behind those labels.

Focussing on process of researching as opposed to the outcome of research.

***Going meta* in relation to a way of working with mathematics teachers**

Attending to the *process* of talking about teaching and learning mathematics as well as the content of what is being said. Seeing comments as *types* of comments in addition to the content of the comments themselves and commenting at a *meta*-level.

Supporting the seeing of *process* and *type* in others and remarking when this seeing is shared. Commenting on what is the same and what is different and inviting others to do the same.

Commenting on instances of observing learning (either in teachers or in teachers' descriptions of their students).

Overarching guiding principles for *going meta*

Supporting learning and supporting the observing of learning (in self and in teachers).

~~~~~
~~~~~

9.2 *Going meta* in relation to this study

In this section I offer a looking back, a *meta*-commentary on my thesis. In section 3.7 (see page 52), I presented a set of criteria for evaluating research studies that utilise creative analytic practices, namely: *substantive contribution; aesthetic merit; reflexivity; and impact* (Richardson & St. Pierre, 2018, p. 823, emphasis original). In this section, I return to consider each of these criteria, offering a renewed perspective, having got to the final stages of this study.

The criteria, *substantive contribution*, is about how this thesis contributes to the understanding of teaching mathematics teachers and the process of becoming a mathematics teacher educator. In this study, I have taken on the challenge of researching the process of learning: learning mathematics; learning to teach mathematics; and primarily, learning to teach teachers of mathematics. The process of teaching mathematics teachers is a complex activity where the layers of learning described in the previous section (section 9.1.2) inextricably come together. The process of researching learning is yet another layer (of learning), which again, through the course of this study has become intertwined with the other layers. Rather than pulling these layers apart, into separate categories of knowledge, or lists of competencies, I have aimed to show some of the relationships between the different layers, and some of the parallels across them. I have also worked on articulating distinctions between what I have been referring to as three methodological levels of becoming a mathematics teacher educator (specifically: the process of learning to teach mathematics and mathematics teachers; researching how I am becoming a mathematics teacher educator; and a way of working with mathematics teachers). In terms of these three levels, there are, again, multiple overlaps and parallels, yet the process of making these distinctions became more and more meaningful over time. Since each methodological level is articulated as a set of associated actions, per methodological level, each sets of actions offers a useful tool for people within education who may want to focus on the level that is relevant to them.

Thus, one contribution this thesis makes, within the community of mathematics educators, teacher educators and educational researchers, is its potential to be useful to

different individuals or groups, for different purposes. For instance, a teacher or teacher educator working on their practices may find the first of the three levels to be most useful. A researcher, interested in researching their own learning, or the learning of others, may find the second level of particular use. A teacher educator, or somebody directly involved in the professional development of teachers, may want to refer primarily to the distinctions made in relation to the third level. Therefore, this study contributes pragmatically, by offering a framework/tool for different individuals and groups within the educational community, to work on their practices (teaching and/or researching). I can also imagine how engaging with this study could by a useful activity for a beginning teacher educator, coming to terms for themselves with the new complexities that they are facing.

Another contribution this thesis makes, is methodologically. Both in terms of having a clearly articulated narrative-enactivist research methodology presented as a high-level set of principles (derived from a combination of narrative theory and enactivism, see section 7.1, page 126, and particularly table 7.1, page 127), as a basis from which learning can take place and also be observed. I view this research methodology as one set of research "find-ing(s)" (Brown, 2015, p. 193). Not the type of research findings that provide the answer to a research question (for instance a list of things I have learned as a mathematics teacher educator), rather, "[t]he find-ing(s) of enactivist research shed light onto the journeys that are travelled in the professional learning that takes place when developing one's teaching" (Brown, 2015, p. 194). My research find-ing(s) are methodological, they offer a way of working as opposed to the outcome of working in a certain way. My narrative-enactivist methodology for becoming a mathematics teacher educator is presented in summary in table 9.3 (see page 268). Figure 9.1 (page 272) is the methodological framing developed during this study for becoming a mathematics teacher educator. Again, I view my narrative-enactivist methodology for becoming a mathematics teacher educator (table 9.3) and the methodological framing (figure 9.1) as sets of research find-ing(s), in relation to my initial research problem, *how am I becoming a mathematics teacher educator?*

As a direct consequence of my commitment to researching process, such that my contribution is both pragmatic and methodological, I have come to view the first criteria, substantive contribution, as directly related to the fourth, *impact*. Impact, as a criteria for this research, is about *real*-ising new ways of being in relation to the research. The process of reading the stories told throughout this thesis could resonate with readers' own lives; the process of reading generating new questions, or uncovering old, unresolved ones. My find-ing(s) could move others to new ways of practising (teaching and researching). Most significantly, however, the impact of this research is in me. As mentioned in section 2.1, the most significant products of research in mathematics education are the "transformations in the being of the researchers" (Mason, 1998, p. 357). Through enacting my research, I have changed, and I am continuing to change as I am becoming a mathematics teacher educator. Capturing and expressing some of this change, and most importantly, the processes by which change is taking place, is central to this thesis. In terms of *aesthetic merit*, I have been committed to presenting my data in a way that opens up the text and invites interpretive responses. I think I could have gone further, in this respect, for example, creating some kind of methodological novel; which will happen, at some point in the future, based on this study. I would like to promote further use of art-based research methods within mathematics education research, and also in working with mathematics teachers. In regard to *reflexivity*, I have been able to demonstrate my relationship with my research, not only because I have been researching my own becoming (i.e., as primarily a self-based study), but in terms of developing a methodology that explicitly aims at developing self-awareness as a mathematics teacher educator, working with teachers of mathematics. This study has been necessarily self-exposing, and I have utilised creative methods designed specifically to expose hidden assumptions and biases. It has not always been a comfortable journey to make, at times I have questioned my approach to researching. I have written many stories that never made the cut (so to speak), some of which were painful to write, often involving my life beyond that of working with mathematics teachers. However, every story has played its part, even if not directly. Some of these stories lay in waiting, ready to be made public at another point in time.

9.2.1 Summary of research find-ing(s): A narrative-enactivist methodology for becoming a mathematics teacher educator

Table 9.3 is a summary of research find-ing(s) that emerged through the process of enacting my research methodology for becoming a mathematics teacher educator. Figure 9.1 also illustrates research find-ing(s) in the form of the methodological framing developed during this study for becoming a mathematics teacher educator.

Methodological dimension	**Overarching guiding principle**	**Potential strategies and associated actions (at each of the 3 levels)**		
		In relation to the process of learning to teach mathematics and mathematics teachers	**In relation to *researching* how I am becoming a mathematics teacher educator**	**In relation to a way of working with mathematics teachers**
Using dissonance	Developing an open mind. Trapping judgements and a false sense of accomplishment. Learning about oneself and others. Triggering a range of considered and thoughtful responses in the moment of interactions.	- Retrospectively locating moments of dissonance and identifying the issue triggering the discomfort. - Considering, alone and with others, the appropriateness of the behaviours that triggered the dissonance. - Prompting more thoughtful responses in the moment of teaching. - Monitoring the consequences of actions by paying attention to further moments of dissonance (or consonance).	- In the process of listening and transcribing, staying alert to moments of dissonance and recording those moments for subsequent analysis. - As part of a process of data analysis, retrospectively integrating moments of dissonance in relation to the data being analysed, being transparent about hidden assumptions and biases being uncovered through this process. - Utilising multiple perspectives and sharing own analyses by seeking resonance and dissonance in an expanding community (to bring into question any awareness clued by moments of dissonance).	- Staying alert to instances of dissonance, in the moment, provoking a more thoughtful response, rather than reacting emotionally or judgementally. - Prompting others in retrospectively re-entering moments of dissonance as a way of identifying issues in relation to practice. - Encouraging others to locate their own moments of dissonance in the future.

Methodological dimension	Overarching guiding principle	Potential strategies and associated actions (at each of the 3 levels)		
		In relation to the process of learning to teach mathematics and mathematics teachers	**In relation to *researching* how I am becoming a mathematics teacher educator**	**In relation to a way of working with mathematics teachers**
			- Telling and retelling stories as a way of re-entering the awareness that is clued by dissonance.	
Staying with the detail	Developing (new and different kinds of) insights. Letting go of assumptions and preconceptions. Being open to newness and difference. Releasing self from judgement and becoming curious. Listening like everything being said is true (or listening openly).	- Separating out accounts-of from accounting-for, avoiding evaluative ways of speaking and allowing new insights and awarenesses to arise from the detail. - Trapping judgements and avoiding unhelpful labels that characterise groups of, and individual students/teachers. - Engaging in doing (mathematics, mathematics teaching) as opposed to describing (mathematics, mathematics teaching), before moving to explicitly consider implications on practice.	- Dwelling in the detail of data through the recursive process of listening, transcribing, re-listening, accessing field notes, doing mathematics, reading, telling, and re-telling stories as a way of developing new basic-level categories. Labelling these categories to increase future recognition. - Working on labels for categories of data in multiple ways (e.g., considering a description of the observable behaviour, associated potential strategies and related functions of those strategies).	- Becoming attuned to the different modes of talk and prompting for clarity when ambiguous labels/terms/phrases are used. - Noticing when a slide might be taking place and responding accordingly by resisting the slide or encouraging it. - Identifying opportunities for doing mathematics or doing actively before considering implications on practice.
Finding conviction	Nurturing a sense of agency and fostering a sense of ownership both in self and in others.	- Identifying teaching issues through marking moments of dissonance, and then retrospectively staying with the detail of those moments. - With others, gathering a range of potential teaching strategies for each teaching issue. Responding thoughtfully in a moment of choice,	- Supporting a process of seeing more and differently by seeking multiple interpretations of the same data. - Supporting the emergence of new basic-level categories by integrating theory with the process of data analysis. Returning to the data	- Fostering a sense of ownership, through supporting others in establishing their own teaching issues and avoiding the urge to impose one's own teaching issues on others. - Nurturing a sense of agency through organising the gathering of

Methodological dimension	Overarching guiding principle	Potential strategies and associated actions (at each of the 3 levels)		
		In relation to the process of learning to teach mathematics and mathematics teachers	**In relation to *researching* how I am becoming a mathematics teacher educator**	**In relation to a way of working with mathematics teachers**
		by bringing to mind a range of potential actions. - Avoiding developing and using labels for generalisations that characterise individuals and groups of individuals, and only speaking for yourself. Avoiding modes of speaking about teaching that are inherently limiting.	repeatedly, each time looking for a different reading of the situation. - Allowing labels for observed phenomena to change as new distinctions are made. Staying open to new ways of seeing data that has already been categorised.	potential teaching strategies and associated actions, supporting the emergence of new basic-level categories. Flagging modes of speaking that act to limit possibilities.
Making it real	Expanding the space of the possible by realising possibilities. Committing to not going beyond what is observable, observing one's own actions in relation to the actions of others. Constantly and critically reviewing practice.	- Grounding philosophical ideas by considering how those ideas could directly inform practice. - Realising possibilities by: immediately enacting new practices when this is possible; conducting small-scale research projects; making and explicitly sharing commitments; using writing as a mode of inquiry. - Staying with the detail of teaching experiences and comparing and contrasting those experiences with the experiences of others, seeking resonance and dissonance.	-Seeking explanatory theories that describe and explain phenomena. Creating a credible account of experiences through integrating theory and practice. Allowing theory and practice to emerge together in a recursive process of analysing practices and making sense of existing theory whilst telling and re-telling stories. - Identifying instances of new behaviours and considering those new behaviours in light of emerging categories.	- Creating the conditions for mathematics teachers to learn for themselves. Expanding the space of the possible in teachers by developing a repertoire of possible actions that may be triggered in a range of situations. - Using mechanisms that specifically support the move to enacting new practices such as: prompting ideas to be considered in relation to practice; making and sharing commitments; immediately putting ideas into practice; conducting small-scale research projects; using writing as a mode of inquiry; imagining the playing out of hypothetical situations.

Methodological dimension	Overarching guiding principle	Potential strategies and associated actions (at each of the 3 levels)		
		In relation to the process of learning to teach mathematics and mathematics teachers	**In relation to *researching* how I am becoming a mathematics teacher educator**	**In relation to a way of working with mathematics teachers**
		- Formulating hypothetical actions, and imagining those actions playing out in a range of scenarios.		
Going meta	Supporting learning and supporting the observing of learning (in self and in teachers).	- Noticing, marking, and recording instances of practice attending to potential labelling/categorising of those practices. - Observing own learning through keeping a professional diary and looking back, paying attention to what might have changed. Observing the learning of others through attending to the process/type of talk as well as the content.	- Drawing parallels between the process of researching and the process of teaching and learning to teach (mathematics and mathematics teachers). - Combining telling stories with categorical analysis as a way of seeing more in a set of data and collecting and analysing data over time. - Interrogating labels that emerge for categorisations, telling stories to uncover the meaning behind those labels. - Focussing on process of researching as opposed to the outcome of research.	- Attending to the process of talking about teaching and learning mathematics as well as the content of what is being said. Seeing comments as types of comments in addition to the content of the comments themselves and commenting at a meta-level. - Supporting the seeing of process and type in others and remarking when this seeing is shared. Commenting on what is the same and what is different and inviting others to do the same. - Commenting on instances of observing learning (either in teachers or in teachers' descriptions of their students).

Table 9.3: Summary of a narrative-enactivist methodology for becoming a mathematics teacher educator.

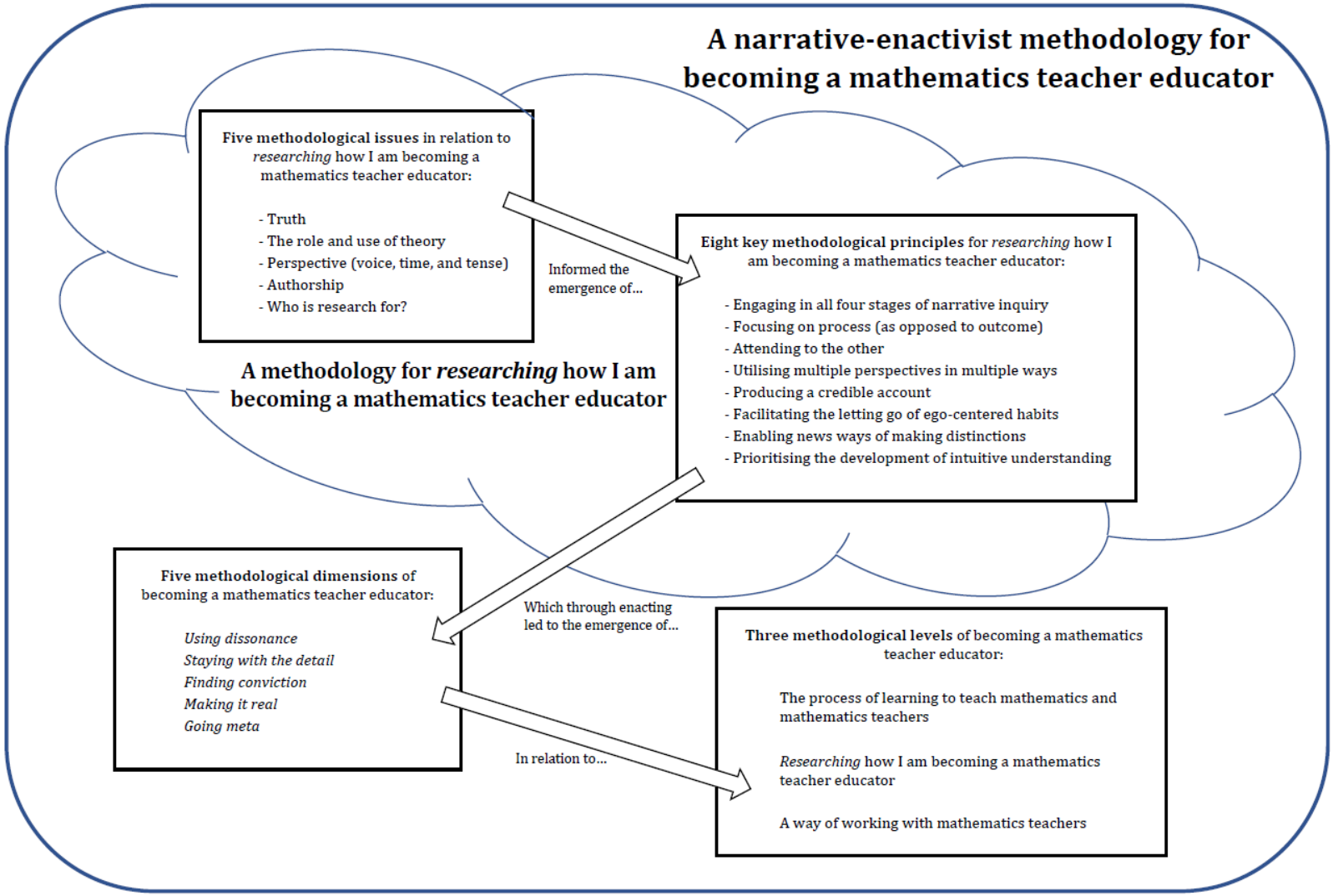


Figure 9.1: Relational diagram showing key components of a narrative-enactivist methodology for becoming a mathematics teacher educator.

9.3 My continuing journey of becoming

In this final section, I share a few of my ideas, looking forward beyond this study, to new research directions. Before that, I want to also acknowledge that, although my dissertation has come to an end, my process of becoming a mathematics teacher educator continues. Working with teachers of mathematics is such a complex activity, I feel like I am only really scratching the surface of how I might be able to support the learning of those mathematics teachers that I have to privilege of working alongside. I will continue to work on how I am attending, committing to critically reflect on my practice with others.

In terms of research, I have become fascinated with ways of observing learning. I would like to continue working on ways of observing the learning of mathematics teachers, and mathematics teacher educators, and also observing the learning of students working on mathematics, extending the ideas I have been developing within this thesis. I would like to further explore parallels in learning between these different groups of individuals. Research in the area of mathematics teacher educator learning is still in its infancy and I would like to continue researching actively in this area, collaborating with other mathematics teacher educators to see more and differently, particularly across different contexts and cultures.

Finally, I would like to continue pursuing some of the methodological aspects that I have only really touched upon in this study. Specifically, I would like to continue developing and utilising approaches to research that draw explicitly on the arts. A broad question I would like to explore further is, *how can research within mathematics education be communicated in a way that moves people?* I will also be thinking carefully about ways in which I can incorporate the arts into the design of the course for prospective mathematics teachers that I teach, such as how to incorporate more storytelling and the use of fictionalising. I have often thought of teaching mathematics as telling a story, how might I further develop this idea and support prospective mathematics teachers in telling their own stories…?

~~~~~
~~~~~

References

Atkinson, T., & Claxton, G. (2000). Introduction. In T. Atkinson & G. Claxton (Eds.), *The intuitive practitioner: On the value of not always knowing what one is doing* (pp. 1–12). Open University Press.

Bakhtin, M. M. (1981). *The dialogic imagination: Four essays* (M. Holquist, Ed., C. Emerson & M. Holquist, Trans.). University of Texas Press.

Bailey, J. (2008). Using narrative inquiry to explore mathematics curriculum. *Curriculum Matters*, *4*, 147–163. https://doi.org/10.18296/cm.0103

Barone, T., & Eisner, E. W. (2012). *Arts based research*. Sage.

Barwell, R., Borden, L. L., Manuel, D., Reid, D. A., Savard, A., Simmt, E., & Suurtamm, C. (2020). Researching mathematics pedagogy. In D. A. Reid, C. Suurtamm, A. Savard, E. Simmt, D. Manuel, L. L. Borden & R. Barwell (Eds.), *Researching pedagogy and practice with Canadian mathematics teachers* (pp. 213–230). Information Age Publishing.

Bateson, G. (1979). *Mind and nature: A necessary unity*. E. P. Dutton Publishing.

Bateson, G. (2000). *Steps to an ecology of mind: Collected essays in anthropology, psychiatry, evolution, and epistemology*. University of Chicago Press. (Originally published 1972.)

Beswick, K. (2004). The impact of teachers' perceptions of student characteristics on the enactment of their beliefs. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 111–118). http://www.igpme.org/publications/current-proceedings/

Bishop, A. J. (1994). Cultural conflicts in mathematics education: Developing a research agenda. *For the Learning of Mathematics*, *14*(2), 15–18.

Bishop, A. J., & Goffree, F. (1986). Classroom organisation and dynamics. In B. Christiansen, A. G. Howson & M. Otte (Eds.), *Perspectives on mathematics education*

(pp. 309–365). D. Reidel Publishing Company. https://doi.org/10.1007/978-94-009-4504-3_8

Bochner, A. (2000). Criteria against ourselves. *Qualitative inquiry*, *6*(2), 266–272. https://doi.org/10.1177/107780040000600209

Bochner, A., & Ellis, C. (2016). *Evocative autoethnography: Writing lives and telling stories.* Routledge. https://doi.org/10.4324/9781315545417

Broadwell, M. M. (1969). Teaching for learning (XVI). *The Gospel Guardian*, *20*(41), 1–3.

Brown, J., Brown, L., Coles, A., & Helliwell, T. (2019). Learning to teach mathematics: The lesson de-brief conversation. In S. Llinares & O. Chapman (Eds.), *International Handbook of Mathematics Teacher Education*: *Tools and processes in Mathematics Teacher Education* (2nd ed., 2, pp. 85–108). Brill Sense. https://doi.org/10.1163/9789004418967_004

Brown, J., Brown, L., Coles, A., & Helliwell, T. (2021). Working with awareness as mathematics teacher educators: Experiences to issues to actions. In M. Goos & K. Beswick (Eds.), *The learning and development of mathematics teacher educators: International perspectives and challenges* (pp. 187–204). Springer Nature. https://doi.org/10.1007/978-3-030-62408-8_10

Brown, J. S. (1991). Research that re-invents the corporation. *Harvard Business Review, 69*(1), 102–110.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, *18*(1), 32–42. https://doi.org/10.3102/0013189x018001032

Brown, L. (2004). It's about learning: From purposes to basic-level categories to metacommenting. In A. Noyes (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, *24*(3), 1–6. http://www.bsrlm.org.uk

Brown, L. (2006). From practices to theories to practices … in learning to teach mathematics and learning mathematics. In M. Bosch (Ed.), *Proceedings of the 4th Congress of the European Society for Research in Mathematics Education* (CERME 4,

pp. 1451–1461). https://www.mathematik.uni-dortmund.de/~erme/CERME4/CERME4_WG12.pdf

Brown, L. (2015). Researching as an enactivist mathematics education researcher. *ZDM Mathematics Education*, *47*(2), 185–196. https://doi.org/10.1007/s11858-015-0686-3

Brown, L. & Coles, A. (2000). Complex decision-making in the classroom: The teacher as an intuitive practitioner. In T. Atkinson & G. Claxton (Eds.), *The intuitive practitioner: On the value of not always knowing what one is doing* (pp. 165–181). Open University Press.

Brown, L., & Coles, A. (2011). Developing expertise: How enactivism re-frames mathematics teacher development. *ZDM Mathematics Education, 43*(6–7), 861–873. https://doi.org/10.1007/s11858-011-0343-4

Brown, L., & Coles, A. (2012). Developing "deliberate analysis" for learning mathematics and for mathematics teacher education: How the enactive approach to cognition frames reflection. *Educational Studies in Mathematics*, *80*(1), 217–231. https://doi.org/10.1007/s10649-012-9389-7

Brown, L., & Coles, A. (2013). Staying with the detail: The use of story as a pedagogical tool within teacher education. In M. Rohse, J. J. Infanti, N. Sabnani & M. Nivargi (Eds.), *The many facets of storytelling: Global reflections on narrative complexity* (pp. 35–44). Brill. https://doi.org/10.1163/9781848881662_005

Brown, L., & Coles, A. (2020). A way of working with teachers of mathematics in collaborative groups. In H. Borko & D. Potari (Eds.), *Proceedings of the 25th ICMI study conference,* 86–93. https://www.mathunion.org/fileadmin/ICMI/ICMI%20studies/ICMI%20Study%25025/BaseProceedings4.24.2020_PageNumbers_FINAL.pdf

Brown, L., & Dobson, A. (1996). Using dissonance – finding the grit in the oyster. In G. Claxton, T. Atkinson, M. Osborn & M. Wallace (Eds.), *Liberating the learner: Lessons for professional development in education* (pp. 212–227). Routledge.

Brown, L., Helliwell, T., & Coles, A. (2018). Working as mathematics teacher educators at the meta-level (to the focus of the teachers on developing their teaching).

Avances de investigación en Educación Matemática, *13*, 105–122. https://doi.org/10.35763/aiem.v0i13

Brown, L., & Reid, D. A. (2006). Embodied cognition: Somatic markers, purposes and emotional orientations. *Educational Studies in Mathematics*, *63*(2), 179–192. https://doi.org/10.1007/s10649-006-9027-3

Bruner, J. (1986). *Actual minds, possible worlds*. Harvard University Press. https://doi.org/10.4159/9780674029019

Bruner, J. (2003). *Making stories: Law, literature, life*. Harvard University Press.

Bruner, J. (2004). Life as narrative. *Social research: An international quarterly*, *71*(3), 691–710.

Capra, F. (1996). *The web of life: A new synthesis of mind and matter*. HarperCollins.

Chapman, O. (2020). Mathematics teacher educators' use of narrative in research, learning and teaching. *For the Learning of Mathematics: Proceedings of a symposium on learning in honour of Laurinda Brown*, Monograph 1, 21–27.

Chapman, O., & Heater, B. (2010). Understanding change through a high school mathematics teacher's journey to inquiry-based teaching. *Journal of Mathematics Teacher Education*, *13*(6), 445–458. https://doi.org/10.1007/s10857-010-9164-6

Chauvot, J. B. (2009). Grounding practice in scholarship, grounding scholarship in practice: Knowledge of a mathematics teacher educator-researcher. *Teaching and Teacher Education*, *25*(2), 357–370. https://doi.org/10.1016/j.tate.2008.09.006

Clandinin, D. J. (2013). *Engaging in narrative inquiry*. Routledge. https://doi.org/10.4324/9781315429618

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research*. Jossey-Bass.

Clandinin, D. J., Huber, J., Steeves, P., & Li, Y. (2011). Becoming a narrative inquirer: Learning to attend within the three-dimensional narrative inquiry space. In S. Trahar (Ed.), *Learning and teaching narrative inquiry: Travelling in the borderlands* (pp. 33–52). John Benjamins. https://doi.org/10.1075/sin.14.03cla

Claxton, G. (2000). The anatomy of intuition. In T. Atkinson & G. Claxton (Eds.), *The intuitive practitioner: On the value of not always knowing what one is doing* (pp. 32–52). Open University Press.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, *23*(7), 13–20. https://doi.org/10.3102/0013189x023007013

Coles, A. (2013). Using video for professional development: The role of the discussion facilitator. *Journal of Mathematics Teacher Education, 16*(3), 165–184. https://doi.org/10.1007/s10857-012-9225-0

Coles, A. (2018). On observing learning. *For the Learning of Mathematics*, *38*(3), 19–24.

Coles, A. (2019). Facilitating the use of video with teachers of mathematics: Learning from staying with the detail. *International Journal of STEM Education*. https://doi.org/10.1186/s40594-018-0155-y

Connelly, F. M., & Clandinin, D. J. (2006). Narrative inquiry. In J. Green, G. Camilli & P. Elmore (Eds.), *Handbook of Complementary Methods in Education Research* (3rd ed., pp. 477–87). Taylor and Francis.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education*, *16*(5), 324–336. https://doi.org/10.5951/jresematheduc.16.5.0324

Cowley, S. J., & Vallée-Tourangeau, F. (2017). Thinking, values and meaning in changing cognitive ecologies. In S. J. Cowley & F. Vallée-Tourangeau (Eds.), *Cognition beyond the brain: Computation, interactivity and human artifice* (2nd ed., pp. 1–17). Springer International Publishing. https://doi.org/10.1007/978-3-319-49115-8_1

Cox, D., & Harper, S. (2017). Using narratives to articulate mathematical problem solving and posing in a technological environment. In E. Galindo & J. Newton (Eds.), *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 985-988). https://files.eric.ed.gov/fulltext/ED581294.pdf

Damasio, A. (2000). *The feeling of what happens: Body, emotion and the making of consciousness*. Vintage.

Damasio, A. (2006). *Descartes' error: Emotion, reason and the human brain* (revised edition). Vintage.

Da Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutiérrez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, present and future* (pp. 461–494). Brill Sense. https://doi.org/10.1163/9789087901127_017

Davis, B. (1996). *Teaching mathematics: Towards a sound alternative*. Garland Publishing.

Davis, B. (2004). *Inventions of teaching: A genealogy*. Lawrence Erlbaum Associates.

Davis, B., & Sumara, D. (1997). Cognition, complexity, and teacher education. *Harvard Educational Review*, *67*(1), 105–126. https://doi.org/10.17763/haer.67.1.160w00j113t78042

de Freitas, E. (2007). Research fictions: Arts-informed narratives that disrupt the authority of the text. *Interchange*, *38*(4), 335–350. https://doi.org/10.1007/s10780-007-9035-y

Derrida, J., & Prenowitz, E. (1995). Archive fever: A Freudian impression. *Diacritics, 25*(2), 9–63. https://doi.org/10.2307/465144

Dewey, J. (1910). *How we think.* D. C. Heath. https://bef632.files.wordpress.com/2015/09/dewey-how-we-think.pdf

Dewey, J. (1922). *Human nature and conduct: An introduction to social psychology*. Henry Holt. https://doi.org/10.1037/14663-000

Di Paolo, E. (2009). Extended life. *Topoi, 28*(9), 9–21. https://doi.org/10.1007/s11245-008-9042-3

Doig, B., & Groves, S. (2011). Japanese lesson study: Teacher professional development through communities of inquiry. *Mathematics Teacher Education and Development, 13*(1), 77–93. https://mted.merga.net.au/index.php/mted/article/view/47

Dorier, JL., & Maass, K. (2020). Inquiry-based mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 384–388). Springer. https://doi.org/10.1007/978-3-030-15789-0_176

Dreyfus, H. & Wrathall, M. A. (2014). *Skillful coping: Essays on the phenomenology of everyday perception and action*. Oxford Scholarship Online. https://doi.org/10.1093/acprof:oso/9780199654703.001.0001

Dreyfus, H., & Dreyfus, S. (1986). *Mind over machine: The power of human intuition and expertise in the era of the computer*. The Free Press.

Dreyfus, H. L., & Dreyfus, S. E. (1991). Towards a phenomenology of ethical expertise. *Human Studies*, *14*, 229–250. https://doi.org/10.1007/bf02205607

Eakin, P. J. (1985). *Fictions in autobiography: Studies in the art of self-invention.* Princeton University Press. https://doi.org/10.1515/9781400854790

Eisner, E. W. (1993). Forms of understanding and the future of educational research. *Educational Researcher*, *22*(7), 5–11. https://doi.org/10.3102/0013189x022007005

Ellis, C. (2004). *The ethnographic I: A methodological novel about autoethnography*. AltaMira Press.

Etherington, K. (2004). *Becoming a reflexive researcher: Using our selves in research*. Jessica Kingsley.

Festinger, L. (1957). *A theory of cognitive dissonance.* Stanford University Press.

Freeman, M. (1993). *Rewriting the self: History, memory, narrative*. Routledge.

Freeman, M. (1998). Mythical time, historical time, and the narrative fabric of the self. *Narrative Inquiry, 8*(1), 27–50. https://doi.org/10.1075/ni.8.1.03fre

Freeman, M. (2007). Autobiographical understanding and narrative inquiry. In D. J. Clandinin (Ed.), *Handbook of Narrative Inquiry: Mapping a methodology* (pp. 120–145). Sage. https://doi.org/10.4135/9781452226552.n5

Freeman, M. (2010). *Hindsight: The promise and peril of looking backward.* Oxford University Press.

Freeman, M. (2015). Narrative as a mode of understanding: Method, theory, praxis. In A. De Fina & A. Georgakopoulou (Eds.), *The Handbook of Narrative Analysis*. (pp. 21–37). Wiley. https://doi.org/10.1002/9781118458204.ch1

Gallagher, S. (2007). Pathologies in narrative structures. *Philosophy (Royal Institute of Philosophy) supplement*, 60, 65–86.

Gallagher, S. (2017). *Enactivist interventions: Rethinking the mind*. Oxford Scholarship Online. https://doi.org/10.1093/oso/9780198794325.001.0001

Gough, N. (1994). Narration, reflection, diffraction: Aspects of fiction in educational inquiry. *The Australian Educational Researcher*, *21*, 47–76. https://doi.org/10.1007/bf03219574

Greene, M. (1995). *Releasing the imagination: Essays on education, the arts, and social change*. Jossey-Bass.

Gusdorf, G. (1980). Conditions and limits of autobiography. In J. Olney (Ed.), *Autobiography: Essays theoretical and critical* (J. Olney, Trans., pp. 28-48). Princeton University Press. (Original work published 1956.) https://doi.org/10.1515/9781400856312.28

Hannula, M. S. (2003). Fictionalising experiences–experiencing through fiction. *For the Learning of Mathematics*, *23*(3), 31–37.

Helliwell, T. (2017a). Salient moments: The potential for using multiple perspectives in mathematics teacher educator learning. In F. Curtis (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, 37*(1), 1–6. http://www.bsrlm.org.uk/wp-content/uploads/2017/06/BSRLM-CP-37-1-07.pdf

Helliwell, T. (2017b). Learning to respond: The use of metacommunication as a mathematics teacher educator. In F. Curtis (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 37(3), 1–6 https://bsrlm.org.uk/wp-content/uploads/2017/12/BSRLM-CP-37-3-06.pdf

Helliwell, T. (2018). Responding to teachers: Learning how to use verbal metacommunication as a mathematics teacher educator. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 75-82). http://www.igpme.org/publications/current-proceedings/

Helliwell, T. & Brown, J. (2019). Formulating our formulations: The emergence of conviction as becoming mathematics teacher educators. In U. T. Jankvist, M. van den Heuvel-Panhuizen & M. Veldhuis, *Proceedings of the 11th Congress of the European Society for Research in Mathematics Education*. https://hal.archives-ouvertes.fr/hal-02422547

Helliwell, T. & Brown, J. (2020). Seeing more, seeing differently as mathematics teacher educator-researchers. *For the Learning of Mathematics: Proceedings of a symposium on learning in honour of Laurinda Brown*, Monograph 1, 9–14.

Hutchins, E. (2010a). Enaction, imagination, and insight. In J. Stewart, O. Gapenne, & E. A. Di Paolo (Eds.), *Enaction: Toward a new paradigm for cognitive science* (pp. 425–450). The MIT Press Scholarship Online. https://doi.org/10.7551/mitpress/9780262014601.003.0016

Hutchins, E. (2010b). Cognitive ecology. *Topics in Cognitive Science, 2*(4), 705–715. https://doi.org/10.1111/j.1756-8765.2010.01089.x

Hutchins, E. (2014). The cultural ecosystem of human cognition. *Philosophical Psychology, 27*(1), 34–49. https://doi.org/10.1080/09515089.2013.830548

Jaworski, B. (1990). Video as a tool for teachers' professional development. *Professional Development in Education, 16*(1), 60–65. https://doi.org/10.1080/0305763900160112

Kahneman, D. (2011). *Thinking, fast and slow*. Penguin Group.

Kerby, A. P. (1991). *Narrative and the self*. Indiana University Press.

Kieren, T. (2001). In focus...in God's image and presence: Some notes based on an enactive view of human knowing. *The Mathematics Educator, 12*(1). 1–7.

Lakoff, G. (1987). *Women, fire, and dangerous things: What categories reveal about the mind*. University of Chicago Press.

Langer, S. K. (1953). *Feeling and form: A theory of art*. Charles Scribner's Sons.

Lather, P. (1991). *Getting smart: Feminist research and pedagogy with/in the postmodern*. Routledge.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge University Press. https://doi.org/10.1017/cbo9780511815355

Lejeune, P. (2009). *On diary*. University of Hawaii Press. https://doi.org/10.1515/9780824863784

Lincoln, Y. & Denzin, N. (2000). The seventh moment: Out of the past. In N. Denzin & Y. Lincoln (Eds.), *Handbook of Qualitative Research* (2nd ed., pp. 1047–1065). Sage.

Lutovac, S., & Kaasila, R. (2014). Pre-service teachers' future-oriented mathematical identity work. *Educational Studies in Mathematics, 85*(1), 129–142. https://doi.org/10.1007/s10649-013-9500-8

Maclure, M. (2003). *Discourse in educational and social research*. Open University Press.

Maheux, JF., & Proulx, J. (2015). Doing|mathematics: Analysing data with/in an enactivist-inspired approach. *ZDM Mathematics Education, 47*(2), 211–221. https://doi.org/10.1007/s11858-014-0642-7

Mandelbaum, M. (1955). *The phenomenology of moral experience*. The Free Press.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Lawrence Erlbaum. https://doi.org/10.4324/9780203053690

Mason, J. (1998). Researching from the inside in mathematics education. In A. Sierspinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A Search for identity* (an ICMI study, book 2) (pp. 357–377). Kluwer. https://doi.org/10.1007/978-94-011-5470-3_23

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge. https://doi.org/10.4324/9780203471876

Mason, J. (2003). Seeing worthwhile things: A response to Alan Schoenfeld's review of researching your own practice. *Journal of Mathematics Teacher Education, 6*(3), 281–292. https://doi.org/10.1023/a:1025127913916

Mason, J. (2010). Attention and intention in learning about teaching through teaching. In R. Leikin & R. Zazkis (Eds.), *Learning through teaching mathematics: Development of teacher's knowledge and expertise in practice* (pp. 23–47). Springer. https://doi.org/10.1007/978-90-481-3990-3_2

Mason, J. (2011). Noticing: Roots and branches. In M. Sherin, V. Jacobs, & R. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes*. (pp. 35–50). Routledge.

Mason, J. (2017). Probing beneath the surface of experience. In E. O. Schack, M. H. Fishher & J. A. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 1–17). Springer. https://doi.org/10.1007/978-3-319-46753-5_1

Mason, J., & Davis, B. (2013). The importance of teachers' mathematical awareness for in-the-moment pedagogy. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 182–197. https://doi.org/10.1080/14926156.2013.784830

Maturana, H. (1987). Everything is said by an observer. In W. Thompson (Ed.), *Gaia, a way of knowing: Political implications of the new biology* (pp. 65–82). Lindisfarne Press.

Maturana, H. R. (1988a). Ontology of observing: The biological foundations of self-consciousness and the physical domain of existence. In R. E. Donaldson (Ed.) *Texts in cybernetic theory: An in-depth exploration of the thought of Humberto Maturana, William T. Powers, and Ernst von Glasersfeld*. American Society for Cybernetics. https://cepa.info/597

Maturana, H. R. (1988b). Reality: The search for objectivity or the quest for a compelling argument. *The Irish Journal of Psychology, 9*(1), 25–82. https://doi.org/10.1080/03033910.1988.10557705

Maturana, H. R. (2002). Autopoiesis, structural coupling and cognition: A history of these and other notions in the biology of cognition. *Cybernetics & Human Knowing, 9*(3–4), 5–34.

Maturana, H. R., & Varela, F. J. (1980). *Autopoiesis and cognition: The realization of the living*. D. Reidel.

Maturana, H. R., & Varela, F. J. (1998). *The tree of knowledge: The biological roots of human understanding* (R. Paolucci, Trans.). Shambhala. (Originally published 1987).

Maturana Romesin, H., & Verden-Zöller, G. (2008). *The origin of humanness in the biology of love*. Imprint Academic.

Merleau-Ponty, M. (1962). *Phenomenology of perception*. (C. Smith, Trans.). Routledge. https://doi.org/10.4324/9780203981139

Morris, D. B. (2001). Narrative, ethics, and pain: Thinking with stories. *Narrative*, *9*(1), 55–77.

Nagel, T. (1974). What is it like to be a bat? *The Philosophical Review*, *83*(4), 435–450. https://doi.org/10.2307/2183914

Nardi, E. (2016). Where form and substance meet: Using the narrative approach of *re-storying* to generate research findings and community rapprochement in (university) mathematics education. *Educational Studies in Mathematics, 92*(3), 361–377. https://doi.org/10.1007/s10649-015-9643-x

Nicol, C. (1997). *Learning to teach prospective teachers to teach mathematics*, PhD thesis, The University of British Columbia, Vancouver, B.C. https://open.library.ubc.ca/cIRcle/collections/ubctheses/831/items/1.0054692

Oslund, J. A. (2012). Mathematics-for-teaching: What can be learned from the ethnopoetics of teachers' stories? *Educational Studies in Mathematics*, *79*, 293–309. https://doi.org/10.1007/s10649-011-9348-8

Polanyi, M. (1966). *The tacit dimension*. University of Chicago Press.

Pimm, D. (1993). From should to could: Reflections on possibilities of mathematics teacher education, *For the Learning of Mathematics, 13*(2), 27–32.

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. In J. A. Hatch & R. Wisniewski (Eds). *Life history and narrative* (pp. 5–23). The Falmer Press. https://doi.org/10.4324/9780203486344-3

Proulx, J. (2008). Some differences between Maturana and Varela's theory of cognition and constructivism. *Complicity: An international journal of complexity and education, 5*(1), 11–26. https://doi.org/10.29173/cmplct8778

Reid, D. A., & Mgombelo, J. (2015). *Survey of key concepts in enactivist theory and methodology*. *ZDM Mathematics Education, 47*(2), 171–183. https://doi.org/10.1007/s11858-014-0634-7

Reid, D., Brown, L., & Helliwell, T. (2017). Emotional orientations and somatic markers: Expertise and decision making in the mathematics classroom. In U. X. Eligio (Ed.), *Understanding emotions in mathematical thinking and learning* (pp. 433–449). Elsevier Academic Press. https://doi.org/10.1016/b978-0-12-802218-4.00017-0

Richardson, L. (1999). Feathers in our cap. *Journal of Contemporary Ethnography, 28*(6), 660–668. https://doi.org/10.1177/089124199129023767

Richardson, L. (2000). Writing: A method of inquiry. In N. Denzin, & Y. Lincoln (Eds.), *Handbook of Qualitative Research* (2nd ed., pp. 923–948). Sage.

Richardson, L., & St. Pierre, E. A. (2018). Writing: A method of inquiry. In N. Denzin, & Y. Lincoln (Eds.), *The Sage Handbook of Qualitative Research* (5th ed., pp. 818–838). Sage.

Ricoeur, P. (1981). Narrative time. In W. J. T. Mitchell (Ed.), *On narrative* (pp. 165–186). University of Chicago Press.

Riessman, C., & Speedy, J. (2007). Narrative inquiry in the psychotherapy professions: A critical review. In D. J. Clandinin (Ed.), *Handbook of Narrative Inquiry: Mapping a methodology* (pp. 426–456). Sage. https://doi.org/10.4135/9781452226552.n17

Rosch, E. (1999). Reclaiming concepts. In R. Nunez & W. Freeman (Eds.), *Reclaiming cognition: The primacy of action, intention and emotion*. Imprint Academic. Published simultaneously in *The Journal of Consciousness Studies*, *6*(11–12), 61–77.

Rosch, E., Mervis, C. B., Gray, W. D., Johnson, D. M., & Boyes-Braem, P. (1976). Basic objects in natural categories. *Cognitive Psychology, 8*(3), 382–439. https://doi.org/10.1016/0010-0285(76)90013-x

Ryle, G. (1945). Knowing how and knowing that: The presidential address. *Proceedings of the Aristotelian Society, New Series, Vol. 46* (pp. 1–16). Oxford University Press. https://doi.org/10.1093/aristotelian/46.1.1

Rosen, H. (1998). *Speaking from memory: The study of autobiographical discourse*. Trentham Books.

Rossholm, G. (2017). Mediating immediacy. In T. Koblížek (Ed.), *The aesthetic illusion in literature and the arts* (pp. 85–106). Bloomsbury. https://doi.org/10.5040/9781350032675.ch-003

Roth, W. M. (2005). Auto/biography and auto/ethnography: Finding the generalized other in the self. In W. M. Roth (Ed.), *Auto/biography and auto/ethnography: Praxis of research method* (pp. 1–16). Brill Sense. https://doi.org/10.1163/9789460911408_002

Sacks, H., Schegloff, E. A., & Jefferson, G. (1978). A simplest systematics for the organization of turn taking for conversation. In *Studies in the organization of conversational interaction* (pp. 7–55). Academic Press. https://doi.org/10.1016/b978-0-12-623550-0.50008-2

Sarbin, T. (2004). The role of imagination in narrative construction. In C. Daiute & C. Lightfoot (Eds.), *Narrative analysis: Studying the development of individuals in society* (pp. 5–20). Sage. https://doi.org/10.4135/9781412985246.n1

Sartre, J. P. (1965). *Nausea* (R. Baldick, Trans.). Penguin Books Ltd. (Originally published 1938).

Simmt, E., & Kieren, T. (2015). Three "moves" in enactivist research: A reflection. *ZDM Mathematics Education, 47*(2), 307–317. https://doi.org/10.1007/s11858-015-0680-9

Smart, P. R., Heersmink, R., & Clowes, R. W. (2017). The cognitive ecology of the internet. In S. J. Cowley & F. Vallée-Tourangeau (Eds.), *Cognition beyond the brain:*

Computation, interactivity and human artifice (2nd ed., pp. 251–282). Springer International Publishing. https://doi.org/10.1007/978-3-319-49115-8_13

Speedy, J. (2008). *Narrative inquiry and psychotherapy*. Palgrave Macmillan.

Steward, D. (2020). https://donsteward.blogspot.com/

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226. https://doi.org/10.1016/s0742-051x(00)00052-4

Suazo-Flores, E. (2017). *Two mathematics teachers' personal practical knowledge: Experiences making curriculum within the 3D inquiry space*. Graduate Thesis, Purdue University. https://docs.lib.purdue.edu/dissertations/AAI10684004

Tahta, D. (1972). *A Boolean anthology. Selected writings of Mary Boole - on mathematical education*. Association of Teachers of Mathematics.

Tahta, D. (1991). Understanding and desire. In D. Pimm & E. Love (Eds.), *Teaching and learning school mathematics* (pp. 221–246). Hodder & Stoughton.

Taylor, C. (1989). *Sources of the self: The making of modern identity*. Cambridge University Press.

Taylor, C. (1995). The dialogical self. In R. F. Goodman & W. R. Fisher (Eds.), *Rethinking knowledge: Reflections across the disciplines*, (pp. 57–66). State University of New York Press.

Tripp, D. (1993). *Critical incidents in teaching: Developing professional judgement*. Routledge. E-book (2011), https://doi.org/10.4324/9780203802014

Tyminski, A. M. (2010). Teacher lust: Reconstructing the construct for mathematics instruction. *Journal of Mathematics Teacher Education 13*(4), 295–311. https://doi.org/10.1007/s10857-009-9135-y

Tzur, R. (2001). Becoming a mathematics teacher-educator: Conceptualising the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education*, *4*(4), 259–283. https://doi.org/10.1023/a:1013314009952

van Manen, M. (1997). *Researching lived experience: Human science for an action sensitive pedagogy.* The Althouse Press. 2nd ed., https://doi.org/10.4324/9781315421056

van Manen, M. (1991). *The tact of teaching: The meaning of pedagogical thoughtfulness.* State University of New York Press.

Varela, F. (1999). *Ethical know-how: Action, wisdom and cognition*. Stanford University Press.

Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. The MIT Press. https://doi.org/10.7551/mitpress/6730.001.0001

Voss, T., Kleickmann, T., Kunter, M., & Hachfeld, A. (2013). Mathematics teachers' beliefs. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers—Results from the COACTIV project* (pp. 249–272). Springer. https://doi.org/10.1007/978-1-4614-5149-5_12

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge University Press. https://doi.org/10.1017/cbo9780511803932

Whitehead, J. (2000). How do I improve my practice? Creating and legitimating an epistemology of practice. *Reflective Practice, 1*(1), 91–104. https://doi.org/10.1080/713693129

Winter, R. (1988) Fictional-critical writing: An approach to case study research by practitioners and for in-service and pre-service work with teachers. In J. Nias & S. Groundwater-Smith (Eds.), *The enquiring teacher: Supporting and sustaining teacher research* (pp. 231–248). Routledge. https://doi.org/10.4324/9780203058855-23

Wittgenstein, L. (1975). *On certainty*. G. E. M. Anscombe & G. H. von Wright (Eds.). (D. Paul & G.E.M. Anscombe, Trans.). Blackwell.

Young, J. E. (1988). *Writing and rewriting the Holocaust: Narrative and the consequences of interpretation*. Indiana University Press.

Zazkis, R., Liljedahl, P., & Sinclair, N. (2009). Lesson plays: Planning teaching versus teaching planning. *For the Learning of Mathematics*, *29*(1), 40–47.

Appendices

Appendix one: Glossary

British Society for Research into Learning Mathematics (BSRLM)

A tri-yearly UK national mathematics education conference day conference.

Department for Education (DfE)

The Department for Education is responsible for children's services and education, including early years, schools, higher and further education policy, apprenticeships and wider skills in England.

Dienes blocks

Concrete representations of numbers (usually wooden or plastic cubes, rods and flats) that are in exact proportion to each other, so they can represent all powers of tens, such as ones, tens, hundreds and thousands.

Early entry

Early entry refers to situations in England in which students complete a national qualification before the intended entry date. In the case of GCSE (General Certificate in Secondary Education) this is usually one year early, at the end of year 10 (age 15 years) rather than year 11 (age 16 years) as is normal.

Five-number summary

A set of descriptive statistics that provides information about a dataset. It consists of the five most important sample percentiles: minimum, maximum, sample median, and first and third quartiles.

Gap task

A project term used to refer to specific tasks that participating teachers were asked to complete between workshops.

Iterative methods

A mathematical procedure for generating a sequence of improving approximate solutions from an initial value.

Japanese Lesson Study/Lesson Study triad

Lesson Study is a form of professional development for teachers that originated in Japan. It is a model of practitioner research where a triad of teachers work collaboratively to focus on a particular area of their students' learning. Lesson study can be described as having the following four phases: goal-setting and planning – including the development of the lesson plan; teaching the research lesson – enabling the lesson observation; the post-lesson discussion; and the resulting consolidation of learning, which has many far-reaching consequences (Doig & Groves, 2011)

Key stage 3

Phase of schooling from years 7-9 (aged 11-14 years) in England.

Lead practitioner

A lead practitioner can have responsibility within a school (and more widely) for the development of teaching and learning within a particular subject area.

Lesson de-brief conversation

After a lesson observation, where the prospective teacher is observed by the school-based mentor and the university tutor, there is a three-way "lesson de-brief conversation" (Brown, Brown, Coles & Helliwell, 2019) focussed on the lesson taught.

Maths Hub

Maths Hubs are funded centrally by the Department for Education (DfE). Maths Hubs are described as mathematics leadership networks which consist of schools, colleges, and other organisations with mathematics education expertise from across the hub's area. Maths Hubs work in partnership with the National Centre for Excellence in the Teaching of Mathematics (NCETM). At that time, there were 35 strategically situated Maths Hubs (there are now 40) across England.

Median (resources)

A collection of online mathematics education resources (designed by Steward), found at: https://donsteward.blogspot.com/

Mixed ability

A term commonly used in England to refer to a mathematics class that has not been grouped by prior attainment. Students in the group will have more varied prior attainment than when groups have been set.

National Centre for Excellence in the Teaching of Mathematics (NCETM)

A national organisation, funded by the DfE, that was set up in 2006 to support teachers of mathematics by providing mathematics specific professional development.

Non-specialist

A teacher who is teaching a subject that was not their trained specialism.

NRICH

NRICH is a collaboration between the Faculties of Mathematics and Education at the University of Cambridge. NRICH provides free online mathematics resources for ages 3 to 18. According to the NRICH website, they aim to: Enrich and enhance the experience of the mathematics curriculum for all learners; Develop mathematical thinking and problem-solving skills; Offer challenging, inspiring and engaging activities; Show rich mathematics in meaningful contexts; Work in partnership with teachers, schools and other educational settings to share expertise (see https://nrich.maths.org).

Ofsted

Ofsted is the Office for Standards in Education, Children's Services and Skills. Ofsted is an independent government department that reports directly to Parliament. Ofsted is responsible for inspecting those services providing education and skills for learners of all ages.

Post Graduate Certificate in Education (PGCE)

The PGCE is a one-year postgraduate course that leads to qualified teacher status and master's level credits. Prospective teachers spend around one third of the course attending university and the other two thirds of the course in local secondary schools.

PGCE Subject session

These are sessions run by the PGCE university tutors with the cohort of prospective teachers. The focus of mathematics subject sessions varies, but usually involves doing mathematics together and considering pedagogical and/or teaching practice issues.

Programme for International Student Assessment (PISA)

An international study that was launched in 1997, first administered in 2000 and now covers over 80 countries. Every 3 years the PISA survey provides comparative data on 15-year-olds' performance in reading, mathematics and science.

Project lead

The maths hub project lead is usually a mathematics teacher or member of the maths hub who is not a participating mathematics teacher. They organise and run the project and associated workshops.

Qualified Teacher Status (QTS)

This is a status that qualifies you to teach in any school in England.

Safeguarding

Safeguarding means keeping people safe from harm. This includes: Protecting children from maltreatment; preventing impairment of children's health or development; ensuring that children grow up in circumstances consistent with the provision of safe and effective care; and taking action to enable all children to have the best outcomes.

School-based mentor

A school-based mentor is a mathematics teacher who has responsibility for mentoring the prospective mathematics teacher when they are on school placement.

Starter

A term used to indicate the first task/activity at the beginning of a mathematics lesson.

Structured activities/structures

Structured activities are structures that are designed to promote mathematical reasoning. They are not content specific, rather they offer a framework for the activity itself. Examples of a structured activity or structure include: Here are some calculations - what could be the questions?; sequencing solutions; always, sometimes, never; What do you know?; What questions could you ask?; and hide and reveal (see appendix 3h, page 412, for the complete set offered at the initial workshop).

Subject knowledge enhancement course/Subject specialism course

This is a course for non-specialist teachers designed to improve subject knowledge.

Top set

Schools often group students in classes depending on their prior attainment. The class with students having the highest prior attainment are often referred to in England as the top set.

Trends in International Mathematics and Science Study (TIMSS)

Provides trend data on the mathematics and science achievement of U.S. students compared to that of students in other countries. TIMSS data have been collected from students at grades 4 (students aged 9-10 years) and 8 (student aged 13-14 years) every 4 years since 1995.

Appendix two: Context one

Appendix 2a: Notes from initial discussion with Sam

We talked about your new role (lead practitioner) now being focused on the teaching and learning process rather than outcomes which you were very energised about. This new role includes 'exhibition' lessons where NQTs observe, and you follow up with discussion at a later time when you meet with them. You also do 'mini teach' on Fridays where you demonstrate how a concept may be taught which teachers come along to.

You are also in a coaching pair and had an initial conversation with your partner. From that came the issue of wanting your students to be more 'independent' you also talked about 'co-construction', 'resilience' and 'retention of key concepts'. Creating a culture where it is 'ok to make mistakes' and where students are 'less passive'. We talked a little about the school focus on being 'ready to learn' and how this system may impact on the lessons.

There was an interesting anecdote about the 'best mathematician in year 10' who hadn't realised how the equation of a graph relates to the co-ordinates on that graph – in particular that when you solve simultaneous equations the intersections will occur when the co-ordinates are the same for both graphs and that this always works. I found this fascinating.

You talked about the fact that you have a background in science – rather than mathematics – and how this meant that you didn't take the understanding of mathematics for granted. You felt this was beneficial to you when explaining concepts. You are keen to build children's confidence through success in mathematics. You wanted students to be allowed to fail to strengthen their understanding. This led to the following questions/issues…

Is my tendency to 'jump in' proving to have a deleterious effect on their learning?

If I am not going to 'jump in' how do I build a culture where it is safe to make mistakes and they build resilience?

Feels like a perfect starting point!

Appendix 2b: Partial transcript of post-lesson conversation with Sam

Black text used directly in main body (Sections 4.1, 4.1.1, 4.1.2, pages 57-67 & 74-77). Red text was not used directly in main body. Some sections not transcribed – marked in blue as "section not transcribed":

Time	Name	Transcript
Section not transcribed.		
09:58	Sam	I think we teach statistics really badly.
10:00	Tracy	You mean as a process?
10:02	Sam	Totally! I don't think we teach statistics with enough real data. So, when it comes to a question where the students have to analyse two different data sets, like they were doing in the lesson today, the data doesn't mean anything to them. The first box plots we ever looked at as a class were the boys' mock exam results versus the girls' mock exam results, and it really meant something to them. The upper whisker for the boys' box plot was huge the top quartile for boys was really spread out, whereas the same section for females was much shorter, and not as far along the scale.
Section not transcribed.		
13:29	Tracy	So why did you choose the data that you used today? The ninety pieces.
13:32	Sam	Oh. Because it was just there, so it was quick.
13:35	Tracy	But I think it was nice having that level of complexity.
13:42	Sam	I always have in mind, what is going to push those two [pointing towards where two students were sat] a little bit further, so when I saw it was point two seven five, I thought yeah let's give it a go. That is going to sort that table out.
14:00	Tracy	I really liked the ninety pieces of data. I think the only thing a few of them seemed to struggle with was finding the forty fifth and forty sixth pieces, just because they were trying to count off the board [the set of data (consisting of 90 separate pieces) was displayed on the board as a list], so they were getting lost in the list of numbers.
14:25 14:48	Sam	Traditionally I've taught box plots from stem and leaf diagrams, it's so much easier to teach from a stem and leaf because you can just strike the pieces of data off from both ends, and you've still got your data there. They don't need to know about stem and leaf diagrams anymore though, so it's a case of finding a new way to teach it and it's trickier from a list. I think that one [pointing to the initial question involving 90 data points] is probably harder than anything the students will have to do in an exam
14:55	Tracy	Right so everything is covered by that one question.
15:00	Tracy	So will you go back and use that data again?
15:04	Sam	I don't know, the range goes from thirty to one thousand four hundred and forty-one so it would be quite difficult to draw that accurately on their exercise-book paper. I think I will do one tomorrow with nice traditional whole number values for all those things; then I will do one that is a little bit more complicated but where they are all halves; and then I'll go back to that data I think.
15:44	Tracy	I saw a nice lesson on box plots a few years ago where the teacher had got some data from the school on rewards and he had it organised by year group or by house. The students had a connection with the data, like you were saying with your mock exam data, yet they didn't need to worry about collecting it for themselves. I realise they need to know how to do that as well, but actually the sophisticated bit is forming the statements about what is different about the distributions and why, which you talked about in your lesson. This teacher had a series of box plots, without any scale, and asked them what's the same, what's different. It seemed to generate a really rich discussion. He might even have asked them to suggest possible datasets that were represented by the box plots, before revealing to them what they actually were.
17:00	Sam	Someone in our department does a human box plot lesson, so they line them up, with whatever it is, by height possibly, so they physically have to bunch together to get on a scale.

Time	Name	Transcript
17:30	Tracy	That's nice isn't it, you could draw a scale on the floor, use some tape maybe, get the students to move around based on different prompts or criteria. You could ask students to articulate what they notice.
17:38	Sam	I could maybe do that tomorrow, see how energetic I feel. I might end up with something awful like twenty-nine kids, no hold on, that might not be so bad actually.
Section not transcribed.		
20:56	Tracy	So, is there anything you would do differently?
21:00	Sam	I might have had an easier set of data for some of the kids.
21:14	Tracy	Than the ninety? I'm not sure. It felt to me like if it was made bigger on the screen and it was just the data on its own without the stuff around it that made it look more complicated.
21:24	Sam	Yeah. It did have a few clues though didn't it.
21:28	Tracy	Yeah that was quite nice.
Section not transcribed.		
24:48	Tracy	So, is there anything *else* you would do different?
25:00	Sam	Um, well, I would like to think I could find a way to talk less but have the same opportunity for discussion before they actually get into a question.
25:15	Tracy	Why do you want to talk less? Is that a general thing?
25:20	Sam	Well, I don't know; I think it all links to getting them working harder than me.
25:26	Tracy	Right, OK.
25:30	Sam	But then, I think it's quite an easy thing to criticise how long a teacher talks for. There are some people in this school that have a bit of a thing about this. We have been told recently that the teaching should all be done in the first ten minutes of the lesson.
25:55	Sam	Apparently, we need to get the kids doing more, more often, more involved, more active, and I think somebody might have potentially said there was too much passiveness. I think the view is that when the teacher is talking, the kids aren't active, and they're not engaged. But I'm thinking and I'm watching and I'm checking all of the time, I'm questioning kids, using their names. I think they are involved. It's interesting, maybe I'm just biased with my kids.
Section not transcribed.		
27:10	Tracy	And what do *you* think about that?
27:17	Sam	I don't think my kids look switched off: I think they engage with the white board; I think they've got pens in their hands; I think they are taking notes; when I ask them a question, they have all listened to the question, they might not have an answer but that's not because they haven't been listening. So, I think it is quite an easy thing to say, "If you talk for less, your lessons will be better". Actually, if you take what happened when I said, "Now do question one and question three", with the exception of three groups who just didn't read the question properly, they all knew what they were doing. They got on with it; it was quite a good level of challenge. So, in one sense you could argue I judged the amount of teacher talk just right.
28:30	Tracy	So, can you unpick what you meant by finding a way to talk less, because I had felt that this was something that had come from you. You mentioned wanting the students to work harder than you. So, is it the amount of talk, or the type of talk, because I think what you said was, something about discussion, getting them to discuss more?
29:20	Sam	Yeah, getting them doing more modelling maybe, more examples, more explanation, and I did try it actually, two weeks ago. We were doing something, I can't remember what it was, but it was very obvious to me that [5 students] had completely grasped every element of what I had been talking about, didn't need any more revision of it, so I found the hardest exam question I could find and they got full marks on it. So, I gave them my textbooks for the new syllabus and said teach yourself iterative methods and then teach the class, that was what I asked them to do. They taught themselves, they found a method that worked, they played with their calculators and

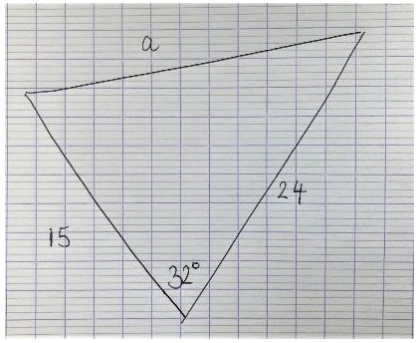
Time	Name	Transcript
		I think they had a really good fifty-minute session, actually, thrashing it out. I then gave them another half a fifty-minute session to plan what they were going to do and then deliver it to the class. I thought this will get them working harder than me. Dangers flash up though, they're not teachers.
30:35	Sam	As soon as they put an example on the board I thought, oh I can see where this is going, they didn't know though, and the example did exactly what I thought it would do and it went really badly. A lot of kids were saying, "I don't get it", "you're rubbish at explaining", kids that wouldn't say that to me. So, I ended up having to teach it again anyway. How can you be responsible for the quality of the initial teaching of the topic whilst still letting them be involved?
31:21	Tracy	So, what other strategies are there, where you are still teacher if you like, but you're not doing all the talking or, or questioning?
31:35	Sam	I think actually, it is not a bad strategy, I just needed to invest some more time in it. So, rather than saying, just you five now teach everybody, I should have got the rest of the class doing something where they didn't need me, and then asked the group to teach me before they teach the class, then I could have given them some feedback. In fact, I did a lesson where I gave the class a whole exam paper where each question was on a different piece of A3 paper. They then went around with post-it notes, visiting each question, making helpful hints, bits of maths that may be useful, maybe even begin solving some of them, so everybody saw every question. I then gave them a whole lesson where each pair were allocated one question each and were asked to answer the question and the following lesson, every pair modelled their solution to the rest of the class. It could have been amazing if I had given everybody a blank copy of the exam paper to focus their listening onto. It was very good though, very student-led, I was able to go around to each group, ask them where they were going to start, give them some feedback, and because there were so many rich resources for them to use it worked really well. If you are going to do it, it doesn't work if you are only going to half commit to it. I have introduced impact marking[19] here and I had a lesson here where I put all of my I am threes, that's the people who really understood it, paired them up with the I am ones. It completely fell apart. I hadn't done enough scaffolding of what that meant. So, the next time I did it I spent some time with the I am threes explaining which questions I wanted them to go through, what the key learning points were, and how to check if their partners had got it. That time it worked really, really well. So, I guess the moral of the story is, if you are going to do it, you can't skimp on the time that it takes, I think the learning they got out of it was significant because it wasn't from me.
35:08	Tracy	I suppose, the lesson I saw that I was talking about involving box plots, the teacher had provided the stimulus, so he had done all of the thinking about what would generate some ideas so that in the lesson it was about him managing a rich discussion where students were able to share those ideas.
35:30	Sam	Was it a group discussion?
35:35	Tracy	A whole-class discussion, based on two box plots.
35:40	Sam	I think well managed whole-class discussion is an invaluable tool for the teacher, but is a really hard skill, because you've got to have all of your behaviour management sorted out; you've got to be prepared to think on your feet, so, what happens if somebody asks me a question that I don't know? So, there's all that subject knowledge, all that pedagogy, and I think it is easier for somebody to say whole-class discussion is not a useful tool, than to say, whole-class discussion is a useful tool when it is skilfully managed, but it's not often skilfully managed, so, think really carefully about whether you want to do it or not. It's almost like, let's get rid of those

[19] Impact marking is a marking system used by the school where students rate their own confidence at the end of the lesson as 'I am 1', 'I am 2' or 'I am 3' (with 'I am 3' as most confident) and make this known to the teacher, either by use of a coloured pen or by placing their exercise books in different coloured boxes at the end of the lesson.

Time	Name	Transcript
		things because it's easier to say, there's too much teacher talk in lessons, too much group discussion, not enough individual work. It's easier to say that than to actually unpick the fact that it can be an extremely powerful tool, if it's well used. That balance of too much teacher talk; I think that particular person might possibly say there was too much teacher talk in my lesson.
37:06	Tracy	But what do *you* think?
37:10	Sam	Well, I'm not sure there was too much teacher talk because once I set them off, and they got on with the questions, their focus was phenomenal, you know, they were talking about the maths. So, was there too much? Probably not, but that depends on who you ask, yes, if you don't want more than ten minutes teacher talk at the beginning of the lesson then there was too much.
37:50	Tracy	But I'm interested in what you want, because you brought it up yourself, there does seem to be something there for you.
38:00	Sam	Maybe there is something there, or I guess I could ask what my reason is for wanting to talk less. Is it because I feel I ought to, because we are being told to talk less? One of the things I pride myself on, with my kids, is that they will get the absolute best of me for the fifty minutes I'm in there. They might not get their books marked particularly brilliantly. I suppose in some ways I feel that I am the best person to teach this, so maybe it is me who does all the talking.
38:40	Tracy	Yeah, I think I would have the same worries about giving the students something to teach. We've spent a long time thinking about teaching mathematics, it's a really complex thing, and I know we say teaching something can reinforce the learning of that thing, so we get students to explain thing to one another, but that is different to teaching.
39:00	Sam	It is very different. When I brought impact marking into the department, I was very clear that the intention was not for the 'I am ones' to be paired up with 'I am threes' on a regular basis so that one group could help teach the other whilst the teacher can focus on the middle. But I think it can be valuable at times. Like I think that exam activity was valuable. What I was trying to break down was this idea that they have in their heads that it's because I'm good at maths that I can do those questions, I wanted to show them that actually the entire exam paper could be done by the individuals in this class, and I don't have to do a thing. I've done my bit, between all of you the knowledge is in the hive, your collective knowledge is enough to get through this paper.
40:05	Tracy	So maybe that's the link to this then, it is their collective knowledge, like you say, with you included in that, who is it they feel has the knowledge in that time where the whole class discussion is going on.
40:30	Sam	If you let go of the discussion too easily and too early, maybe it's just my inner control freak, but I think the really good quality learning breaks down quite quickly, because there is not enough there to build it on, not enough of a foundation. Whereas the conversations that they were having about the box plots were good, high quality discussions because there had been enough fertiliser, enough ground-work.
41:01	Tracy	OK, so you gave them a prompt or a source of discussion, and you gave them a few minutes to talk about it, it wasn't you telling them at every point what to do, explaining the procedure.
Section not transcribed.		

Appendix three: Context two

Appendix 3a: Full transcript of feedback session 1

Time	Name	Transcript	Phase one comments	Phase two comments	Label
01:30 01:45	Tracy	Welcome everybody, for the next hour or so we will be sharing our experiences of doing the gap task. It is important each person or pair has time to share, to get into some of the detail of what happened in the lessons as well as the process of working with one another and with your wider departments. You will, for example, need to explain to one another which structure you used and how. If you can bring to mind any particular moments that have stuck with you from the lesson or lessons then you can share those and you may have notes to refer to as well from any post-teaching discussions you were able to have. As listeners, try not to ask questions to begin with, whilst the person speaking is having their time, but do make a note of them so they don't get lost, you will get the opportunity to ask them. I don't mind who starts but whoever wants to start, just begin by telling us what happened. We can start anywhere.			*Detailed description*
03:08	Sam	I'll go if you want			
03:09	Tracy	Go on then [laugh]			
03:10	Sam	[laugh] ok so, we were really lucky that Paul had time to come out and bring us up to speed, problem solving is a big issue for us. That whole problem solving, fluency, mastery, all these reasoning things they're all kind of threaded together so I had more clarity, I think, after the session with Paul, that **it was alright that they threaded together, that you didn't have to make them distinct things**. So, I decided, I have a top set year ten and we had just done sine rule, cosine rule, and area of a triangle without knowing the perpendicular height. I went for the structure, here are some calculations that a student might have performed based on this stimulus [Sam holds up a diagram of a labelled triangle, 		But is anything distinct and would it be helpful to work on what is distinct about reasoning from these other 'skills'?	*Distinctions* ***[Dissonance]***

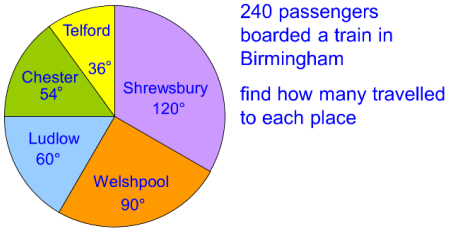
Time	Name	Transcript	Phase one comments	Phase two comments	Label
03:53 04:33 05:14 05:30 05:34 05:52 06:04 06:33		and a separate list of associated calculations, such as $15^2 + 24^2 - 2(15)(24)\cos 32^o$ and, $\sin^{-1}\left[\frac{24\sin 32^o}{15^2+24^2-2(15)(24)\cos 32^o}\right]$. I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? **I picked that structure because of all of the structures that Paul showed me, that was the one that made my head absolutely bend round. I thought, well if it makes my head bend, let's see how good my year tens really are. It completely split my class massively.** The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure. I don't think I had made a good enough job of making sure they understood the structure. **On reflection, I think I should have done something much simpler** first. I should have gone for something much simpler so they understood the idea before throwing in some quite horrific maths on them. I actually I did it and then two days later I taught it and some of those calculations I was thinking why on earth did I do that I can't remember what that's all about. So, but that was quite interesting because you know I think with them I'm kind of almost at the stage where they're testing my maths um and it was quite good for them to see you know I can't remember what that calculation was all about so let's try and pick it apart. I actually started with the same triangle and asked them to come up with a minimum of three possible examination questions that would need the sine rule, or whatever, to solve. That was fine. That was good, **but there was a massive jump** between that and the structure using the eight calculations. **In terms of the whole department** we have a lot of non-specialists. I mean, I'm a non-specialist, I'm a Physicist, I only did a subject knowledge enhancement course two years ago but my maths is ok**. I think a lot of my department would really struggle with opening themselves up mathematically to that so I think we probably have to teach the structure with something slightly simpler** and then encourage them to stay a little bit out of their comfort zone but not to the extent where they are completely freaked out. It took me ages to plan it. The first time I did it, I did it actually with Paul in the room, do you remember? I'd done it wrong, because I then thought I'd come back to it the next day and I thought I'd try it for myself and there were two of the calculations where the actual the basic maths didn't make any sense, so it was interesting. **It was quite hairy at times. The first task took about five minutes. The calculations task took about ten** so it was quite a meaty starter that could have gone into a whole lesson if I'd felt like it I think**. I think I should have allowed more time. I think I should have actually unpicked each one of the calculations**, but as it was the class were forming that kind of horrible situation where some got it, and others really didn't get it and were really cheesed off that they weren't getting it. So **I had to put some emergency repairs in place**. That's me.	Is this something you value? Something you find difficult is worthwhile maybe? What does this mean? Those that can and those that cannot? What is wrong with being challenging? Encourage them? Who the non-specialists or the students? Is this limiting? Hairy? I recognise that feeling. Emergency? This feels a little drastic, although I think I know what she means.		*Slide (from 'account-of' to 'account-for')* *Should/ought /had to* *Slide (from 'account-of' to 'account-for')* *Unresolved* *Speaking for others* *Should/ought /had to* *Teaching Issue* *Slide (from 'account-of' to 'account-for')* *Should/ought /had to*
06:57	Tracy	Has anybody got any questions?			

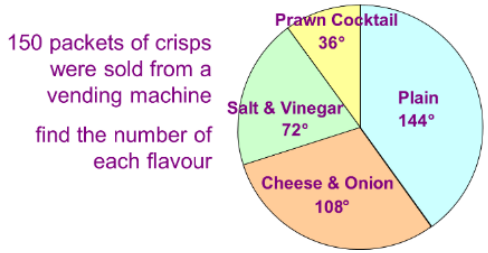
Time	Name	Transcript	Phase one comments	Phase two comments	Label
07:02	Beth	I was interested in what you were saying about the split of the class, **why do you think there were pupils who were really backing off from it?** What was the barrier there?		But what did Sam mean by split?	*Asking why?*
07:11	Sam	I think about a year ago, I was in a Saturday session with one of my teach first people who had just done some reading by some guy about mathematicians, really good mathematicians, **being either inchworms or grasshoppers. My grasshoppers who just fly from one concept to another had absolutely no problem with it at all. The inchworms who are more procedural were saying, I don't know what you want me to do. I understand that if I've got a triangle then I can take the sine, but where do I start with this thing that is completely different? So I think it might have been that. I don't really know.**	Challenge dichotomies	The danger of dichotomies perhaps it doesn't matter that I had not heard of these terms. One meta here could be that I reflect on the danger of dichotomy. Also the response didn't really answer Beth's question so perhaps one strategy for me is to check in with Beth to ask her if indeed this did answer her question or if she has a follow up question. Something I have been reminded of the difference between talking about a model of learning and enacting a model of learning. In my writing I am enacting this model of learning while simultaneously supporting the teachers' learning - Perhaps this is somehow a criteria for this research. This process that I am engaged in now around becoming sensitised to the distinctions that I am making has a purpose of supporting me to support the teachers in becoming sensitised to their own distinctions so my hypothetical responses need to reflect this. For example being explicit about new distinctions for me. Pointing out when teachers are making a distinction or asking a question of clarification which might lead to a distinction being made.	
07:48	Mia	**But doing activities like that is good for them then surely**, so they might battle against it at first but if they see more of it, start to get used to it that's gonna make them better.			
07:56	Sam	I think yeah I think that but **I should have done something to allow them to access the structure because they had to access the structure and access the maths.**			*Should/ought /had to*
	+	Um hum			
	Sam	and I think if I had broken that down a bit			
08:06	Joe	I was gonna ask, did you teach this on its own first or **did you do the structure as the way of explaining it?**	Structure to teach the maths or maths then structure.		*Distinctions*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Sam	No I had previously taught it about three weeks ago			
	Joe	Oh ok yeah			
	Sam	so it was kind of lets come back and see what you can remember			
08:19	Simon	So, with the structure how did you set it up in terms of discussion?	Nice question.	I feel like a meta-comment about each of these questions would be useful – what types of questions are these, what are they doing – perhaps to establish some norms for the group… This is such a nice comment and evidence of teachers supporting another teacher. Simon has located the issue but Sam is describing and rephrased it and is prompting for more detail about that particular moment that led to that particular issue.	*Detailed description*
08:24	Sam	I asked them to pick one that they could understand and one that they thought was wrong from the list of eight.			
	Simon	Cool.			
	Tracy	So there's some wrong ones, some ones that don't….		Even though I asked this clarifying question I didn't really hear the answer or I didn't hear the significance of the answer.	
08:34	Sam	Yeah, it would not give them anything useful, I wrote the answers over here, when I realised I had cocked it up I thought I better write this down so yeah there are numbers from here but they wouldn't actually allow them to access anything, and my IT skills are not up to putting that on a computer so that was a photocopy with my little um visualiser on it.		There is something here I hadn't caught before even though I've listened many times which is that Sam set up the structure with a variety of different calculations but she wanted the students to find the two that were nonsensical so there was a closed question in the sense that there was a particular solution to this but what about the range of other calculations it is a bit like find the one that doesn't belong.	
09:04	Simon	Mia and I followed a very similar structure.			*Unresolved*
	Sam	Did you?			
09:07	Simon	We did arcs and sector areas, so I just put a circle up, can you see that there [Simon points to an image including the stimulus and calculations shown]. I gave them these different calculations and said, "right, what do each of these formulas represent on the diagram?" Some of the calculations didn't actually represent anything in relation to the diagram and	A detailed description of a moment from the lesson I observed.		*Detailed description*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		in those cases, they had to either sketch or at least try and explain what that formula could be showing. So, it was a similar structure to yours [Sam]. Mia and I approached it quite similarly, didn't we?			
	Mia	Pretty much the same.			
09:35 10:05	Simon	Yeah, just pairs, talk about it, write something down, discuss with the people around you. That was the most powerful thing, the discussion that came out of it was really, really good. There were a few nice questions like, "if it is three hundred and sixty over two hundred and fifty-two, could that be used to do anything useful?" "What's the point in that?" We just tried to keep the numbers very, very similar. Some of the students were saying, "well maybe it's just a smaller circle". Someone raised the point, "oh there's pi times three point six squared, would that be the area of a smaller circle?" So, I said "well, would it be half the area of one that's got a radius of seven point two?" It just stemmed off from there. But I think it was good putting some in there that were not relevant to the diagram that was quite powerful too.	Shall we pause and have a think about that?	Simon is getting into some detail here and I have the added layer of having been in that lesson. Having been in the lesson what Simon is talking about is potentially less mysterious for me than for others and he's talking quite quickly. It would be helpful for me to slow him down. There are opportunities here for doing some maths together even if just for a minute. What can be said about this, why was this a motivating comment for the students?	*Doing mathematics*
10:28	Sam	Can I ask how you put your pairs together?			
	Mia, Simon	As they were sat [laugh]			
10:33	Sam	Ok [laugh] yeah. Because I had a real tussle about do I reseat them cause at the moment they are sat by ability			
	Mia	Oh ok			
	Sam	Do I reseat them, someone really able with someone less able, and I figured probably not because the discussion would be completely different			
	Simon	Mine are quite jumbled up anyway so			
	Mia	Because both of our lessons were quite loud, there was really loud um but loud group discussion wasn't it			
	Simon	Yeah it was			
10:52	Mia	There was hardly anybody off task in either of the classes.	I get the impression this is a difference.	Observation of behaviour (as different to normal? I could have asked if that is different to the norm)	*Teacher observes change in behaviour*
	Simon	No			
	Mia	And so there was quite a lot of stuff going across pairs as well so it wasn't strictly pairs, they were really quite excited weren't they by		I wondered where he was going with that comment or train of thought, but he was talking about the fact that pairs weren't isolated from others perhaps what he is saying is that the seating wasn't significant.	
	Simon	They were yeah			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
11:03	Mia	They really liked the structure.		Similar to another comment "they really didn't like…" But his one doesn't stand out so much as it is less limiting? Negative? I could still ask "how do you know?"	*Speaking for others*
	Simon	Yeah			
	Mia	Um, we did that and then we did um couldn't remember whether this was another structure we talked about but it was missing um missing information. But we put some um, not on purpose, but the best one was actually one of our mistakes wasn't it.			
	Sam	[laugh]			
	Mia	Was this one here with an angle of 20 degrees, arc length of 8 centimetres and a perimeter of 20 centimetres so most of them used the 20 and the 8 to work out that the radius was then 6 and then they used the angle of 20 and the radius of 6 but the arc length wasn't right then so those three together don't work so that ended up being some of the best discussion didn't it			
	Simon	Yeah it did			
	Mia	In both of the lessons			
11:47	Simon	Yeah agreed. They're quite high ability sets that we were doing it on so um well like one of my students said like 'oh we thought we'd check' but like that, that was such a big thing, that he's actually thought, he's got all the answers so he could just move on, but he's just decided he's gonna go back and actually check it out which is quite good…. Yeah.		This is the first time Simon talks about checking later on when he talks about what reasoning is it feels like he's talking about this lesson again. I wonder if his image of this lesson is so strong for him that he draws on it since later he talks about checking he talks about asking questions and that's exactly what he's described here.	
12:06	Alex	Um, myself and my colleague did pie charts, I did it for top set year 8, they hadn't previously done pie charts			
	Ellen	We did it with two classes one was top set year eight and mine was mixed ability fairly low ability year seven and the only thing we changed was one, one calculation wasn't it			
12:32	Alex	Yeah so the numbers			*Patterns/ connections*
	Ellen	Everything else was the same			
12:36	Alex	We did a matching exercise. So, there was one pie chart where all the slices had the same area. [Turning to Ellen] what was the second one we all thought they were gonna go for, but they didn't?	An opportunity to do some mathematics maybe?		*Doing mathematics*
	Ellen	The one when they were all different			
12:48	Alex	They were all different. This one [Alex points to the first pie chart on the resource shown that he is displaying on his laptop screen]. Then there is one where the frequency is one, two, three, four, five and they didn't look at that at all.			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
13:02		 They saw this one here [Alex points to the fourth pie chart] where it's got the largest sector and then looked for the largest number from the tables. When we were going around asking them why they were choosing, the discussions were really rich. It helped that the resource was well designed. It only took me a little while to find it, it was a Median resource. So we used this [pointing to the resource again] and I thought it was really rich. They had that and they had to match each of the letters with a number. I think we sort of explained the next task briefly [showing a different resource] and then they went away  and really went for it. Because they hadn't done pie charts, some of them had done it in primary school, and some of them may have done it in year seven, we went through the workings and they had the discussion; "do we need to find the last one or do we just take it away from the full amount?" That was a really rich discussion. We then, it was a bit like the flour lesson, do you remember the flour lesson?			*Slide (from 'account-of' to 'account-for')*
	Simon	There were formulas?			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
14:00	Alex	So we gave them the workings to put in the correct order to find the number of passengers so we blanked out the passengers, one of the calculations was wrong, the last one, so the actual answer was wrong, um which we changed for the year seven mixed ability.			
14:15	Ellen	**Because they wouldn't like that**			*Speaking for others*
	Alex	Because they wouldn't have liked that it was wrong, um, again they had really good discussions between each other about why they chose them in that order, what the calculations actually meant, I don't think I was as explicit in my explanation to have that discussion **so we sort of adapted that** so my colleague was more explicit that they needed to think about the, um, what the workings actually meant so I think we sort of learnt from that as a pair, um and then this one was amazing, so um, they worked out the 36 over 360 so again the numbers helped was the easiest one to work out because they could see it was equivalent of a tenth			
15:00	Tracy	So what was the question that went with that?			*Detailed description*
	Alex	It was how would you find each of these sections, what would be the easiest starting point?			
15:07	Tracy	Nice			
15:08	Alex	 So, then a kid said, "well I know that thirty-six over three hundred and sixty is the equivalent of a tenth". Then someone else interjected and said, "well then, if that means a tenth, this [pointing at the prawn cocktail slice] will be fifteen". "OK perfect, we'll write this on the board". "Which will be the next easiest to find?" Then someone else said, "it's got to be seventy-two, because that's double the amount". Then someone else said, "well if you've got seventy-two, we know what one four four is, so take that away". Then someone else said, "well you can take it away from the rest of it to find what the hundred and eight was", and someone else said, "why would you do that? Why don't you do thirty-six plus seventy-two makes one hundred and eight?" Then someone else said, "well you could also do one hundred and forty-four minus thirty-six". So, the numbers are really important.	A lot of detail given.	Beautiful detail. Evocative – I felt like I was there! This is why asking them to paint an image of the classroom is so important. An interview practice as well as a de0brief practice. This seems to be about RULES BECOMING DISTINCTIONS to me. Since it is so important to see the detail – there is something about some of these descriptions that are evocative and others that don't give me an image – with an image I can then evoke feelings and memories – resonance and dissonance.	
15:53	Sam	So, the choice of numbers is really important in terms of the reasoning.		From the detail a new awareness has arisen here about the importance on	*New insights/ awareness*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				numbers. Significance of numbers was also mentioned before by Simon.	
	Alex	… helped the discussions and again it was			
15:58	Ellen	What was the one on there where you had somebody working out ratio or			
	Alex	Um			
	Ellen	did something completely different, I can't remember who it was			
	Alex	I can't remember, um but yeah, in your class one kid who is really high ability but really has issues with behaviour, sort of			
16:22	Ellen	He was allowed out of isolation to come to the lesson because I told him it was really important so he got dispensation to be out of isolation, only his twelfth this year [laughter]			
	+	[laughter]			
	Sam	Academic year or calendar year? [laughter]			
	Ellen	Academic year [laughter]			
	+	[laughter]			
16:37	Alex	And then he ran with it so he again saw the 36 over 360 was a tenth			*Detailed description*
	Tracy	Nice	Recognition – is this OK?		
16:42	Alex	and then was like double, double…			
	Tracy	yeah			
	Alex	…take it away, and then again the other one next to him sort of said there's a different way of doing it and again they explained that, so again I felt that this, **the numbers and the resources helped the conversations**			
16:56	Ellen	There were a few students in Alex's class when I was observing who didn't engage quickly **even though it was a top set class**. I had to get them going, and they were saying "we don't get it", but they were fine. The split was much, much, wider in my class, because obviously it was mixed ability and there were some children there who don't know their four times table. But they all had a go, none of them just sat there, did they?		Inherited language? Noticing a pattern – use of 'split' Does split mean the same thing to both teachers?	*Inner/Outer*
17:25	Alex	yeah			
17:26	Ellen	Even if they could only pick out little bits, **they all thought there was something they could access… but the ones who are being bored to death with the mixed ability** were really happy to have something that they could fly with			
17:46	Alex	Um, so yeah I think the main thing was the numbers were helping the discussions.		This comment seems to be as a result of the getting into the detail – a new awareness a useful 'accounting-for' and repeating of a previous awareness.	
	Ellen	And your class got onto the last task which mine didn't			
	Alex	Oh yeah			
	Ellen	Which was			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Alex	Oh hang on yeah sorry			
	Ellen	There was a			
	Alex	Oh yeah			
	Ellen	The same as you did, a bunch of calculations what does each one mean? What would it find?			
17:59	Tracy	Was this all in one lesson?			
	Alex, Ellen	Yes, yeah			
	Tracy	wow		Surprise	
	Ellen	But mine, mine didn't get onto that because obviously each bit was that little bit slower because of the ability			
	Alex	Um, again, they sort of struggled with the hundred divided by two, of that was it, one kid said, that was the thing we talked about, so a hundred divided by two, he said um hockey and tennis added together is a hundred and that was the mean of the number of people that played tennis and hockey			
	Simon	nice			
	Alex	He's coming to GCSE catch up and doing GCSE papers and he's in year eight. Because his brother is really on maths and his brother told him so at parents evening I had to tell him to just calm down [laughter]			
	+	[laughter]			
18:40	Alex	and learn more year eight stuff before he does year eleven, um but yeah that was amazing that he came out with that, um			
	Tracy	yeah			
18:46	Ellen	And there was a lot of discussion about which one gave you one person and which one gave one degree and there were two children arguing about that			
	Alex	Yeah that was the one where they sort of struggled on, whether it was three sixty over four fifty and so			
	Tracy	And was this like a whole class discussion?		My criteria?	
19:01	Alex	Um			
	Ellen	In groups			
	Alex	In groups yeah in pairs			
19:04	Tracy	So the arguing was kind of in a group			
	Ellen, Alex	yeah			
	Tracy	wow			
19:09	Alex	**So I think the numbers helped**, um, yeah and that was it really		A potential characteristic of an effective structure… given this has been acknowledged so many times – it might be	

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				worth pausing to consider more stories in this category or other strategies.	
	Ellen	But my class didn't get onto that but I am going to do exactly the same thing with my year nines next week, it was just			
	Alex	It just helped			
	Ellen	It just, we couldn't do it with a joint observation because we teach year nine at the same time			
	Alex	yeah			
	Ellen	so I had to do year seven even though they weren't the ideal class for this			
19:30	Alex	And I did year eight and they hadn't done pie charts but			
19:32	Tracy	And so how did you find the process of kind of watching, did you plan it together while you were here?			
	Alex	Yeah planned it together			
	Tracy	And then kind of later on			
	Alex	We planned a tiny bit here and then took it back to school, planned it at school			
	Tracy	yeah			
	Alex	Um, I taught and then we had a conversation afterwards, tweaked it, err, for my colleague's um lesson			
19:51	Tracy	And so in terms of the rest of the department, have they, do they know this is happening			
	Alex	They do, we've talked on it briefly, but another teacher came in and observed me, I said to him does he want to do it for his year nine class so hopefully he said he might be doing it tomorrow so I might drop in tomorrow. We've got a CPD session booked in, in two or three weeks' time where we can talk a bit about um			
	Ellen	The other things we are			
	Alex	The other things we are picking up today, so the first session an then obviously questioning today			
20:23	Tracy	Great			
	Alex	So that also, I will probably share the lesson after another person in our department has done it tomorrow that I'll then send that lesson round			
20:34	Tracy	Does it feel like a structure you can apply in lots of different contexts. In fact you've got multiple structures haven't you going on there	I think I am trying to summarise, to point out some similarities and differences.	Meta – about the lesson	*New insight/ awareness*
	Alex	The day after our meeting I did which one oh this structure [putting things in the right order] on completing the square and solving by completing the square			
	Tracy	And that was the putting it in the right order one		Labelling for the others – since we are focussing on use of structures – flagging choice (e.g., using multiple structures)	
	Alex	Putting it in the right order and seeing if the, if the workings are correct			
	Tracy	yeah			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Alex	So I think that's something that you can run with quite easily. Now obviously something, this matching task, you have to, again **it's all about finding the small examples, small number of examples that actually pick things out**		Another potential characteristic of an effective structure – a categorisation / criteria	
	Tracy	yeah			
	Alex	So it's being able to, if you've got the luxury of			
	Ellen	But you can easily do that with things like area of a triangle and area of you know all sorts of things			
21:21	Alex	Yeah, so it's sort of transferable but its finding those, like I was lucky that I found, I found Don [laughter]			
	Tracy	That Don came to the rescue [laughter]			
	Alex	[laughter] Don came to the rescue with about three different PowerPoints and I just cut and paste his three PowerPoints			
	Tracy	great			
	Alex	Into this			
	Ellen	And I put them on flipcharts			
	Alex	yeah			
	Tracy	[laughter]			
21:42	Alex	Yeah there was something we came onto even at the end so I think one's thirty seven point seven percent and one's thirty six point six percent so they're gonna say the thirty seven percent's more but would they have looked at the top saying the actual amount			
	Tracy	Thirty seven percent of what yeah, nice			
	Alex	Yeah so that was it			
	Tracy	Fabulous			
22:03	Alex	Questions?		Example of teacher picking up a meta	*Teacher going meta*
	Tracy	Any questions		Before me	
	Alex	Open to the floor			
22:09	Mia	That was very thorough		Acknowledgement of the detail	*Detailed description*
	Simon	That was pretty thorough		Yes there was something about the detail that Alex gave that was really useful – particularly the dialogue between he students and the same/different between his lesson and Ellen's	
	+	[laughter]			
22:09	Alex	Well I thought we'd rather do one lesson well rather than trying to get in into every lesson		A meta-comment here about the power of staying on the detail could have been a useful utterance. This may address the laughter too.	

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Ellen	And actually that was quite hard getting that done, because as I said I wouldn't have ideally picked that, I would have ideally picked my year nines,			
	Tracy Alex	hummm			
	Ellen	But the constraints of timetabling, and, you know the last time we came was just before Christmas when everything was up in the air, you know, we said we planned a little bit before Christmas, by the time we got back after Christmas I was lucky Alex had a recollection of what we planned, it was gone for me			
	+	[laughter]			
22:45	Tracy	It's interesting though that you got it to work in the two completely different groups	What was my motive for this statement – am I trying to force an awareness, challenge a preconception? It is clear to me that it will work with different groups	My comment was to challenge a theme about 'ability' and what was possible – to force an awareness potentially. This potentially provoked another awareness of sameness across the two groups from Ellen and some detail.	*Inner/outer*
	+	mmmm			
	Ellen	Well the other thing was they both did the breakdown of this task, both groups broke it down exactly the same way they both started with the same, the biggest, then the pairs and left the one that was all different to last, we thought that would be the second one.			
23:07	Alex	One student went for the largest first he noticed it was the largest sector and he looked down and he saw there was a seven there.			
	Tracy	Right			
	Alex	So that was his way, but yeah pretty much most people even because it was the first one that was the first one there perhaps if it was there [pointing] they probably wouldn't have noticed it, they all saw that they were equal, had a discussion about well what do fifths actually mean, well five equal parts.			
	Ellen	One student was constantly trying to change it into a percentage but another student wasn't			
23:42	Alex	If that wasn't there [pointing], would they have chosen that one first, I don't know. Would they possibly have then chosen the largest sector if that wasn't the first one**. Because people like doing things in order don't they.**		Alex is talking about the detail of the 'task design' the order in which information is presented on the task that the students were working on. It would be really interesting to pause and look at this together – to consider alternatives and how each alternative might change the response to the task or what might be learned.	*Speaking for others.*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	+	Yeah			
24:03 24:43 25:29 26:16 27:06	Joe	Ok, so um, we were looking at substitution, as a topic and kind of an all or nothing person so I didn't think the idea of just doing one, I essentially taught the last two weeks, all my lessons on that unit have been, apart from the first one where I introduced the concept, has been very heavily geared around the reasoning structure, and then I did it reflecting, I thought actually on a daily basis I used the kind of, this is the wrong answer, call it my nan's answer and get them to unpick the mistakes they make, I use multiple choice questioning a lot, and I make all my unit assessments diagnostic questions and I've started more, getting them to unpick all the wrong answers and that's really helping in terms of response but I've found with algebra, always, sometimes, never was a wonderful structure, so you give them a statement along the lines of **'2a is bigger than a' is this always, sometimes or never true? 'a over b is smaller than a' is this always, sometimes, or never true?** And we got them to prove some of these statements. We also, as part of the do now part of the lesson, all that was around negative numbers to kind of point them in the direction of try different things, try negatives, umm, and what we found was a certain task in particular was **'a over b is smaller than a'** was a really interesting one because I did it with two separate classes and one class was fairly unstructured and I let them plug in what they could I said right you may use integers, negatives, decimals any permutation of it, and we got things like, um, children were dividing by zero and they were getting math error and they were getting to that and we had a whole group who were just at the end were obsessed by the idea of zeros going on forever and infinity. Really engaged and was really interesting. And, um, **what happens when you divide a positive by a negative if say a was negative and b was positive what would happen, you know, or double negatives in the situation or even dividing by a decimal** but what we found was, when we, when I did that without giving them values, they got lots of different opinions, it was really great but it was quite hard to consolidate so the second time I did that activity I gave them a range of values positives, negatives, decimals, negative decimals, zero, and I got them, and I also thought you know in terms of the reasoning, did a bit of reading and reflecting and it was actually, **if I want them to make a statement, I need, they need to know how to say it, so how do they start it, a over b is bigger than a when and you give them , and they write down the conditions that allow that**, and I felt that was really, really enabled deep learning, on that. Another task we gave them similar expressions, similar things said right find the biggest possible solution using these constraints, um, things like, a minus b but you give them negatives, they've got to understand to take away the negative and start with the biggest positive and subtract, some really rich discussions and real depth. I remember, I did the first lesson on this, I had a PGCE student, he said, I thought that would take you two minutes but we managed to get half an hour of really great learning out of it and it blew his mind almost. So it's finding, giving them some constraints and saying what's the biggest solution what's	This felt like it needed slowing down – opportunity to work on some maths together. I wonder what is different about doing the maths as a group for 2 minutes vs what happened here. I was in the lesson so was clear about the task – who else was?	Less detail in this account more of a scatter of everything that he did (which was a lot!). If I was going to do this again I could get him to slow down and talk about one or two things. In setting it up – I could point to the importance of getting to the detail. So we need some context – enough to understand what was happening and then talk about something in detail… to get to some new insights or issues. Doing mathematics as an alternative to explaining mathematics teaching – they each do different things. Joe speaks for over 5 minutes without a pause.	*Detailed description*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
27:46 28:55 28:55		the smallest solution again was incredibly powerful, so I did that, I did one task on just expressions and then I did it, a similar thing on formulae and what I did the second task was get them to think about the strategies, how you, how can you do it in one, how can you make sure you get the right answer first time, which a lot of the more able children really liked as a challenge, but it was still accessible to the less able because they can still pick values play around they can still get to the right solution and unpick it, it's just it enabled that depth of access and understanding. Another one I loved was, with algebra, same different, given two statements, expressions, formulas, ok what's the same, what's different, so 2a, a what's the same, what's different, well they both have a but one is multiplied by two. Um, formulas was different brilliant, was absolutely superb, you gave them I think it was 8t + 5, c = 8t + 5 what's different? Expressions versus formulae, and then we gave them a worded formula and a kind of algebraic formula and got them to pick apart what was the same, what was different. Where was the meaning the same between the letter and the word, um, which really helped them understand and I kind of took this a bit further into some of my other classes so I had a year nine class who were doing rearranging of formula and actually what I did was I gave them the whole sheet of working out rather than go through it line by line and said right at each stage what has changed what has stayed the same and what has changed and get them to pick apart that actually the variables don't change um or the values don't change, what has changes is the location, the operation getting them to understand they are actually just manipulating rather than calculating was a really nice extension, so I've gone all in on this. The last thing we do, the end of every lesson we get them to reflect on their learning, and I was looking at a lot of their sentences and they're 'oh I learnt today how to calculate' and things like that which I thought was not very useful **so I've got them really unpicking what they've been doing in lessons at the end and giving some examples**, the most important thing is to remember that two times a is a 2a for example if a is four, two times four is eight and getting them to really. I'm going to stop talking now because I've gone on too long [laughter].			*Change in practice*
29:23	Beth	We had the same starting point and our lessons went like this, so Joe kindly lent me two of his classes because I'm not teaching at the minute, and we started with the same structures which was the, here are some expressions, here are some numbers can you make the biggest value for each expression so you know if it was two a they had to substitute a large positive number if it was a takeaway b they had to substitute a large negative number to make it bigger and then we just, the classes kind of went in very different directions. From then on, so, **I was amazed** by how much time we could spend on one tiny concept so we gave them, for my second lesson I gave them this and they had to match up what was the same and the idea was to go onto this which was just looking at the fact that a minus b is the same and minus b plus a what actually happened is they thought that, I'm gonna go back, they thought that these two were the same, **so we spent**		Then Beth offering a difference – and more detail – homing in on one lesson – an example of when there is so much detail but across many lessons – hence difficult to do anything meaningful with it – versus one lesson	

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		about twenty minutes exploring why minus b plus a is not the same as minus a plus b, go I get them to substitute, and so they are practising substituting of negatives and that surprised me because that was not what I was planning to do, I had like a whole lesson planned.			
	+	[laughter]			
30:20	Beth	But then we went into this one, and it's always, sometimes, never true and what was interesting is that you [Joe] kind of did that as a different lesson where you'd gone on to division and we'd discussed whether we should phrase it differently, so you'd [Joe] phrased it as a minus b is bigger than I think or you phrased it. We looked at whether phrasing it positively made for a different discussion and we kind of expected that the kids would be like yeah, course it is but actually they still weren't like that, they still took a lot of substitution and a lot of trial and error before they were convinced, um, so, there was a whole lesson planned but we actually just didn't get onto that we literally did those three slides and it took a whole hour. Um, what was really nice was at the end, there were two moments that really stood out for me, one was a kid going I think I was wrong Miss and talking about why that is really important to identify that your assumptions aren't always true and the other one was, well two other things, one was they got obsessed with using zero.	I saw some of these lessons. What does seeing the lesson do in terms of what I then hear in the conversations? Do I make assumptions about what others are hearing? Perhaps it is least important that I hear the detail.	New structures – I could be recording these visibly as we go and making notes next to them (e.g., based on the characteristics that have been effective).	*Distinctions*
	+	[laughter]			
31:10	Beth	they realised how zero was, so I'd be trying to get them to substitute other stuff like, try some of this stuff because zero hasn't… you know it's a special case here, maybe try some other numbers [laughter]			
	+	[laughter]			
31:19	Beth	But they really, they were starting to cotton on to what kind of numbers would give special cases and that was a really nice thing, looking at how their trial and error was working, and the other one was, um, so after this lesson where we were looking at these two and also looking at whether that was the same as minus a plus b getting them to write sentences about why this is always true and why the other one was only sometimes true, we had to say sometimes because they would try zero and zero obviously worked um and it was really nice that it was quite a mixed ability class and I've actually never taught mixed ability before and there were some kids in there who were really struggling with arithmetic and their explanation wasn't quite as mathematically sound but they had this sense that the b was the minus and so although their explanation wasn't quite as nice and articulate and mathematical there was still this sense that the b has the negative and the negative stays with the b so I felt like it was a slightly more superficial understanding but some quite important algebra was going in so that was really nice, but it was just so fascinating how these two classes went in totally different directions.			*Detailed description*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
32:19	Joe	I think it really revealed who the proper high, even the, the high attainers are, not necessarily the ones who always do best on the test but the proper mathematical thinkers are in the class really evident on that.			
	Beth	Um, yeah.			
32:33	Paul	Can I ask a question? You talk about different levels and you know what you said before when we talk about reasoning a lot of the time we think about proof where we go into the algebra and doing that, if you were looking at your mixed attainment group could you refer that to those five steps and start seeing they were also reasoning but on a different level or…		Linked to making distinctions. When is it most effective to introduce frameworks of existing categories… before an experience? After an experience? Both? Can we critique the framework? What is missing? How does the framework match or not match our experiences (how does it compare?). Reminds me of using a framework in PGCE sessions and Med sessions. Using a framework to analyse an experience vs. developing a framework based on our experiences. When do somebody else's categories become meaningful? What is the process of making them meaningful (this seems to be exactly what I am in the process of doing myself!)	*Existing frameworks*
	Joe	Yeah definitely			
	Paul	It's just that question of did that change maybe your perception of reasoning for those students			
33:03	Beth	Definitely, for me it was really interesting because I didn't know the kids particularly, I had like their data on paper and Joe had kind of warned me this kid might need a little bit more help and you could tell who the TA automatically homed in on.			
	Joe	Yeah [laughter]			
33:18	Beth	But for me they were a blank canvas and so from the first lesson, I was really surprised that the lower attaining kids that maybe find maths hard actually doing much better in the second lesson, I think I'd slightly adjusted how I approached it, maybe made the modelling a bit clearer, so they knew exactly to write down but they were still able to access it, but you were right there was a different level of reasoning for the kids who'd spotted that it was a rule, that came out in their explanation. It was also, because I didn't know the other adults in the room, obviously I knew you two, but I didn't know the TAs that I was working with, that was really interesting, two different ones, one each lesson and watching, like one was literally holding the pen and writing for a child and the second lesson, the same child was so much more successful and they did not have someone sat with them all of the time, and actually just, they had a kid who was really engaged and really interested and			*Teacher observes change in behaviour*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		they worked really well together it just really revealed to me how the adults can really influence how the kids are working in the lesson, our assumptions about them. Not that we shouldn't, obviously you know your kids and you know when to put things in place they won't access but maybe sometimes we assume they'll need something that they don't.			
34:18	Joe	I think there's a really good point you made there about the getting used to it I think the more I did it the better the outcomes were it was quite hard at the start to kind of get my head around it but the more I did it the more they got used to thinking. The easier it became, the better learning we got.		What is the 'it' Bouncing around – sticking to one issue before moving on it a distinction I now make	*It/that/this*
34:38	Tracy	What gave you the sense that the learning was better, what was it they were doing that meant that the learning was better?	I want to see what they have seen… how do you know?	I see this as wanting to know what their distinctions are or potentially making their distinctions visible to themselves so that they are able to see this in the future (i.e., a process of labelling) – so I could look for more examples of labelling and what happens in those moments? Prompting for detail – what was observable? To recognise the same or similar phenomena in the future (in teachers and in self) – in relation to this… I think I have been doing this instinctively.	*Teacher observes change in behaviour*
	Joe	Well, for example, when I was giving the students the biggest, smallest task, they were reasoning more about which values to use in the lesson when I had given them a selection of values to choose from, so they could select in order to yield the result they wanted. The first time I did it, it was very much trial and error. Whereas the second time was very much more about strategy.			
	Tracy	Oh OK.			
	Joe	And their different approach was really noticeable.			
35:05	Tracy	So same structure both lessons, but in the first you let them choose any numbers and in the second, you gave them a selection to choose from?			
	Joe	Yeah, so how they approached it was different and the speed at which they got to the correct answer and the accuracy of calculation was much stronger in the end. The students were not writing four examples per expression and stumbling on the answer, they were saying "right I think I'm going to get the right answer by following this strategy" and getting there much more quickly.			
	Tracy	Yeah it's fascinating			
35:30	Beth	For me it wasn't the fact that there was lots going on like in the first task where there was literally 2a, a-b, 2a+b and a set of about six numbers they had to substitute a mixture of		Example of a parallel between learning maths and learning to teach – detail.	*Teacher going meta*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		small, large, negative, and positive and there was just a lot of ideas happening there and then the second lesson we were literally focusing on one thing and that just meant they could really delve in and they were just less distracted by some of the other rules and that's why the learning was better. We did less maths but more maths.		A recognition of the power of staying in the detail in the teaching of mathematics. Teacher meta – getting to/articulating a new awareness (evidence of teacher learning?)	
	Tracy	Less maths, but more maths, I like that… fab			
36:07	Miguel	Shall I go?			
	Tracy	mmm			
36:09 36:23	Miguel	So I had er sort of top, second to top year nine class and I was just briefly teaching them um percentages like percent, you know finding percentage increase by doing a multiplier, um, and what I did was I taught that the previous lesson, I think they basically understood it, and then I put a question on the board which was something like a thousand and twenty percent and I was trying to explain, imagine if you saw this as like the headline of a question and then you miss out the rest of it, what would the questions be, to have these, this was the workings out for the answer, and, **I think what struck me first of all was, I think because they've never seen, they've rarely seen that structure before, they were a bit confused by it** and I tried to explain it, with another, like a simpler example, so I said you know, let's say Bob had five pounds, Bill had four pounds, if you had five plus four what would the question be? And they say well you'd be finding out what they had in total and I said well what if you had five minus four what would it be and they said well you'd be working out the difference between them, like you know, how much more has Bob got than Bill. Um, So I think they sort of got it um but I think the main thing they struggled is that they hadn't, they hadn't seen that structure before and I don't think I explained it as clearly as I would the next time round. I really liked the idea of it, I think I added, I think I made it unnecessarily complicated because I realised that I'd done not only what's the question but I put a mistake in one of them so then to ask someone what's the question, oh and by the way this is wrong.		The same issue as Sam – so given this is a common issue – this could be noted and a focussed discussion leading to strategies to entail. A missed opportunity.	
	+	[laughter]			
37:27	Miguel	I think is a bridge too far and to begin with I sort of, because I'd be worried that they'd just sit there and, because they basically tend to be spoon fed. The way that I naturally taught, is that I would explain something and then give them the work, and I've tried to move away from that. My attitude used to be, how can I explain this in the clearest possible way so that the students have to think about it as little as possible to understand it and my attitude now has become what is the least I can possibly do to explain it to them, without deliberately making it confusing, so they can make the rest of the leaps themselves.		Clear distinction being articulated – change in practice referred to. An example would be useful here – to make it real for others.	*Change in practice*
	Tracy	That's lovely		When Miguel makes his clear distinction, his change in practice, I recognise this as a change (it is explicit) but why say "that's	
	Miguel	So they can make the rest of the leaps themselves			
	Sam	That's nice			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Tracy	That's really nice isn't it		lovely" – where did that pleasure come from? Because it was a clear articulation of something I have carried myself as a teacher? A recognition of sameness, the forming of an idea I have had around for a while, a shared value. Who is to say that what Miguel meant and how closely my interpretations were to his own sense of what he was articulating, but there was a palpable moment in the session were many of us seemed to appreciate what was being said, to resonate. I could have asked Sam "what do you take from that Sam?"	
38:04	Miguel	So I gave them the question, I said right you're not allowed to talk for the first three minutes, and the reason I do that is because if you've got a choice between, um, doing nothing and working on something just the fact that it's really boring doing nothing they tend to push themselves to work on something, otherwise, even a top set will be like look at it, I don't instantly get it so I'll talk about it until sir explains it.			
	+	yeah			
	Miguel	Um, and I also want to put up the resilience and the fact that in the exam they're not going to get the choice of don't get it they're gonna have to be faced with nasty things that they don't understand so I sort of said no talking for the first few minutes, now you're allowed to talk about it, um, and quite a few of them got it, um, but I think it was just the fact they hadn't seen it before, I mean I asked at the end, I said to them considering it's the first time I've taught it and it's a bit ropey my explanation and you haven't seen it before, what do you think of the overall structure, like, doing it this sort of way, people genuinely didn't like it			
	Sam	hmm			
38:52 39:08 39:20	Miguel	But I still think that's just because they hadn't come across it before and I'm, I really like the idea of it and I was thinking about what I've often done, when I've taught a subject, I often give them like quick questions on the board, so I've taught like them adding fractions, I'll have, let's say, five quick questions on adding fractions starting on easy ones and gradually building up and I'll give them maybe, I don't know, between ten and twenty seconds to answer each question, and I'll go through them at the end and say right, take a note of the hardest thing you understand and I'll often use those partly if there's part of my lesson where I've got ten minutes left and I don't want to start something new, um, but I want them to use their time productively and also realise that the main problem with people doing well at maths is **recall** so if you teach something and don't revisit that for a year the chances are, almost certainly that they will have forgot it. When if you go over it,		Getting to a future action … So **making it real** is about moving from know-what to know-how? moving to action. Afterall, I don't really know something until it is what I do… the separation between holding a meeting and talking about stuff to becoming part of your being is one purpose I can support. Another one of these bridges is working on the mathematics actively –	*Commitment*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		even briefly they've got a chance to remember it. And I was thinking actually it would be quite a nice structure to do with those quick questions so if I kept them, you know, once I had taught a subject and you'd do, I don't know, let's say these are the answers what are the questions you could keep that slide and then six months later you could say to them right these are exactly the same questions I gave you six months ago, you might change the numbers in them, um, and then see how, it would be a good way of getting them to recall their understanding of it. But I think the main, I think in that particular instance the main problem was that I was a bit shaky on delivering it but I thought overall I liked it I thought it was good, it was a good idea. So, any questions? Have I exhausted you with that rant?		doing not just saying – closing the gap – making it real. Listing strategies is therefore only 'real' when it is integrated into our beings/our practices… one step closer to making a commitment. 'Recall'	
40:20	Simon	**You know you said about the not revisiting it for a year thing, that's a really big thing for us too**, we've changed our scheme of work completely for year seven straight in with algebra and our last department meeting, we always seem to have like 15 minutes at the start where we are trying to develop an idea or resource or question that is linked to mastery or reasoning around linking back to what we have done before, so for example like we were talking about ratio weren't we and we were trying to link algebra into um ratio um and we are trying to build that throughout so that they don't have to not revisit for a year they're constantly doing fractions, Pythagoras use fractions, whatever, anything and its like that kind of um, what's the word, like cyclical, like just like continuing spiral of like coming back to it.		Common issue ('recall') – labelled first and then discussing.	*Patterns/ connections* *Teaching issue*
41:04	Sam	That is something, **because we've got so many non-specialists**, how do I make this more difficult? Fractions, negative numbers, surds, you know just throw those things in.			
41:20	Simon	But then again though, this is what we were saying, we did it for year eight didn't we, and we split off into groups. I think we found trying to **plan a question that links other stuff into that topic for low ability was something we found that particularly difficult**. I don't know whether that was just because we weren't really thinking, or it was just the nature of ratio or whatever, but we've just found that that was harder**. I don't know what other people have found but we're just trying to avoid not revisiting for a year basically and just trying to build in lots of different things. Maths is a connected thing rather than we're gonna do ratio then we're gonna do algebra, then we're gonna do something else. They're all the same, you're teaching everything all the time, that's what we're trying to build in.**	We could have spent five minutes here working on this issue as a group. This seems to be a big idea – what mastery looks like in one school – you're teaching everything all of the time – what might this look like?	Simon is asking for stories/suggestions here – he has gotten to an issue that he wants to work on with support from the others – I would like to intervene here (soon at least).	
42:03	Miguel	And sometimes I can see that it links that's fine but often I'll say right, this is nothing to do with the subject I'm just gonna give you a question on something we've done six months ago. I mean I was really taken aback when it was my second to top set year ten class last year, who you know are pretty capable, and I said right well you know if we are doing division and you haven't got a calculator what do you do, and there was like deafening silence, I was like, do you know bus stop, remember bus stop, and they're like, "no, no, no".			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
42:22	+	laughter			
42:23	Miguel	You know and it's not that they didn't not that they, you know, they hadn't done it for like five years so surprise, surprise, they had forgotten it, so I'm often aware that those things, you know you can see that can't you, often quite a capable pupil, some really basic things like that are completely lacking like negatives			
42:36	Alex	It's quite good, sometimes, there's a kid, last year I taught year, year nine, top set year nine and they were amazing, this kid never had a calculator so I'd go through the answers and I'd go through the answers that he had look and he would help me going through his answers because he never brought his calculator but he knew his fractions and, instead, so I was like amazing, so if this ever came up on a harder paper, on a non-calculator, that's exactly how you'd do it			
43:01	Ellen	With the higher ability children there's often a danger that people skip over the basics			
	+	Yeah			
43:08	Ellen	Because they assume that they've got those and then it leaves a hole, like bus stop, they think, because their recall of division is quite good but then they can't do the more complex problems		Why do they need bus stop?	
43:22	Simon	It reinforces that but it also extends it as well, so like with the surds and Pythagoras thing, that extends their knowledge of surds it doesn't just consolidate, that's like, different contexts, awkward numbers, …			
	Sam	Expanding brackets, expanding double brackets, throw a third in there			
43:34	Simon	Yeah expanding, its chaos isn't it exactly and that's like that's kind of the point		At this point I really want to get to some concrete intentions.	
		GAP IN TRANSCRIPT HERE – mainly project lead talking			
44:50	Joe	And on that subject when we were doing the substitution was nearly all number reasoning well actually what we were doing was what happens when you subtract a negative more generality what happens when you divide by a decimal we reckon the divergence happened because the class that I teach what was a share class one was a whole class I teach the whole time I just nailed on negatives 4 weeks prior so when it came to doing the negative questions they were more confident whereas the other class had been taught negatives by different teacher and it hadn't been taught in the same kind of way I would have done it and they diverged and they couldn't necessarily keep up			
45:27	Beth	Something I noticed even in my two lessons that I taught them was their use of language it was something I tried to be really careful with. It was minus three add seven and the child wrote minus four as the answer so they said minus three add seven is four but you've got a negative and a positive so it must be minus four. Such a classic. So I was really hammering, it's adding a negative that's the same as subtracting rather than it becoming		Beth talking about language then to substitution as a structure "a structure for exploring negatives".	*Distinctions*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		negative and it was really fascinating to think realistically they've had a term and they've already mis-learned that topic and it was not that they were taught it incorrectly it's that the use of language had perhaps allowed them to slip into some poor assumptions. But my lesson was ok. And substitution itself was a structure because it was a structure for exploring negatives.		New distinction by Beth – seeing substitution as a 'structure' – a new label from the project. My role is to draw attention to these new labels (listening *for* new distinctions) potentially. "so what do you see a structure as doing, what is a structure to you?" A new distinction for me – not always looking for how to 'connect' topics (the current issue being discussed) but looking for topics as structures – you need to learn the structure. Maybe this is one interpretation of Gattegno's subordination – just articulated differently – a new way of seeing a familiar idea.	
46:20	Alex	I think it also helped at the start of my lesson that I explained why it was important for reasoning so I went back to a starter I did two and a half weeks ago where someone had scored a gold after 30 minutes and 31 seconds and I asked the class what minute it was in in and this is what happened this kid here said it was 31 minutes and his explanation was because it was closer to 31 minutes than 30 so if I had taken it without explanation next time when it was 30 minutes and 5 seconds he would have written the answer 30 he would have thought he had been correct but he would have had a shock that he actually got it wrong so we had to draw back to that one example every time so trying to find that one example where you can give them this is why we need to reason because otherwise we're going to make mistakes later on when we haven't truly grasped the concept			
47:18	Sam	Counterexamples			
	Alex	If I had accepted his answer without explanation then I wouldn't have known he didn't understand the concept either.			
47:32	Beth	Can I ask something, this is a very general question but I did a little survey with some primary staff to ask what people thought reasoning meant and it was amazing how many of them had the word explaining in their answer, they were all like, reasoning is explaining, its saying what you did, I was wondering how people felt about that because to me there is more to reasoning than just explaining, I was wondering how everyone thought about it.	An interesting questions – moving away from describing own experiences to more general points. When is the right time to do this?	Beth asks what is reasoning (coming back to Sam's original point) – beyond explaining. Fruitful work on making distinctions – a variety of distinctions being made, one leading to the next…	*Distinctions*
47:52	Simon	It's about asking questions, in my head if a student is reasoning they are asking themselves questions they're trying to justify their thinking but also be accepting of why I don't know taking into account different perspectives and ideas, it's not just explaining			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
48:17	Sam	No explaining is more descriptive isn't it, an explanation is more of a description of what I thought rather than why I thought it maybe?		I definitely have these as different things – one is what the other is how/why…	
	Beth	I just thought of a question where for example if it's something add 10 good reasoning would be just do that in your head and add 10 but if they explained the column method for that they've maybe not reasoned very well because they've chosen a really inefficient method so you can explain bad reasoning.			
48:45	Paul	I think that maybe a bit of a faux amis. So the idea of, it's like when we hear the term differentiation, as teachers, we think are we talking about attainment, or are we talking about mathematical differentiation. And you know there are two differences with one word, I think, in terms of reasoning, if you are looking at literacy, literacy based subject, or for example, to get a grade A in RE, you would have to show one side of an argument, the second side of an argument and explain your reasoning throughout those. Whereas we're talking about mathematical reasoning, that's justifying and proving, and so, is it may be that case that it is the same word being used in very different ways.		This is an example of teachers, having shared their own stories and experiences, to making commitments? What would be the extra thing I would do differently/additionally – maybe a move to review/commit… This is something I have changed in my practice – a new distinction – the need to commit, to articulate, to become – who knows why things happen but I would imagine these conversations being influential – so much richness – but what of it becomes real?	
49:34	Lucy	The classic question would be if they if there's a statement in an exam paper, and then the student would have to say why that statement is wrong, and they would have to use what they know about that subject, and show some examples, hopefully.	Did I really not notice Lucy had not said anything until now? Perhaps I should set up contributing expectations?	Lucy – first instance after 49 minutes! Likely a result of my 'setting up'	*Inclusion*
	Beth	That the showing of examples is as much reasoning as the explanation there, if it's an appropriate thing.			
49:57	Tracy	I suppose you're RE example, you've got one side of the story, you've got another side of the story, that kind of does feel like what you're doing when you're reasoning mathematically.		I try to make connections between RE and maths and go back to Sam's original point – this feels pointless – perhaps because it is not meta – something else might have been to use Paul's distinction to check in with others? Do other's see these as different? Is anything the same? Is it useful to think about reasoning from different subject disciplines? What other faux amis are there? Check in on understanding about this term…	
	Paul	It's just that it's literacy based. It's in a wordy way.			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
50:14	Tracy	So you can reason mathematically on paper, you can reason mathematically verbally, it can be in words, so it's not an easy question, what is reasoning. It's your thing, there's problem solving, there's fluency, they are not obviously distinct are they.		… Making it real is another of these distinctions – linked to the above – making it real becoming more finely distinct – different ways of making it real. E.g., doing something (e.g., planning a lesson, performing a teaching strategy, an action) Making sense of it for yourself – hence the need for questions, to pause, reflect back in own terms, seek clarification, resonate, lead part of the conversation.	*Existing frameworks*
50:38	Sam	Which I find very difficult.		Sam acknowledges wanting distinctions	
	Tracy	You want to categorise them.			
	Lucy	What the guys said on Saturday, the AQA guy, he said, problem solving is more open ended, whereas reasoning, there's usually a right or wrong answer, but then you have to say why, say why you're right		The AQA guy gives yet another perspective – the examiner perspective which is *no less valid* it is just a different perspective (that I immediately respond negatively to and I would like not to have!)	
50:52	Ellen	It's making sense of		Interestingly - they had been presented with '5 steps of reasoning' in the previous session - but nobody has talked about this - not meaningful to them (yet) – this feels like the time to bring that back as a further possibility… This 'timing' speaks to the sense of having experienced something before then using a framework OR the sense of coming back – in a 'reflexive' way – seeing previous ideas with new insights. I would like to have captured some phrases/behaviours from classrooms (some categories – maybe starting from stories)	
50:54	Tracy	Making sense of, that's nice			
50:55	Joe	Why is it correct, justify your answer			
50:56	Simon	Is it realistic? yeah			
50:58	Beth	Because I was thinking about when we did substitution, a child that substituted a negative number into a minus b to make it bigger, their number choice was reasoning, they explained it beautifully as well but they had already reasoned by choosing a negative number not just by explaining, it would have felt wrong to say they weren't reasoning until they explained it, because they made the right number choice		'Beth' sees doing/intuition as reasoning? This is a distinction worth pointing out – I hear it as this maybe because I have been thinking about doing vs. articulating myself	

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				– Beth does not privilege the articulation? I could have checked this out with her.	
51:19	Paul	Because they're explaining their reasoning, the reasoning is not the explanation.		Very nice subtle distinction	
51:23	Joe	The calculation justifies their answer I suppose			
	Simon	If you set up a task that is to do with reasoning then my thinking is that it should provoke curiosity so students should be asking questions around it, it shouldn't be just like a… they can explain, I don't want to use the word explain, they can say why it could be that or why it couldn't be going back to the RS, both sides thing, but it should be more like, oh here's a problem, what about that, what if I did this, why did you do that, and it's the questions that are coming from it, that's what…		This feels like an email	
51:59	Tracy	That's lovely		"lovely" – another recognition of a distinction, a recognition (re-cognition) of a new distinction to me – a new way of seeing … to recognise – recognise a distinction.	
	Simon	… I think is the main…			
52:01	Tracy	And that's what you talked about, you talked about the boy who said if I double the radius…			
	Simon	Yeah			
	Tracy	… do I double the area			
	Simon	Yeah			
52:10	Tracy	So he's gone that step further			
52:12	Joe	I think maybe that could be hugely intimidating for non-specialists, that open-endedness, not having the right answer, for somebody who is much less experienced.		Challenge the assumption about non specialists - share story of TA or non-specialists and what they have to offer.	*Speaking for others*
	+	Yeah		There are more members of the group who confirm this assumption is more widely held.	
		GAP IN TRANSCRIPT HERE – mainly project lead talking			
56:15	Sam	We've taught quadratics traditionally so badly So we'll do a week on factorising a week on completing the square a week on drawing the graph. We never actually say here's one solve it and solve it five ways and see what you get. It's the shock on their faces when they realise It's the same answer and you just realise how badly it's been taught, that they are that shocked that the answers are the same. Then that reinforces roots and all sorts of things.			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
56:45	Tracy	Lucy, I'm conscious that you haven't had your time.			
56:47 56:57 57:47 58:07 58:30 59:30 59:33 59:58 1:00:05 1:00:40 1:00:45 1:01:23	Lucy	We only had a couple of days so we did something completely different. But I did want to ask question at the end of yours Because it was something that I came across. I did adding fractions with year sevens and what I did was because I liked the proportion questions with different workings out but in a random order So the kids had to choose the correct order and then figure out what the question was. So I did a similar thing with a couple of red herrings in mine. So a few boxes had equivalent fractions in for 1/2 and 2/3 and then the first five times tables for twos and threes and then a wrong answer of 5/6 and 3/5 and then a correct answer and then got the kids to try and put it in the correct order and figure out what the question was. But some barriers I came across, **because they are mixed ability year sevens**, first of all, there were a couple of students who couldn't access it at all they don't know what the question is asking them, they can't remember the method for adding fractions. **My initial thought was when do I jump in and how long do I leave them to suffer.** Then there are obviously the brighter kids who know what the question should look like so they figure it out pretty quickly within five minutes. **So then you have to** ask them what the next steps would be or to make up a similar question. Or what a harder question would look like, or a wordier a question. For the middle of the group the barriers were adding denominators, which they frequently do, or not knowing their times tables which is a major problem because they didn't know where the equivalent fractions were coming from. I did a fraction wall, so ½, 2/4, 3/6, and the 2/3, 4, 6, 3/9 and then I did circle the right ones so that they could see a pattern from there which didn't require much knowledge of the times tables and because it was just twos and threes anyway. What I find challenging is knowing when to step in. Because it's mixed ability. For the weakest kids I probably leave them for a few minutes, and I usually put a timer on the board, so I know how long they've had thinking about it. I find it really difficult not helping them. I tried to break it down and give hints to those that were stuck, to try to get them to recollect how they would do it. We were talking earlier about different methods of doing one question, I want the students to give me more than one method, but typically at the end of the lesson, and only when I know everybody has understood. Otherwise, I feel that it makes them a lot more confused than they would have been. If you have taught them one way of doing something, and they are still grappling with that method, to have somebody else come and show another way; "I haven't got my head around this way yet, and you are showing me something else". I feel more confident when a student comes to the board to offer a different method once most of them have understood the first method. They find it hard to explain what they are doing, to put it into words, some of them are not fluent enough to explain why is it 7/6 and not 5/6 and where did I go wrong, so for some of them, when they came across what the question was, I would have said it's 3/5 because I am just adding across, so some of them struggle to say why is it correct that way or why isn't it a different way. **You have to ask them leading questions** to get them to respond		Lucy takes us through some detail and then a series of teaching issues that do not get resolved before moving to the next issue. This is an example of detail-issue-strategies in one extract. A stream of potential teaching issues called out but unresolved.	*Detailed description* *Inner/outer* *Teaching issue* *Should/ought /had to* *Teaching issue* *Inner/outer* *Unresolved* *Teaching issue* *Unresolved* *Teaching issue*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
1:01:57 1:02:11 1:02:16		and to hopefully say the right thing. It was loud there was lots of talking, some of it probably not relevant, but from the conversations that I heard I think it was the more confident ones that were actually explaining the work a lot better than the least confident ones who were just saying this is how I would do it, but I got confused because of the equivalent fractions. If I had more time, I would have made the questions a bit wordier and possibly put a few more hints on paper for the least able kids, because it was mixed ability. To stop me from actually going over, and instead just encouraging them to read the hints. I would have liked to have differentiated it more possibly.			*Should/ought /had to* *Teaching strategies* *Inner/outer* *Teaching issue*
1:02:22	Beth	Is this the first time that they have seen adding fractions?			
	Lucy	No it's a recap they have done it before.			
	Ellen	And they are all supposed to come from primary school completely fluent in fractions.			
	Lucy	Well most of them were in September, but we are now in January so a few of have them have forgot.			
1:02:45	Tracy	Some of you have spoken about the fact that the structure felt difficult for them because it was the first time they had seen that structure and that might be part of what you are describing maybe?		Making a comment about a 'common' issue However it seems to provoke people to try and answer 'why' … commenting on a similarity/common theme then leading to hypothesising why... not useful in this situation. An alternative would be to have taken a notes of the issues that Lucy did articulate so well and collect stories and strategies for those issues.	*Patterns/ connections*
	Lucy	Yeah, I'd never done it like this before			
	Simon	I don't think our group did			
1:03:02	Ellen	I just thought they just took it in their stride, it was just a different lesson			
1:03:10	Simon	Do you think that could be about school culture, or ability? We did our lesson with quite high ability classes so maybe they were more willing to try, or maybe they didn't have a fear of being wrong, I always try to have a no fear culture.		Hypothesising – when is this useful?	*Asking why?*
	Ellen	Maybe because they are more used to variety			
	Beth	Or how close to your normal teaching practice that is.			
	+	yeah			
	Alex	I think they liked it because it was different didn't they			
	Ellen	I didn't think it was that different.		Different vs variety - so not different, so what is the same is that there is always variety!	
	Beth	Maybe that says it all, that is didn't seem different. It didn't seem different to the kids either.			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
1:03:37	Tracy	It is interesting Ellen that you taught a mixed year seven group and you didn't have the same barriers that you have described Lucy. It didn't feel like there was something like adding fractions that got in the way of the students accessing that task.		I am swaying this back to having the attention on 'Lucy' – refocus on the issue she was describing and my comment also says something about my own attention on barriers to accessing a task – making something accessible was a big thing for me in my teaching – so I would guess this is why I noticed that in all of the many words that Lucy spoke.	*Inner/outer*
1:03:53	Ellen	Maybe it was because the task was slightly easier, or maybe it's because I'm always saying because, or why? Maybe they are used to, how do you know? that's my usual one, how do you know?		More hypothesising	
1:04:08	Alex	The kids know that if I can't hear them, it is because they haven't responded with a because.		More concrete	
	+	[Laughter]		Energy increases when a *strategy* comes out.	
1:04:16	Sam	That's nice			
1:04:18	Alex	So after a while the kids will say, "oh that's what I need to say, the reasoning behind it". So again, maybe it is because we already have that established.			
1:04:30	Beth	It's interesting to consider if certain structures might match up to certain topics. Maybe the structure was just slightly more aligned with the topic in that instance. So there were just less barriers for the kids to get through.		Beth says something in direct response to my question.	
1:04:51	Tracy	Feels like a good place to stop, thank you so much everybody.			

Appendix 3b: Full transcript of feedback session 2

Time	Name	Transcript	Phase one comments	Phase two comments	Label
00:03 00:47 04:00	Tracy	OK, so let's just spend a couple of moments gathering some thoughts, if you want to jot things down. You are each going to have some time to reflect on questioning particularly in the way that Paul has described so you've got some prompts there if you need prompts to get you thinking. But essentially, it's about each having time to reflect on what has been going on between now and the previous workshop focusing on questioning ideally but if something else has begun happening around the structures or anything else around reasoning then this is an opportunity to share those things. Then I suppose the role of the listeners, everybody else, is to support that person in their reflection, so not to find solutions necessarily but maybe just supporting the person to get into greater depth or you can ask a question if something is not clear, so listening and asking questions. Just spend a few minutes now to gather some thoughts. If you want us to do some mathematics because something has come up then feel free to share some mathematics with us that we can work on together. It is going to go quiet now while you are thinking, and then whoever is wanting to start, in a few minutes or so, they please just start. [01:33-04:00] *Teachers are making notes* Ok, who would like to begin?		Some changes from my previous setting up: Giving time to collect thoughts individually before launching into the conversation. An emphasis on role of the listeners, asking questions for clarity, speaker getting into greater depth, on opportunities for doing mathematics actively.	*Change in practice*
04:03 04:45 05:10 05:58	Alex	Um, yeah, I don't mind. As part of a team meeting we took a question, what are the angles, so looking at angles on a straight-line um and sort of explain to them what had happened in the meeting, er, previous when we, er, did the BIDMAS question. So, at the moment with year seven we are teaching geometry and measures , um, and the next sort of bit in the curriculum that people were getting up to was angles on a straight line, erm, so we set up into three groups and thought about the script and what could go on throughout the lesson, and explained to people about what makes a rich question trying to make it as open as possible and when closed questions are appropriate and when to use more open questions. The context is, we have a couple of new members of staff in the department, and a PGCE student as well. As a result of this, we said, when you come to doing your planning, you don't have to draw it out on an A3 sheet like we did in the last workshop. But do have a think about what paths the lesson could take. What sort of questioning might come up and how you could approach that? Also, sort of saw the different ways that teachers would in fact actually teach the topics, so it was quite interesting to spark that debate, about how they thought the students would, which way the students would go, because previously that's how they've honed the conversations down. So as a result, teachers said, that, I don't think that they were previously pinging back like the old table tennis to students but, I think they were then getting other students more involved in conversations, getting them to be involved in the conversations just as a facilitator. So I	What is the evidence of this?	It would have been useful here to repeat back what I interpreted from this point. To check for clarity and understanding.	*Distinctions* *Importance of context* *Change in practice*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		think, myself included probably would accept, like yes, amazing, um, without getting them… the number one discussion point or thing people were going to take away and try was convince me so it was quite powerful, convince me, or, how do you know it's right, or, can you convince the person next to you why it's right or can you convince the person they're wrong, or, how do you know they're wrong so it's having the, them having those meaningful conversations, getting them to reason with each other and reason with you about why they are correct, and possibly, why has someone picked out that misconception, why is that important to raise it now, because obviously, if you are unaware there's misconception you're gonna be doing it for the rest of your life pretty much.		What gave you have sense? Did each teacher feedback?	*Teaching strategies*
07:23	Ellen	And there were some misconceptions that we all came up with, like everybody had the same ideas, and then there were some that I've never had that answer			
07:32	Alex	Yeah, so it was really interesting to have those conversations and, as I said, not doing this [pointing to script] all the time but in your head just thinking about which ways could those conversations in class come up so that you're more prepared to facilitate those conversations. So, yeah, we had a 50-minute meeting, got into groups of three, and then I just floated round the three groups, Ellen and myself took the meeting. Did the one question, at the moment meeting wise we've condensed our meeting into fifteen minutes, so now our 50-minute meetings…	Is this reflection? There is no real detail. What is the purpose here? What is the discipline? One possibility for the other teachers – this might offer a model of working, if the discipline is others asking questions	Story of teachers preparing themselves for future scenarios. Connected to making it real for me – hypothetical dialogues.	
08:15	Ellen	The admin side…			
	Alex	… the admin side…			
	Ellen	So that meeting is completely about CPD			
08:20	Alex	… collaborative, collaborative planning and CPD, so every week, so every other week we have CPD, but on the off week, when we have our team meeting…			
08:31	Ellen	We're still doing CPD			
08:31	Alex	… that is CPD			
	Tracy	Right			
08:34	Ellen	So we're just picking a topic…			
08:35	Sam	You meet once a week as a team?	What I'm listening to is what is different to my own experience – so this example of surprise shows that this is happening here. One purpose of the conversation is just allowing the space to ask questions.		

Time	Name	Transcript	Phase one comments	Phase two comments	Label
08:35	Ellen	… yeah,	Is a discipline needed? Am I enforcing the way I set this reflective discussion up… What is reflection as a group? What is the purpose?		
	Sam	We have 6 a year			
	Ellen	Oh			
	Sam	wow			
	Ellen	I don't know which one I'd rather have			
08:42	+	[laughter]			
08:44	Alex	I don't know, yeah, somewhere in the middle, but that was it really. So we are breaking that down, so when we now do our meetings we are gonna have people just sort of thinking about what potential questions there are, so, breaking it down, we've got fourteen teachers, so we'll probably have six or seven in a meeting planning a lesson, but we're then gonna have two people responsible for thinking about what potential questions could come up			*Commitment*
09:09	Ellen	Um, yeah, and what we've done is we've put people, split people into key stage three and key stage four, and planned the actual lessons going much deeper than we would have gone if it was one person just doing it their way			
09:26 09:32	Alex	Yeah, so it's actually collaborative planning rather than, ooh you take that topic, you take that topic. That's not collaborative planning, that's just someone planning for you			*Distinctions*
	+	[laughter]			
09:42	Alex	And you adapting it to your, to the way you teach and adding to it, um, so that was that, any questions?			*Teacher going meta*
09:43	Paul	Can I ask a question?			
	Alex	Yes			
09:45	Paul	Only because I know about how the curriculum was designed and that idea of, there was a flow, so that idea of changing how a topic might have been taught to look at that deeper understanding, how did, did you bring that into the questioning or did people kind of go off on a tangent and decide to follow a different path or	I have no idea what this question means? There may have been an opportunity to intervene here – "what do you mean by flow?"		
10:05	Ellen	Well it was only one question which was from, from the um…			
	Paul	From the flow			
	Ellen	From the flow, yes		What do you mean by 'flow'? I should not be embarrassed about not knowing, or assume I am the only person who does not know	

Time	Name	Transcript	Phase one comments	Phase two comments	Label
10:11	Alex	Yeah, so in the future it is them thinking about how they can use it in their lessons as a tool. I'm sure people think about questioning but sometimes people don't think that much in advance as part of their planning process…	I could have said, "How do you know?" or "What gives you that impression?"	As well as developing their own practice, the members of this group are also supporting others in their departments – so the challenge here is how to get them to reflect on this without making assumptions, speaking for others. Encouraging evidence etc.	*Speaking for others*
10:27	Ellen	And also, we are under quite a lot of pressure sometimes so that the, the important bit of the lessons is what is in their books…			
	Alex	hmmm			
10:35 10:43	Ellen	…and actually, sometimes the important bit, bringing these conversations out and, and you know, I can remember there being times in the past where if you still at the board after ten minutes in an observation that is poor teaching whereas this is taking much longer but more kids are getting involved and, and the way that you are at the board is changing as opposed to like, so, so, in fact if anything the time at the board is longer but the usefulness of that time…		"How exactly is it changing?"	*Change in practice*
	Alex	Is richer isn't it		"How is it richer? What do you mean?"	
	Ellen	… is richer yep			
	+	hmmm			
11:09	Tracy	So, I'm interested in your sense that it's thinking through not just the questions but how you expect the students to respond. Is that the shift that you are trying to get…	I'm not sure if this question was well understood	My question was not clear - I wanted the focus to return to learning to teach using the approach of the gap task.	*Asking why?*
11:20	Alex	As a whole school there has been a shift in onus on developing literacy and numeracy, so, we've linked the work on this project to that whole school focus. So we won't just accept an answer of, say, seventy-two. Why is it seventy-two? Can you expand on that? Trying to get the students to use full sentences. I think that initiative has helped the work we have done during project, because there is a wider school emphasis on expanding answers and developing oracy in the classroom.		The significance of school context. Inner/outer – making things your own	*Importance of context* *Inner/outer*
12:01	Ellen	Yeah, and we're not in a real rush to get yes that's the right answer, right now you practice some	I notice here the energy in Ellen's voice		
	+	Hmmm			
12:06	Ellen	You know, why is it the right answer, who else, that's the right answer, how did you get that right answer, did anybody get that answer in a different way			*Teaching strategies*
	Alex	yeah			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Ellen	Did anyone get a different answer, using this, you know that sort of thing			
12:18 12:33	Joe	Can I add in, it's just I've noticed very similar things, one of the questions I put down was, I found that a lot of students can give me the right answer and intuitively know it they real struggle to either orally articulate it or there's a big focus on writing and actually I was just thinking how I would love to explore how we can get students to express their reasoning in different ways, so maybe pictorially, things like that. It's a real hurdle at the moment, I need to think about training them.	If there is something you hear from someone else that you want to comment on – they are working together in some way – rather than on separate issues. Carried over from the previous session.	An example of an idea being sparked off - something powerful to be harnessed - perhaps speaks to why we do 'groups' the way we do - links to listening to others' stories... stories that lead to stories. A new (but related) category nonetheless, did the previous thread get closure? Is that important? "Training them, what do you mean?"	*Teaching issue*
12:47	Paul	That goes in really well with what we are doing today actually			
	Joe	Perfect			
	Tracy	That's nice			
12:50	Sam	Because it's kind of, I think they object to it particularly because it's maths.			*Speaking for others*
	Joe	Yeah			
	Sam	So, why do I have to write something when this is a maths lesson, it's numbers, why words?	It occurs to me that an issue has been identified here, so perhaps one of my roles is to be explicit when an issue has been raised… make note of it, form a list of issues to work on as a group? Could be the basis of me coming out to visit?		
13:00	Joe	But there's also a case to think why do you have to write a sentence are there other ways you can express it?			*Teaching strategies*
	Ellen	Yeah because the sentence doesn't have to be in the English language it could be a maths sentence			
	Joe	yeah			
	Sam	Yeah, or a diagram… or a graphic representation			
13:12	Maria	It's really interesting that you said that because I work with a non-secondary ready class, and um, they are very much, they give me the answers, and I just, you know, I say to them are you sure, can you show me how, and you know they have just started now to do pictorially			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Joe	Oh	I have no idea what this might be a picture for, should I be slowing this down? Asking about a specific case/example from this class?		
13:27	Maria	You know, drawing baskets, drawing eggs, drawing, which is fantastic			
13:31	Ellen	And do you find with the lower ability that if you say are you sure it, they think that they're wrong then			*Inner/outer*
	+	yeah			*Unresolved*
13:37	Maria	That's my little test, I say, are you sure, and they're like no and its wrong then			
	+	yeah			*Teaching issue*
13:41	Beth	Not just low ability			
	Ellen	No			
	Beth	I was teaching top of foundation, maybe would have been a higher class in last year's GCSE the other day and I had to every time I said why? or are you sure? or why do you say that? I had to like, the panic, I had to go…			
	Ellen	yeah			
	Maria	… because you're right			
	Ellen	yeah			
	Maria	because they wouldn't like that			
	Ellen	no			
13:57	Miguel	Do you think that with taking notes then, I know you said there are other ways of doing, I say to my lot you do need to be able to take notes, all I say is it doesn't matter if it is dreadful to begin with, it will only get better, but I also think that is you, if you explain some of the importance of taking notes outside maths or outside even academia, then they're more receptive to it, and um, I just say they've got to make a start, and also if you ask them, if you ask for complete silence usually boredom will drive children to work.	What I notice here is that this discussion is a bit sporadic – new points being raised without addressing the initial points maybe? There needs to be some discipline here. Where did 'taking notes' come from?	I have lost who it is that is 'having time' here.	*Unresolved*
14:21	+	[laughter]			
	Miguel	And if you…			
	Tracy	[laughing] I remember you saying this last time, yeah		It would be really useful to pause and just try to articulate the teaching issues that are being explored here as it is easy to lose the thread.	
	Miguel	Then they'll write something down, you can see you know, they pick up the green pen and get on with it, so yeah			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
14:32 14:38 15:15	Paul	I think, what was really interesting from this cohort to last cohort is, um, at the end of workshop one, the previous cohort were really worried that there wasn't enough work being done and what they meant was the work was in their books, and then with things that have come from Ofsted and the ideas that now actually not all of your work has to be in your books and its perfectly acceptable to have a whole verbal lesson or that understanding or those conversations or more time at the board, it's not about a teaching method, it's about children's understanding, and that came in around the end of workshop two - beginning of workshop three last year and all of a sudden they said, it didn't even come up in workshop three, they talked about it briefly in workshop two but by workshop three they just accepted work does not mean it all being, loads and loads of things written into their book and having that justification in their department and the big thing was then about the school recognising that, you talked about, you know, oracy having a larger part within the school and that also then being accepted so having those conversations about different ways of learning within the school if that makes sense, so it's really interesting this shift between cohort one who that was the first hurdle they kind of came up against, whereas you guys are just sort of mentioning it in workshop three as maybe a side-line so I just think that's really interesting…		What Paul said made me think about the importance of school context - so what are the school foci for improvement at the moment? The importance of context needs making clear, there are no right answers - we work within our own contexts and our school contexts so being explicit about these contexts is an important thing. Enough context is needed to make sense of each other's issues etc. Also, that different groups will develop different foci of concern.	*Importance of context*
	Sam	Definitely, there's a massive shift			
15:56	Paul	… a shift in education or a shift in the way people have started thinking maybe			
16:02 16:08	Sam	Even so much so, that we had an inset day yesterday, so we are really having a good week and our head actually said so after, because it was about different ways of marking, live marking and impact marking which is what we do which I'll talk about later when we do questioning, and he actually said I'm now at the stage where if I don't see marking in books I'm not fussed about it and that is such, you know that's a year on, well even two months ago I would never have thought he would have said that because it was very much, I don't know, back to the walk, what can we show, what can we show, what can we show, but he followed that up with but what I will do is sit down with children and say so, where are you, what do you need to do next, how's your teacher got you to this stage, how is your learning different from the learning of other people in the classroom. So, I think that's, that's really positive that he's not looking just at what is very easy to measure, or that there's no marking in this book, or there's not enough work in this book, to something that's actually more difficult to measure… so holidays for us then	We are at a distance from the classroom here and have strayed into whole school/management issues which does not feel particularly helpful other than using the group as a sounding board maybe. Questions I might have as reminders: How does this help the person who has had time? Is this a new point or a point connected to one from the original story? Is this a question for the person who has had time?		*Inner/outer*
16:58	Tracy	So one of Paul's prompts was to think about how you measure mathematical reasoning because it's not something you can measure easily		This is a reminder of why the questions need to come from the	*Teaching issue*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				group. Why would they be interested in this question?	
17:10	Miguel	Isn't that what a test is? Sorry for saying that…	I have an emotional response to this – I remember feeling bothered by this in the moment and still each time I listen to it, because it goes against what I value. However, I need to approach this type of comment without judgement – as this may be a time to pause and just go around and think about what other ways there might be for measuring or assessing a student's ability to reason mathematically… How much is it my role to get teachers to question their views…? These are not PGCE students, they are experienced teachers.	"That might be one mechanism" .... "what other mechanisms might there be?" Interestingly, I now feel less of an emotional response to this comment. It seems to be more about listening now than judging.	
	Beth	Depends on the question design I guess			
	Maria	yeah			
17:15	Miguel	Yeah but that's what a test is, I mean that's what GCSEs are anyway somebody gives you a job because of your GCSEs they see you can mathematically reason to some extent anyway			
	Tracy	That might be one mechanism but I'm not entirely sure what that does in terms of…			
	Ellen	It depends on the questions doesn't it, and they are moving towards more where you can't get the right answer unless you can reason, so, so that's why we have to concentrate on being able to reason because knowing the method won't get you there unless you can reason with the method			
17:46	Maria	yeah			
17:47 18:04	Sam	Even just, the paper we have just done, paper one question one, higher paper, was, which, it was multiple choice, which of these is the correct formula for the total number of angles in a polygon, and we've taught 180(n-2) and it wasn't it was 360-180n, **and the number of kids who couldn't actually work out that all you do is multiply out the bracket…**	So this feels like an *issue* so do I stop at this point and collect similar stories or strategies?	Some work could be done here to try and articulate the issue that this example is speaking to.	*Teaching issue*
	+	[mummering's]			
	Sam	… just that first question…			
	+	[laughter]			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
18:11	Sam	… it was really tricky, just horrific			
18:13	Miguel	You do wonder quite how much, um, I know because it's early days we haven't had that many examples of the papers, um, when we get enough they start repeating themselves, you can always revise for a test, I mean like the eleven plus, if you haven't seen it before you are gonna be stuffed, but you can revise for it and they're gonna have, they are eventually going to start repeating those type of questions, there's only so many times you can do that, I'm not saying you don't need to teach mastery…	I have just noticed that there are 3 voices that have been absent up to this point (Simon, Mia, Lucy)		*Inclusion*
	+	[laughter]			
	Miguel	… what I'm saying is that, the idea that you can gauge genuine understanding as opposed to practice for a test is always going to be a bit false when you've got like a test behind you, that's what I'm saying			
18:47	Joe	In terms of judging that, I think there is a huge case for professional judgement in relation to kids reasoning. The reason is, if I look at the classes that I have been focussing on their reasoning , then the children who are consistently the best at reasoning, the ones who are able to make the connections, are those who typically fall in around quartile three in terms of their attainment. They won't be at the top, they won't be in the middle they'll be kind of half way between, um, but that doesn't discount from the fact that whenever we're in class and we're doing this they're reaching conclusions faster, they're helping the other children get there, so there's, I think, it's, there's a tangible way you can measure it in a test, there's also a huge amount of intangible ways you need to kind of think about it and then, I don't have all the answers here but I think about how you can build on that	This sounds like a lovely research problem/dissertation title or MT article… noticing this feels like a shift for me in terms of what I am hearing. As a teacher, this just wouldn't have 'crossed my mind' so why is it I hear this now… what has happened to me so that my attention/the *way* I hear what has been said has changed??		*Research opportunity*
19:30	Sam	So, sorry, has the reasoning moved them, because you said they were kind of halfway between half and top			
19:36	Joe	yeah			
19:37	Sam	The fact that their reasoning is getting better, is it moving them towards the top?	This feels better – in that this question is directly related to an *issue* that has been described		*Asking why*
19:41	Joe	I think it certainly is, but what I would counter it with is that the student who consistently gets the top mark on the test is not necessarily the student who is the best at the reasoning.			
	+	yeah			
19:50	Ellen	There's a lot of memory involved	"What do you mean by memory?"	An interesting comment - highlights a few things... comments being said and not heard.	*Distinctions*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				"Do you see reasoning and memory as different?" How philosophical/theoretical is ok? Responses linked to asking why.	
	Sam	Is it because they can't reason or they can't articulate how they can reason?			
	Alex	I got someone that got 97% on a test, the only reason he lost marks is because he actually found the answer without a calculator rather than estimating			
	Paul	umhum			
20:04 20:12	Alex	But he's so, so high order level of thinking, he can't, he can't explain to me how he gets answers, and that's the one thing… I've got him sat next to the person that's struggling but can articulate really well to try and see if they bounce off each other, because this is gonna take, he only year eight, but he can probably take, well he is close to taking GCSE now, but he genuinely can't explain anything…	What do you mean by high order level of thinking?	Certain categories seem more useful than others. Those that are at the level of action or concrete, and usually arise out of the detail.	*Distinctions*
20:33 20:39	Maria	We are trying live marking in our school at the moment and we're trying to incorporate the reasoning within that, and, you know, similar kind of experiences, we're getting **high ability kids that really, really, struggle on a one to one with the teacher, you know, how have you got this? what might happen if? and they just really, really struggle don't they.**	What do you mean by live marking? Again, there fells like an issue has been identified here… linked to classroom culture, expectations	So important to establish the context – so that everyone in the group can understand what is being talked about. Who is having time? Unstructured.	*Importance of context* *Teaching Issue*
	Sam	Yeah			
	Maria	Yeah			
20:55	Simon	I did a lesson period five last week which was probably the best lesson I've put together for a while		Then Simon shifts to a story... a particular moment, concrete and real, which feels in a different place to the preceding conversation.	
	+	wow			
	Simon	[laughing] with a bottom set			
	+	laughter			
21:02	Simon	No, but it took the least amount of thought but actually it was probably the best because they didn't physically write anything down, right, I put them in to groups of threes, it was my bottom set year eight and we were doing angles on a straight line, they know opposite angles, that's about it, so they each got given a table, they got given some, what's that tape, like, Sellotape, but you can kind of write on it…		This now feels connected to the previous set of comments, a potential way forward given the issue described. It would have been useful to label this as a potential idea in relation to the issue and then collect more.	
21:23	+	Masking tape			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
21:25 21:38	Simon	Kind of like masking tape, so I gave them each a roll of masking tape and they just put loads of lines, kind of like… that [shows sketch to group], just all over their tables, then I gave them a black board pen and they had to measure the angles and then they had to go round in groups, they each got given a mini white board, and they had to write down anything that looked good on someone else's table, anything that was not so good, and anything else they generally spotted that they thought was interesting, because we hadn't talked about angles in a quadrilateral, and is there more than one way of coming up with the answers that they got, so, like, there was one point where I think someone said oh yeah well they're opposite angles and someone else said well, because I talked about like if you cover your hand over then that's like a straight line.	This detail feels really important, what the students said could really be a focus	It would have been useful to capture this detail	*Detailed description*
22:04	Sam	Oh I love that. I'm doing that.		A recognition	*Commitment*
	Simon	Um… but they didn't write a single thing down, the entire lesson, they literally just drew on the tables, and talked, it was great, they loved it, because they're bonkers so they were just like… draw on the table, it was quite good, um, but in terms of like reasoning…			
22:22	Sam	So you didn't give them any rules at all, like did you insist that there was a set of parallel lines in there or?	Simon was cut off just as he was about to talk about reasoning – this is another reason why each person needs some time then questions could come later		*Unresolved*
	Simon	I haven't done parallel lines with them, they are bottom set year eight, they literally, they can barely measure angles so I just thought it was a fun way of doing it…			
	Sam	No, it was really nice		recognition	
	Simon	… and it just kind of like spiralled off really, um, but yeah, groups of three something like that, three or four			
22:41	Tracy	You were about to say in terms of the reasoning.	So I noticed in the moment that Simon had been cut off so why wait until now to come back to it – why not immediately?		
22:44	Simon	In terms of the reasoning I just thought it was good because I just put a few like kind of prompty questions up on the board just things I wanted them to talk about and then we were talking, we were going 'round each table, talking for like five minutes on each table, yeah, is there more than one way of doing that was kind of the biggest one I think, because there were lots of different ways they could have done it. They were all coming up with all these different ways and why that one was this, then, they'd be like, well, the corner of the table is a right angle so then now we can do this, and quite good, um, but it required no thought, that was the best thing about it [laughing] really.	So, this is a really lovely question that got students in his classroom to talk mathematically. Is there a place here for sharing other questions/prompts like this? Perhaps making a list of other	Capture the prompty questions like "is there more than one way of doing that?" Moves between an account of what happened to accounting for – this feels like a useful shift. Potentially next step would be to articulate the	*Slide (from 'account-of' to 'account-for)*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
			similar questions/prompts would be a good activity. So what is it about some questions that means they lead to rich discussion?	phenomena (e.g., getting students to explore mathematical properties) and then collect further stories/strategies.	
	Sam	Just a few baby wipes afterwards [laughing]			
	Simon	Yeah, I literally just got some spray and was just like right…			
23:25 23:47 24:31 25:12 25:24 26:11	Lucy	Yeah, I had a lesson with my year eights the other week and it was on ratio, dividing into ratio, and some of them are quite reluctant to write things down, so I gave them mini whiteboards and they were showing their thought process on the mini whiteboards, but I was trying to get them to write some of it in their books as well, because I'm still of that thinking that you need to show, because we are, we have been told that you need to show lots of work in their books anyway, but when I was at the front I questioned them about, um, say things like, um, what does, you know, two to one mean and some of the misconceptions would be, you know, writing it as a fraction, er, so two over one, um, and then I was, you know, explaining two to one is actually the total is out of three so how would you write it as a fraction, and then labelling each of the number, so like Melissa to I don't know Sahim, um, and then, um, so, I was asking them, um, like a wordy question, for example, two to one, divide 126 in the ratio of two to one, if Melissa gives a quarter of her share, how much would she have left? So then they have to think about what's her share to begin with and then what's a quarter of that, so, pulling out their knowledge of fractions of amounts and then sort of answering the question step by step, because they look at the whole thing and their put off because there's a lot of words, so I really had to stand up at the front and break it down for them and they were able to answer it as I was breaking it down and it was quite nice because other people were helping so if someone got stuck then somebody else would give them a hint, you know for example, I said what's a quarter of 84 and somebody else said, I know that half of 84 is 42 so a quarter is, so they were breaking it down for somebody else. I think the questions, I wouldn't say that I necessarily planned the questions they kind of came about organically based on the responses they were giving me and based on the mistakes that were coming out of their questions. I was drawing out the misconceptions on the fractions and then making sure that they were answering the questions and when other people were giving them hints I would say, for example, when somebody said a quarter of 84 is 21, I said how do you know and then I encouraged them to use pictures, so draw a circle and then show everything as 84, split it in halves and then split it again, so using different methods. There is one child in my group that doesn't like answering questions, and I don't, she has really low self-esteem and I'm not quite sure. I know we propose no opt out, but I don't know how to get her to answer a question without, sort-of,…	So at this point it would be much clearer and easier to just come to the board and show the group. Account-of vs. accounting-for. What is the observation that leads to this evaluation? Is this now an issue to open up to the wider group?	"How would you each answer this question? What different representations would you use? What is the questions was…?" A very explicit question/issue - an example of Lucy talking her way to an issue.	*Inner/outer* *Doing mathematics* *Slide (from 'account-of' to 'account-for)* *Teaching issue*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
26:30	Tracy	So the issue is that she won't put up her hand to answer a question?			
26:32	Lucy	If I ask her to answer a question, and she feels that she is going to get it wrong, she'll say no I'm not answering it. Even though the person next to her is quite confident and would give her a hint. She still doesn't want to.			
26:43	Ellen	Could you avoid only asking one person to answer. If they've got mini whiteboards and they all write an answer down, then she's answering it along with everyone else?	An offering of a strategy here – this would be a good time to open up wider and make a list of strategies against this issue: Issue - Strategy		*Teaching strategies*
26:52	Lucy	Yeah, I guess so yeah, but I think in her head she's just convinced that she's going to get it wrong even though she's got the ability.			
26:59	Alex	What's wrong with getting things wrong? Just celebrate the fact that people are getting things wrong. That's an opportunity to learn.			
	Lucy	Yeah, yeah, that's what I say			
27:07	Ellen	That's going back to my old, oh what's the program called? You know the one, I used to use it.			
	Alex	Was it QI?			
27:20	Ellen	QI that's it! Where the wrong answers come up. I was hoping someone would say that. I used to use that a lot. I've forgotten about that.		Leads Ellen to remembering a forgotten strategy and sharing of strategies more generally.	
	Lucy	Oh yeah, absolutely. Like with the fraction misconception, when someone says something like that, I say, "oh thank you, because this is the mistake that people often make", and then I celebrate it. I even make mistakes in front of them, so I try to celebrate mistakes.			
27:40	Alex	I use Joe's Nan, and they don't even know who Joe is. It's not Joe it's his Nan that gets things wrong20 [laughter].			
	+	[laughter]			
	Tracy	So not Alex's Nan, Joe's Nan [laughing]			
	+	[laughter]			
	Alex	It's not Joe it's his Nan that gets things wrong			
27:52	Lucy	But yeah			
27:53 28:17	Paul	Maybe if it's about self-confidence and it is a serious issue for that student, then if you have a particular question, or you've planned a particular question that you want to ask that student, then you give her lots of time. Say to her, at this time in the lesson, I'm going to ask you this, so have a go on your whiteboard, and we can have a look at it beforehand. If she's worried about being wrong, then that conversation is maybe between you and her,	Strategies are still coming out from this conversation. This feels more useful – that an issue is being addressed rather		

[20] In the first feedback session, Joe told the group that he often refers to his fictitious nan in reference to making mistakes.

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		but maybe that would actually help build up her confidence of you being her teacher, and, you know, enable her to speak.	than just recounting or only celebrating the successes		
	Lucy	Yeah, yeah, yeah			
28:32	Paul	… so something like that with that particular student, you might find that the time, the time you give her gets shorter and shorter to the point where it's only a few seconds maybe, you know that and celebrating the wrongness of stuff			
28:45	Lucy	Yeah, because another student in that lesson when one of the, someone was observing me, and he was really, he felt really on the spot, but he was still answering, I was really pushing him, he would give me an answer and then I would give him another question, follow up question, and he was, you know, visibly sweating but he still carried on answering, and she was, you know, they were all watching him, and I said, I know that you're nervous because, you know, sir's watching you, but he was fine, I know you can actually answer the question, and he persevered with it. Um, yeah.			
29:18 29:22 29:37 29:44	Joe	I was just thinking back to a session I went to at the maths conference at the weekend and a lot of what we are discussing now here is very talk based, and is there a case with some of the things we are modelling to promote reasoning, we say a lot less, just show them, break it down into manageable steps, so I did this thing, copied somebody at the conference completely, linking area of rectangle to area of triangle, I taught that normally last term, it didn't go down very well			*Teacher going meta* *Change in practice*
29:47	Tracy	What do you mean by normally?	Yes – in the moment my reaction was the same as what I would do on reflection.		*Slide (from 'account-of' to 'account-for')*
	+	[laughter]			
29:50   30:06

Time	Name	Transcript	Phase one comments	Phase two comments	Label
			rather than saying what you did, do it!		
31:03	Tracy	Less talk from…?			
	Joe	From teachers but also potentially from students, there's that interesting point which is sometimes if you are listening and you are thinking it's two different things whereas if you can just isolate the thinking you can really get it to a stronger point			
31:18 31:54 32:22	Miguel	I've started doing this more and more recently, I think I said this last time but I used to often think, I need to explain something really clearly and then give them the work to do once I've explained it, and point out the pitfalls before they do it, where now what I do is give them the work but ask them in silence to give it a go, and the reason why that's good is that they're gonna be faced with an exam in silence without any help so they need to get used to it, um, but also it draws out the mistakes they made, so I always think if someone points out an error that you might make, you go, well I'd never get that wrong, but actually if you're allowed to get it wrong yourself then you realise once you did get it wrong, just a bit like if you give someone a puzzle with the answer it seems so obvious but obviously it's not obvious because most people couldn't do it, so what I'm tending to do more and more is give them, give them questions to do without any help at all and then usually what happens, if I go round the class marking the work, particularly if I've got like an answer, like my own copy of the work myself, I have a work sheet, I'll circle things their getting wrong with a few notes on it then when I go through it on the board I draw out things they've got wrong, and often make the mistake that they made, so say I'm gonna make a common mistake here, I go through it making the mistake and I say put your hand up when you spot what I've done wrong, and then I talk through it, and also when I'm going through the answers rather than going through the method and giving out the answer, I'll start by giving the answer at the bottom, like I'm not really interested in it, and then, and then talk about the method, because otherwise I think the danger is I think the pupils are thinking what's the number so I can tick it, where if you give the number and say I'm not interested in that, that's just the answer, let's go through it, I think it redirects their attention towards the process and the reasons behind it	This feels now like a slightly different thing to what Joe was talking about. The link is silence but Joe seemed to be talking about sense making around a concept being led by the teacher. This feels more like having a go at something to make mistakes and learn from them. So is it unhelpful to allow the conversation to do this drifting or could there be some sort of enforced staying with ideas? Or are these ideas close enough given the link of silence…? What might I be writing on a board as the conversation runs? What gets distilled onto a board is based on what I hear. Do I have a framework/structure in mind or do I write verbatim?	Miguel - gives a different take on his change in practice as articulated during session 1 - less evocative maybe but this time he is demonstrating how an issue can evolve. From Lucy's getting a child to contribute came strategies for creating a classroom culture where it is ok to be wrong and where the answer is not the most important thing - also I notice Lucy didn't get closure in her issue. Is this sharing strategies or is it something else? It would have been useful to return to Lucy to get her to articulate if the conversation had addressed her issue and in what way (what she takes from the conversations, what commitment might she make?)	*It/that/this* *Change in practice* *Unresolved*
32:43 32:55 33:17	Beth	Yeah, I did something quite similar, it was year eleven, not key stage three but, we were talking about the multiple-choice questions at the start of the paper. I circled the right answer and said, "why is this correct?", so they didn't need to think about that, they knew which one was correct. Then I gave them the question again three more times with the other three answers circled and said, "now you need to tell me what the question could have been if this was the right answer". It was really interesting because they did things, I didn't expect them to do. It was a ratio one, it was what fraction of the drink is orange? I thought they would just change the type of drink each time, but they also started reordering the ratios as well, which for me, showed a better understanding. I was a bit	So perhaps the theme here is getting the students to make sense/focus on the reasoning/do the work… this perhaps confirms the need for the teachers to describe the issue/theme and not me. Hearing what I hear not what they mean.	Use of multiple choice questions. There are some lovely ideas and stories of practices emerging and they are crying out to be captured, but also, to be labelled and attached to a particular issue. Are people having time?	*Drift*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		nervous, is it the right thing to do? Just give them the right answer and hope? But actually, it worked really well because it really did help me to understand what they understood.			
33:27	Sam	We've got a thing in year eleven at the moment where all of our starters is at least one multiple choice question, one, three or five multiple choice questions	Now the link is multiple choice…		
	Beth	Are you AQA by any chance [laughing]			
33:34	Sam	[laughing] so we're going through that, and sometimes I just put one and say right I want you to right questions that would give me the other three			
	Maria	yeah			
33:41	Sam	So I don't give them the right answer but I think actually in the future I think I will, I'll say that's the one now you right the questions		This section of the conversation demonstrates to me, how a conversation can drift. Could I track the issue as it evolves? What could be made explicit about the process of the conversation? I feel the need so strongly now to group and to collate and to do so publicly. I feel the need to get the teachers to get themselves to their own moment, given the issue, that the issue speaks to for them - or at least a purpose - so that they recognise it again in the future and can bring forth the new possibilities that this conversation brings. Hence writing it down - making the connections explicit. Even the issue might not be 'common' but the process of articulating/labelling the issue for themselves is so powerful.	*Commitment*
	Joe	Or another one I got off somebody else at the weekend, which was write your own multiple-choice question, go through a few with them, ok well, why did you get these three answers, right ok, make your own now			
33:56	Sam	Yeah because anyone can write a multiple-choice question, writing a *good* multiple-choice question			
	Joe	Yeah, a diagnostic question			
	Paul	Negative variation	I'm not sure anybody knows what this is – I don't!		

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	+	yeah			
34:06	Paul	Can I ask a question with regards to lots of you talked about work. Could we learn anything from primaries with regards to reasoning, so, if we're not talking about writing in their books, a lot of primaries still make evidence, they'll keep evidence just as you would in your own mark book, whether that's photographic evidence or a note as to what happened that lesson, I don't think it's something, if your SLT are happy and on board and completely forward with the idea of different lessons look different in books and on work but if they're not then would that maybe support some of those gaps.		An external teaching issue? How do we 'show evidence'?	*Teaching issue*
34:46 34:58	Maria	D'you know I've gotta say, SLT have been fantastic with us because we're actually doing working walls now, so everything is going up on display what we've been doing in the lesson, so even, you know, we haven't got, necessarily, the content in our books but if anybody walks in they can see it is there, this is what we've done.			*Inner/outer*
35:04	Tracy	Are these white boards? Or are they pin boards?			
35:07	Maria	We've got a working wall, so they're display boards, and we call them working walls, they're all on the same colour throughout the maths department and that is standard procedure, SLT know when they're bringing in visitors, you know, they will come in they will find our working walls and they will.		The idea of working wall is raised by Maria and I ask a question. It's also in my mind about vertical surfaces. A genuinely new idea for me.	*Distinctions*
35:22	Sam	And its things like pictures of kids' books, photographs of what they've done on whiteboards, when they've used flipcharts stick that up there, they look a complete mess, well no that's not true they don't look a mess.			
	+	[laughter]			
35:32	Sam	But that, you know, if you look at that, sort of, architype, what five years ago would have been a good display, that you put up for parents evening and stays there all year, you know, at the start of a module they are completely empty, it just says key stage three working wall but then by the end its full of brilliant stuff.			*Change in practice*
	+	…			
35:48 36:04	Sam	But the kids use them, they go back to them so in an algebra unit my teach first person developed this thing called dangerous mistakes, talking about celebrating mistakes, dangerous mistakes, dangerous mistakes, and she took all these dangerous mistakes and they are all around the classroom and the kids go, right no that's that, you've just done that dangerous mistake, um, and it really feels that displays work to support the learning.		A nice example of a description leading to a meta comment or a slide.	*Teacher going meta*
36:08	Maria	And we've got, we've got questions linked to this gap task now, we've got questions that students have come up to ask other students, and that's on our working walls as well, so we're displaying those, to the point where our SLT have changed our learning walk drop in notes so we've now got questioning really stretches the more able learners, questions were really supportive of our least able learners, so they've actually tweaked all our learning walk drop-in notes, apparently it was thousands of pounds to do this, but…	I think this is interesting but what is the place for it here. Is the purpose for others to consider new ways of working in which case maybe this is fine.		

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Sam	… who cares [laughing]			
	+	[laughter]			
	Maria	We're happy, so yeah, they've been great			
36:50	Mia	I was gonna say, I use learning walls but in a slightly different way, I have put sticky back plastic stuff over the top, pretty much like that display board there but with sticky back plastic over it, and so write with white board pen on them, so I've got like a learning objective section, so you write all the different objectives down, so we know what we've covered, key words, formulae, so I just write them with a white board pen…			
	Maria	nice			
	Mia	… So yeah, it does look a little bit untidy but it's a *working* wall			
37:14	Sam	It's a *working* wall		There seems to be a process here of the teachers establishing what is meant by 'working wall' in their different contexts – with an emphasis on 'working'	*Distinctions*
37:15	Mia	So, and then I just wipe it with a baby wipe at the end of the unit			
	Simon	I've got the same thing but it's like, what have we struggled with, so I just put that as a bubble, it's just a mind map of ideas that have come from different year groups of different things they have struggled with, might be a misconception, why is this? I don't know could be anything really, something to look at when they come in	The theme here is working walls, how they can be used… there seems to be quite a lot to say about this from different people. Will this all be lost though? The more I transcribe the more I think about how so much of all of this conversation is lost whereas if I was able to capture this on a flipchart then it may be something to keep the conversation focused but also to refer to later on…. Getting some commitment as well that some ideas will be tried (if there is motivation to do so!)		
37:37	Tracy	So, back to questioning		I then refocus on questioning - it's a rule!	
	+	[laughter]			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
37:42	Mia	Can I pick up on another point you said earlier about, because there is something I saw from a teacher when I was training that's really, it's mostly with algebra that I've really used it but he puts like a table up, it's like a, you know a really simple drawing, and he writes things here and the kids have to guess what he's gonna write here	Another opportunity to do some maths together… why is this so different to just being told about it? Experiencing the function game – in the doing of the mathematics something happens		*Doing actively*
37:58	Paul	The teacher game			
	Mia	Is that what it is, is it?			
	Paul	The teacher game			
	Tracy	The function game?		Function game - why not play this?	
	Paul	The function game			
38:02	Simon	You don't say anything			
38:03	Mia	But you don't say anything, and if they get it wrong then I just put an unhappy face on the board and if they get it right then happy face, but the kids really enjoy it, um, so it works quite well.	A missed opportunity here maybe? Perhaps in future meetings, coming prepared to share one mathematical activity would be part of what we always do?		
38:15	Tracy	**That is linked to your [looking at Joe] not talking**, because I was interested in that too, that you didn't sound like you were just about the teacher not talking, so what does reasoning look like if there is no talking from anybody?		Then a refocus on reasoning (not speaking) so I don't stick to refocusing on questioning – I break my rule maybe because it is not important to me?	*Patterns/ connections*
	Joe	As in, kind of, student sat here looking at what I'm doing, that			
	Mia	Yeah and there's nothing else to do because they're silent is there so	But I don't think this is quite what Joe was talking about		
	Joe	Yeah, making that connection between what I'm doing, what I'm writing, what the result is, it's getting them to get to that conclusion, I mean it's sounding awfully like discovery learning it's…			
38:50	Paul	Yeah but, sometimes, is that always wrong, is having different methods and methodologies			
	Joe	It's kind of breaking, or having lots of steps, into tiny steps, so that they're able to make the jumps on their own, that kind of, that kind of…			
39:04	Paul	Sounds like teaching for mastery that does.			*Distinctions*
	+	[laughter]			
39:08	Tracy	I really like that sense of their doing reasoning regardless of whether they're talking			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
39:10 39:45 40:10 40:33	Joe	And then you just, you just click another slide and you've got three other problems that are just very linked and one that may be linked into what you're going to do next, and you're just trying to guide them through those steps but I think it was said, their cognitive load is low but their cognitive effort is high, so they're not having been thinking oh I can't, oh there's three lines of thing, help what do I do, it's, they're not having to think about that so they can devote their energy to actually making the connections, and the reason I like this is, I did that lesson and then the lesson after, I did an NRICH task, it's the one where they give you two rectangles. One of the side lengths is ten and you have to figure out what the missing length is for the perimeter and area to be equal. After that lesson and admittedly with a mini white board to help them kind of make some conjecture the majority of my class just breezed through it. I was anticipating, they were going to take forever, they would say, "oh what do I do?" But with a little bit of guidance; "how about you try a different side?" "Does it have to be a whole number?" "OK you're very close what should you do now?" Just little question prompts like that. The result was that when they came to having to apply it, area and perimeter in a bit more of a reasoning context, they were able to do it, so [pause] my next task is going to be show them the formula for how do you get the perimeter and area equal and then see if they can postulate you can't have, because one of the results they came up with actually was a zero by zero rectangle [laughter]	Another activity lost on us – so quickly explained. Of course, we can't stop every time or this discussion would carry on forever, so when to stop and when not to…? What do you mean by just breezed through it?	Cognitive load vs effort a new distinction for me - again framing something I am familiar with but in a different way.	*Doing mathematics*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
41:07	Sam	Teaching in silence is a really nice idea, I'm going to give it a go.	So this might be a 'take away' something you are committed to doing/trying and reporting back on the following time? I could keep a note of these and make explicit that at any time you would like to try something that you note it down and we go around at the end and hear each other's take aways.		*Commitment*
41:11	Joe	Yeah, from what I thought would be kind of do and review of something at quite a low level and I'd have to really go over here's how you do area, here's how you do perimeter, actually it then turned into they did it all themselves, and you know in the class you get hands up all the time, it wasn't sir help me, it was sir look at this, look at this, look at this I did it			*Teacher observes change in behaviour*
41:32	Tracy	Oh, so that's nice, so the difference is in hands.	I think at the time I was thinking about how you know there has been a change, what do you notice in your classroom. Joe noticed that the hands going up were for different reasons than previously. Something you could look at as data if this was, for example, an action research project.	Hands - pointing to something observable - a future trigger.	
	Joe	yeah			
41:36	Tracy	The reason for the hands going up is very different			
	Joe	I found it!			
	Tracy	And hands are something maybe that are measurable, it's back to this, you know what can you measure, so what are they putting their hands up to do what are they putting their hands up to say			
	Joe	Sadly, I'm not able to sight body language as an AFL technique [laughing]			
	+	[laughter]			
	Tracy	Why not?			
41:57	Joe	It doesn't go onto a form nicely.	This is an interesting response, taking measuring as being		*Inner/outer*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
			done to (by a visitor?) maybe rather than as a teacher – linking to the learning walk conversation maybe – perhaps I was unclear about what I was talking about.		
	Tracy	no			
	+	[laughter]			
42:00	Paul	But you could talk about emotions and emotive maths and have that as part maybe of an AFL, because one of the things is if you have the maths, and as they're working through it their feelings as they went through, because without, without struggling you're not going to get that oh! moment, so, it's looking at how the emotions changed as they went through that process possibly, and that would, that is a way of measuring how they've progressed in that lesson		Paul then connects this to emotion... another interesting connection and then a statement about without a struggle there is no moment (and I'm not sure I agree with this) it's a very certain claim – "can anyone think of a time where you learned without a struggle?"	
42:38	Tracy	I reckon you could do a research study on the reasons kids put their hands up.			*Research opportunity*
	Joe	I'd have to write stuff [laughing]			
	+	[laughing]			
42:54	Tracy	So, other stories			
	+	…			
42:59 43:29 44:03	Sam	I was actually, I was gonna say, picking up on your emotion thing, you know sometimes when you somebody just says the right thing at the right time and you just think yeah that's just it, I was struggling with my PGCE student, getting her to engage and then I was on the teach first mentor day last week, and somebody put a slide up, and it was about learning can't happen without emotional engagement and I'd never really, and it was because, and I'm a scientist, I should know this, apparently when you are emotionally involved you produce Dopamine and another hormone which I can't remember, and it is Dopamine that's essential for the laying down of new synapses in the brain, and I never put those two things together, and actually that's what I went back to her and said, look it's about, so actually, as a consequence of that I've taught a lesson on wave particle duality because I'm a physicist and John [pseudonym] whose the professional tutor has taught a lesson on rocks to maths kids because it's completely, you know, bonkers, but just that, I'm trying to get that idea over to the student that it's about your emotion and if can transfer that emotion then that excites the kids but any way, so on that, well what did we do, we went for almost a completely different view to, rather than whole department training because I'm the professional, er, sort of whatever I am	What I notice is that I don't really know what this is about – why did Sam teach WPD – was it to demonstrate something about emotion? A question here may have been appropriate although it is away from the stories that I was asking for. I need to be more disciplined.	"yeah that's just it" a reflection on her own learning. Sam makes a connection to emotion.	*New insight/ awareness*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Maria	Lead practitioner			
44:07   44:42 44:46   45:17

Time	Name	Transcript	Phase one comments	Phase two comments	Label
			result is more detail which might be a good result?	distinction though, from my own teaching?	
	Sam	yeah			
	Tracy	that's when she's going to say it is exciting?			
47:10 47:24 47:56	Sam	Yeah, so she asked the question, and then she put it on the whiteboard and then she sat down and said I'm going to give you a learning story or something like that, a maths story, so if I have eighty skittles and divide it by two is that the same as having eighty skittles, sorry, divide it by four, no two, of god whichever way round it was, I'll get it right in a minute, no sorry, was it eighty divided by four is the same as eighty divided by two plus eighty divided by two, that was it, and if I've got eighty skittles and I divide them by four is that the same as eighty skittles divided by two and then eighty skittles divided by two and then she said I think I'm gonna let you talk about it. And so then they had this little chat and then she pulled out the disagreement, I say little chat, the noise level went like that because they were so excited, and she spent ages bringing out all these wrong answers, so much so that I was going stop, stop in my head I was going stop, they are convincing themselves that eighty divided by four is the same as eighty divided by two plus eighty divided by two but she just went with it for such a long time, um, and it was a completely different response when we watched that bit and then we watched the next bit of the video which was when she found somebody who had given the opposing answer and she just put, one person, didn't take any other people who'd agreed, and just asked that one person to explain in a lot of detail so she'd taken lots and lots of wrong answers, um, I'd loved to have seen the bit where she'd pulled it around because I was so, there was a poor little boy at the back, and he was so convinced he was right, and my emotional side was, you can tell that boy, and you can tell that boy, and at the end of it he was going yeah alright, but you just knew that in his heart he was gonna go home thinking this is the real world and this is maths [laughing] um…	How much does the 'emotional' side get in the way of considering different ways of working from a more neutral position? I don't see this the same way, but then I didn't watch the video, even so, how do I open up further possibilities. I wonder if it would be worthwhile here stopping to think about why the teacher in the video may have chosen to spend so long collecting incorrect answers and explanations. Real world being one where everything makes sense?		*Detailed description* *Slide (from 'account-of' to 'account-for')*
48:48	Tracy	Fascinating, so you think he had reasoned so much that he had convinced himself he was right when he wasn't?	"How do you know he was convinced? Have you ever been convinced of something and then been persuaded otherwise?"		
48:56	Sam	I mean… you shouldn't be judgemental should you because it was a fifty-minute lesson and we saw two five-minute clips and there are some rules about IRIS film club that you are not supposed to be judgemental. But it was really interesting watching somebody else, so it wasn't me and it wasn't them, it wasn't my IRIS it wasn't their IRIS, it was a neutral person, just er, on questioning, yeah it's called the empire talks back if anyone's interested.		I now have a new awareness about why it is so important to reconstruct moment/video clip *together*. Same with visualising. Same with transcribing and why it is so difficult. Same with multiple perspectives on data. To expand what is possible to perceive alone!	*Teacher going meta*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
49:20	Tracy	And I think if you're working with your departments on video, it is important not to be judgemental and try and stay with the detail of what's actually happening			
	Sam	Indeed… Alex what were you going to say?			
49:33	Alex	I was wondering, if you've tried it with the students, if you have, how does it work with SEN students? because I've got..	How does *it* work?? What is *it* here?	What is the 'that'? So important since I'm not sure ultimately what is being talked about for the whole of the next few comments.	*It/that/this*
	Sam	Sorry, how does?			
	Alex	How does it work with SEN students?			
	Sam	How does what work?			
	Alex	Well, the, those discussions…		Still not clear to me	
	Sam	Oh, ok um…			
49:47 50:09	Alex	… that they are so into it that they, some of them potentially think that they're right, how does that work with the SEN students, because I know that I've got an autistic lad, in my top set year eight, that I have to be quite mindful of when I am having real deep conversations because he can then pick up and be fixated in a wrong answer, so I was wondering if you have any SEN students in those lessons and how, **how far do you allow the fixation to go on for?**	I feel very moved listening to this that the label of SEN students is an unhelpful one but that may be due to my issue with limiting students, so what does this mean I offer at this point?	This sparks off a long section of conversation – what is interesting about this conversation is that I have no idea what it is about… not really!	*Teaching issue*
50:13	Tracy	And I wonder if that doesn't just apply to lots of people		Challenging assumption	
	Alex	Possibly not no, yes it might happen to everyone so. That was just my			
	Sam	Personally, I don't teach SEN kids but I do have kids that are fixated on the right answer, some of my really brightest year tens don't want that discussion they just want to know it			
50:27 50:34	Alex	I was wondering how, because obviously we're meant, not, we're meant to differentiate, we do differentiate all the time, how are we going to differentiate in this scenario? For the SEN students who struggle picking up concepts and processes and would it be too much for them?	What does this mean? Why is this scenario different? Would what be too much for them?	"What is the 'it' that would be too much?"	*It/that/this*
50:45 50:56	Maria	Personally, I'm thinking about my NSR group, I've got two autistic children in there and I wouldn't have given them that kind of scenario, mine would have been more visual, you know, diagrams and not so much of the, this answer, is this answer the same as this answer because yeah there's the level of confusion.		I suspect you could apply the same structure (same/different) and be visual, simultaneously.	*Distinction*
	Sam	Cognitive confusion, conflict			
	Maria	yeah			
51:15	Alex	I was just wondering, it just popped into my head really			
51:18 51:34	Mia	That does definitely happen with high ability kids as well, I was just thinking of a time a couple of weeks ago when I was doing conversions and we were doing area and volume conversions, but part of the starter was just simple conversions and a kid from a top set was convinced that to get from mm to cm, you times by ten and even putting examples up he still was convinced no it was times by ten so even though he knows there are ten mm in one cm he still was convinced you times by ten so I don't really understand how to…		But what exactly is being talked about? '*fixating on a wrong method/answer*' maybe? Because I exactly sure – we could have made It clear by labelling the phenomena being discussed and agreeing on the meaning.	*It/that/this*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Tracy	Well it is, isn't it, you are timsing by ten, it's ten times bigger, I guess maybe that's where that's coming from?			
	Mia	Yeah, yeah			
51:53	Tracy	So what do you do then, what do you offer?			
	Mia	By the end of the lesson he was ok with it I don't know what I did to convince him, but he was absolutely sure, he was arguing with people next to him as well, no its definitely times by ten, even though he would know fifty mm is five cm, so clearly you've divided by ten but he was still convinced it was times by ten			
52:14	Paul	Feels like there's some kind of algorithmic idea in their head, I was visiting some science teachers last month and when I asked them what they knew about circles they all came up with pi D or 2 pi R or pi R squared, and I'd go what's that, and they didn't know if it was the circumference or the area or what, but it was just, they had in their head that there was an understanding, and it might well be that what they picked up on was that it was ten times bigger or times by ten, and it's very easy then to see where algorithms go wrong because if you're trying to remember everything in algorithmic form its really easy to make those mistakes and then those concepts can get stuck with you can't they forever.		Paul makes a connection to mis-using algorithms, I bring it back to the issue that I see it as a conceptual / anti-intuition issue (i.e., the classic professor-student problem). I think maybe because I am aware that I struggle with these mathematical situations myself and always need to pause.	
52:57	Alex	It's like those share in a given ratios that, when it came to the harder questions, like, Aaron's got thirty-six things, its shared in a given ratio, still try and share it in a given ratio, even though two parts are thirty-six.			*Drift*
53:17	Joe	I did do something very similar, with the same/different so I had three questions, a share in a given ratio question, second question gets more than numbers the same, third question person on the left gets this much and actually on that one the reasoning structure really helped break down that barrier			*Doing actively*
53:37	Paul	I think Ellen and I saw a lesson, an interview lesson, the other week that was about sharing into ratios ratio wasn't it, but it was…			
	Ellen	Yes			
53:43	Paul	… just done, it was just beautiful, I don't get emotional about watching teachers but I was just, yeah, we need to hire that person now, because it was explained in terms of I want to give this group three times more than this group so it was instantly not using that idea and then looking at representations in order to work with the structures and um you know and definitely not going down the let's see how many parts there are, lets add them together, let's, you know there was none of that in the lesson at all but there were a bunch of year	How clear is this to the other teachers, perhaps again, do it with us so we can experience it.		
54:18	Ellen	It was a story, it was about giving money to penguins and			
	Paul	orangutans			
	Ellen	orangutans			
	+	[laughter]			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
54:28	Tracy	But I'm not convinced your student was doing something algorithmically because I think it's unintuitive that if you're going from mm to cm that you divide, you want to make it…	This feels surprising, in that I had been holding on to this for a while, I wonder in the transcription that I lose the 'real-timeness' of the conversation, so I can't quite re-enter what was happening for me in that moment.		*Unresolved*
	Sam	…Make it bigger			
	Tracy	Yes bigger, so timsing by ten would make sense, there is something conceptually reversed in what you actually have to do in these kinds of questions			
54:52	Joe	I do think students really struggle with dividing, we run timestable rock stars at our school and the feedback I've got from the teachers is the biggest thing it's done isn't so much the timsing it's the dividing			
	+	yeah			
	Joe	It's the biggest impact it has, just get them dividing constantly			
55:05	Tracy	But knowing when to divide and when to times	So at this point I am really thinking about what is going on for the children, I am attending less to the teachers maybe, I am thinking about whether reasoning may get in the way of students becoming fluent. Where should my attention be?	I think I want my point to be clear	
55:10	Mia	I think it's what you said, because I always think of it as you are going from a smaller unit to a bigger unit, so you need less of them, so that's how we sort of talk about it, but you're right, millimetres to centimetres is getting ten times bigger, it is just the amount			*New insight/ awareness*
	Tracy	Whatever happened in the lesson, obviously by the end of it, he had resolved that			
	Mia	yeah			
55:29	Alex	I bet if you gave him, not to explain it, but if you gave him ten questions he'd get all ten right			
	Mia	Oh yeah			
	Alex	He just couldn't explain it			
	+	yeah			
	Tracy	That's the difference is it?			
	+	yeah			
55:38	Tracy	That's similar to your idea that the person who gets the best mark in the test doesn't necessarily know how to …			*Patterns/ connections*
	Beth	How to explain it			
55:44	Tracy	To explain what's going on and is that important or is that just fluency?			*Distinctions*
55:48	Ellen	I always think that when people, **and it is changing the subject slightly**, but when kids have to have scribes in exams, that would disadvantage me terribly. Something happens between here [pointing to her head] and here [holding up a pen] that doesn't involve the rest of me, and if I try to verbalise it, it's sometimes when you're doing a question in class	Another protocol thing…. The conversation wonders…. And the detail is lost maybe. There are many interesting points	This is a really nice conversation but feels like we talk about different things because then it is about division (for Joe) my point is	*Drift*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		and you haven't come across the question before and kids ask you to explain it I have to do it myself first, so that it goes straight from my head to my pen, without saying anything, and then once I've got the answer I can work backwards verbalising.	arising but to what end? What is the purpose of the free-flowing conversation? It is engaging but is it useful? What does useful mean in this context?	still not the dividing itself it's when to divide and when to times. Ellen makes a link to letting your hands do the work and I mention the word fluency at that point. "its changing the subject slightly" – a recognition of this – an observable distinction Ellen is making.	
56:22	Tracy	The saying gets in the way?		If we started with what's reasoning or paused at any time to bring together our distinctions then this conversation could have added to that – especially if we had collected the labels and marked them on the whiteboard.	
	Ellen	Yeah absolutely			
56:24	Beth	For me that's where this idea of planning questioning comes from, in that actually we need to do something…			
	Ellen	But I could not have done the exam using a scribe where the only chance I've got to get marks was by speaking.			
	Simon	It's harder to speak than it is to write isn't it			
	Ellen	yeah			
	Tracy	Yes, it's like something else is going on	Am I getting to involved in the conversation (it is very interesting)… what else could I be doing? What could I be listening for? Who needs to do the talking?	I now hear Ellen as talking about tacit learning and embodied knowing (labels I don't think I had at that time). I wonder if offering these now would be useful? The problem with some labels are they feel static – although it is the meaning that develops maybe. I have certainly found labelling phenomena as a useful mechanism for my own learning.	*Existing frameworks*
56:45	Ellen	And when people have said to me can you tell me, like even my own children they ask me to help them with something. When they used to ask me what to write, I'd say, "oh you have to pass it to me, because I can't tell you what to write". I just have to write it.			
	+	yeah			
57:02	Tracy	And for me that is linked to Joe's idea of showing that set of images, it's as if the words can sometimes get in the way. Why do we need words when we have direct access to something? Students don't necessarily need to verbalise things for themselves, because	I suppose one thing I am offering them is some connectedness – it feels	How does this then become real? Maybe we could collect some more stories or strategies but certainly	*Patterns/ connections*

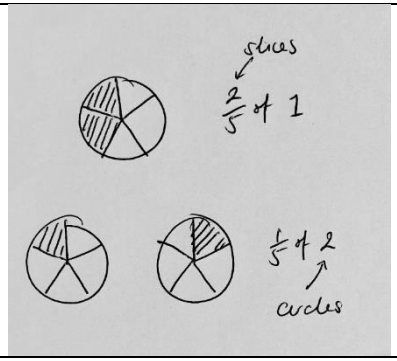
Time	Name	Transcript	Phase one comments	Phase two comments	Label
57:25		the reasoning is there, within the structure of what you offer them. For me, that is connected to Ellen needing to write for the words to come.	throughout this conversation that I have been making explicit links back to other things that have already been said – so I am noticing the connections as well as what is currently being said – so maybe being more explicit… making the connections on a flipchart.	some sort of commitment, for example, to run an activity in silence. Or we could do a silent activity and reflect on the process for ourselves.	
	Ellen	Yeah, I just need to write it			
57:28	Alex	We've got one teacher in our department who's Charlie Chaplin, he hardly ever speaks. He just does all his stuff on the board, because, same thing, he says that if you're having to listen to the conversation, you're not actually thinking, you're just listening. So, he'll do lots and lots of examples in silence and then when he sets the students off on their work, the kids that don't get it keep looking at the board, and the kids that do continue working. He'll just go through example, example, example, on the board, and then he knows when the kids look away that they've got it.			
58:03	Tracy	They've got it			
	Alex	Yeah, and you don't have to disrupt anyone, everyone's working at their own pace. They can get as many examples as they want			
58:10	Simon	It's like if you're happy, ignore me, if not, keep listening until…			
	Alex	But they don't have to listen because, if you listen, the kids get distracted whereas he just doesn't talk…			
	Sam	But ignore me doesn't necessarily mean don't listen does it, it can mean ignore what I'm writing.			
	Alex	yeah			
58:28 58:39	Beth	The only thing that I'm kind of thinking there is, is it very procedural? Are they following a method and what happens when you want to ask a question? Does it follow the same pattern? Is that where your questioning needs to come in **because you haven't checked for understanding of the concept, you've checked for replication of the process.**	I find this comment so insightful and interesting, and I suppose in reflecting on this I come to see that there are many points being made, many experiences to share in the room. Why do I need to share my own? So, am I back to asking what, if anything, do I share?	"Can conceptual understanding be developed through patterns?" "Is process only ever replication?"	*Distinctions*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
58:45	Tracy	But this [pointing to Joe's images] doesn't feel like replication of a process.	"Given all of this, can we go back to the original issue?" Awareness of awareness (wanting to go back) if acting on that awareness – bring back to the original teacher or given these reflections can anybody re-interpret what the original teacher said?	Being aware of awareness… comes from experience but this process of working in detail on these conversations is helping me!	
58:47	Beth	No that doesn't, so it depends on what you're doing	I'm suddenly struck by the potential status of my own views… this feels like an ethical issue (not just one doing research). If I am being ethically mindful, sensitive to others, then I need to be careful of becoming somebody who is seen to know all of the answers (which I do not!)	Perhaps my previous comment was too certain?	
	Tracy	It's what it is you're doing?			
	Beth	Yeah, what's happening			
	Alex	Yeah, yeah			
58:52	Joe	**Going back to the talking thing**, about that Charlie Chaplin, and like Mia's point, just like putting something on the board and they're just thinking without saying anything, I've got a PGCE student taking my group at the minute who will just talk and talk and talk and they lose the clarity, and it's like they actually confuse the situation	Re-focus by member of the group		*Teacher going meta*
	+	hmmmmm			
59:10	Joe	And actually, handing responsibility over to the students needs to happen sooner because not only are they getting a little bit bored but they're also getting more misconceptions coming out of it			
59:18	Sam	It's why you have to choose your words so carefully don't you…	Whose words? I took this as the mentor/observing teacher and I did again initially, but maybe Sam is talking about as a teacher in a classroom. So, whose 'your' are we talking about?		
	Simon	Yeah absolutely			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Sam	because mine's guilty of, so it's the bottom left hand corner, the one that's opposite the right-hand corner, the one that's underneath that, you know the one that's, you know just pick a point and stick to it			
	Simon	yeah			
59:30	Sam	Pick a description and go back again and again and again			
59:33	Simon	**So going back to your video** of that eighty divided by four or two or whatever, can't remember what it was exactly [laughing]			*Teacher going meta*
	Sam	Yeah, yeah [laughing]			
59:37 59:44 59:55 1:00:30 1:00:39	Simon	I put up a prompt the other day, like a stimulus, and I guess what I'm basically asking is like where, **how open do you make things for them to be able to facilitate being able to reason and what's too open?** I did forty percent of fifty pounds equals fifty percent of forty pounds. I put that statement up, and then I walked around. I had a quote board, so anyone that said anything interesting, I just wrote their name on the board and I used that as a discussion point. Out of that, we had a few things that came out, like, "oh it works for that, but what about if it's thirty percent of sixty pounds, or something else?" So, they changed the numbers. They asked, "what about if it's odd?" "Does it matter of it's odd?", "Does it matter if it's even?", "What if I double one value and halve the other, does it still work?" Then it just led off to lots of other really nice questions. But I hadn't deliberately structured a lesson like that before, and just that open, and we kind of lost our way a little bit. but with a bit more…		Articulation of a teaching issue	*Teaching issue* *Slide (from 'account-of' to 'account-for')* *Teaching issue*
1:00:41	Sam	Yeah, I like the idea of quote board, it's really nice	A positive affirmation of something Sam might try	If I take Simon for example he definitely has issues that he wants to talk about and he keeps coming back to them so it feels really important that each person gets to their issue and then they're supported then they can focus on somebody else's issue rather than continually coming back to their own.	*Unresolved*
1:00:45	Alex	I did the same thing, trick the kids. I did a bit of blind answering where they put their heads down, and I said so this is the question, which is more, so they blind voted, some of them said the first one was more and some of them said the second one was more. I said I reckon myself I'm going to go with the majority. You can work it out. Then one of the kids said it's the same Sir. I said your calculator is broken try mine [laughing]. Then another kid was like it's the same Sir. I said what here's another calculator try that one. [laughing].		Is that the 'same' thing? It would be useful to establish here what the phenomena is that is being discussed.	*Distinctions*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		Then they were all, Sir it's the same it's the same! So then it was a race to work out why it was the same. Yeah that definitely works that one.			
1:01:40	Paul	Can I refer back to what you [looking at Sam] were saying about the video and your learning and how that related to the reasoning and the responses in the department as a whole.		A nice connection but the previous issue(s) are not getting closure.	*Teacher going meta* *Unresolved*
1:01:50	Sam	Yeah, so having been able to have some individual conversations with, how many are we? I think eight, so there are still two people left that I haven't yet had an individual conversation with yet, but you might remember we have maths club on a Friday after school, so they have kind of passively picked up but there is a lot of conversation at the moment about talking, questioning, and planning for the activities that are going to follow on from the questioning, and thinking about the types of responses you might get. **Because we've got a lot of non-specialists**.		A really nice distilling of the process and purpose of the gap task. Back to non-specialists. I see this now as related to school context.	*Drift*
1:02:25	Maria	Sorry can I jump in. We had a twilight session, so we literally started off with open and closed questions and whether they fully understand that and it was interesting because I don't think the clarity was there.		Maria starts to talk and she talks about beginning with the difference between opening close questions again this is all about distinctions labels so another opportunity here listen because Simon already also talked about the word open just to talk about what we mean by open?	*Distinctions*
1:02:39	Sam	Particularly with some maths examples. If there is only one answer, is it an open question or a closed question?			
1:02:44	Maria	Yeah. Then obviously, we gave everybody the questioning template which is now on display in their classrooms and just asking them to experiment as much as possible, and seeing where they got with that so we're still trialling.			
1:03:04	Sam	Kind of working this way [gesturing] in this way I would say [gesturing]. We are aiming for there, but it's definitely, we keep embedding it.			
1:03:13	Maria	Yeah, but as I said, the minute we go on our learning walk and take these in now, you know you can hear the questioning from the teacher, you can tick as you go which is lovely.		Check in with Joe who asked the initial question – "is that what you wanted to know Joe?"	
1:03:24	Tracy	Nice… Beth, I'm aware that you were gonna share something.		Beth begins to talk about something on fractions, it's something that she hadn't mentioned to me previously that she wanted to ask the group hence me setting it up.	
1:03:30 1:03:57	Beth	I've been involved in running various bits of professional development recently and one of the topics I've been working with teachers on is fractions. It wasn't meant to be on fractions actually, it was meant to be conceptual understanding and we were going to do lots of topics and then fractions somewhat took over. I had a really interesting response to this [Beth has drawn two images on the whiteboard]. On reflection I wish I had then asked the teachers to have a go at a script, what would happen if a child had come up with this misconception, I really regret not doing it I just asked the teachers to look at these two representations [pointing to a whiteboard]. So, it's a pizza with five parts, two shaded, and then two pizzas cut into fifths essentially, five parts. I asked them, "what is the same and what is different?" and then I asked them "what does the numerator represent in each case, and what does the denominator represent?" I thought that everyone would be quite clear that here [pointing to case 2] it's two pizzas cut into five parts and here [pointing to			*Slide (from 'account-of' to 'account-for)* *Slide (from mathematics to teaching mathematics)*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		case 1] it's one pizza cut into five parts with two shaded. It was amazing how controversial it was. One idea was it [case 2] must be two tenths because there are ten parts there [pointing to case 2], rather than it being two fifths of one pizza and what the whole is. It was really, really interesting. I think for me it was interesting because it showed how fixed our ideas of fractions are and how we might unintentionally limit what children think.			
1:04:43	Tracy	Well do you just want to give us a minute to think about this. There might be other people who…	Slowing down	I think this might be the first time I get everybody to have a stop and I think about something – to do something actively	
1:04:50 1:05:52	Beth	Yeah, so the first question I asked was "what's the same, what's different?" **What I wished I'd asked was "what do you think the kids would say?"** and then let people think about how they would address that [I gesture to prompt Beth back to the doing]. So, any responses?		Beth says what I wished I'd said was what would the children say and I wonder what was her motivation for saying this? The move to thinking about what the students will do feels like such a natural teacher thing to do (a different type of slide maybe). Staying with the maths brings different insights – so actually it is important to separate out sometimes and to let the focus be on the maths.	
1:06:03	Tracy	Well in both cases you've got two sections highlighted.		I respond to Beth's question, whats the same, what's different to get people into this different mode – so that we might develop different insights rather than move the 'the answer' or the 'problem'	
1:06:10	Mia	It looks a bit like a fifth plus a fifth is two tenths, which is a big misconception isn't it.			
	Beth	Because there is ten parts in total so your whole is two, yep, any other responses?			
1:06:25	Paul	I was thinking about what you would define the vinculum, the fraction line, as			
1:06:29	Beth	Yeah, so that was the next question, what does the numerator represent in each case and what does the denominator represent, I think this is where we came quite unstuck, which was good, it wasn't a bad thing, it was interesting, so…			
1:06:43	Tracy	Well do you want to give us a minute to just think about the answers to those questions, what does the numerator represent in each case and what about the denominator?	Slowing down repeating	I have to ask again for Beth to let us have a minute to think about so there is this process of doing rather	

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				than being told – something I have been taught to do myself.	
		[It is quiet, and then a (related) conversation begins – not transcribed]		At this point we are sliding well away from the initial prompt.	
1:08:16	Tracy	Try and stick with this for a minute.	refocusing	I'm quite strong about redirecting this conversation to what we supposed be focusing on but maybe we could say well let's come back to that later. Trying to resist the slide away from doing actively.	*Doing actively*
	+	[laughter]			
1:08:23	Beth	So what does the numerator represent in each case? Does it represent the same thing each time?			
	Miguel	Two bits of pizza			
	Beth	Both times?			
	Miguel	Yep			
1:08:28	Sam	Yep, I would say so, to me the bit that goes on top is the bit we are interested in			
	Beth	OK, so what about the denominator? If this is always two bits of pizza, what does the denominator represent?			
1:08:40	Sam	Arrr, and therein lies the conflict			
	Miguel	How big the top bits are, they tell you, it's like you've got two bits and the five bits is how big it is out of one, I suppose.			
	Beth	OK, so in this case, you think the two are these two? [pointing to the two shaded parts, 1 from each circle – see image below]			
1:09:01	Tracy	I think the two is the two circles. 	Responding in the discussion (ok when working on some mathematics?)		

Time	Name	Transcript	Phase one comments	Phase two comments	Label
	Beth	So here is it, I've got two pizzas, and I split it into one, two, three, four, five parts, so its two things divided into five parts, as opposed to, two out of five parts.			
1:09:20	Miguel	Well, I would say that it's a fractional choice. We've got a whole, we've got one whole, one pizza, and you divide by 2, then you shade it and you've got a half, but when you have two fifths then it gets a bit trickier doesn't it. I think, you've got two, as in the two whole ones, but then when you say split it into five, you need to assume the kids know you mean split each one into five.		Miguel talks about something and then he talks about what the kids should know which feels different way of talking (a slide) about it when we are working on the mathematics. It is like when we ask the prospective teachers to dwell in the mathematics for themselves - it gives them different awarenesses before they get into planning the lesson itself. He is making a different type of slide – a slide from the mathematical insight to the teaching mathematics.	*Slide (from mathematics to teaching mathematics)*
1:09:50	Beth	I could have coloured that one in instead and it may have generated some different responses. Showing they were one part but colouring in the different ones to the ones we were highlighting.			
1:10:05	Paul	You could have two and then split those into groups of five, so you could have had the pizzas then giving you ten quadrants. Or you could share those into two groups of five which would also be a representation of the same thing.			
1:10:26	Beth	Yeah. It made me really reflect on how fixated I have been on this representation when I've been teaching. I don't know if you have ever done the chocolate bar task, when you place one on a table, two on another and three on another and then you ask them to choose where to sit.			
1:10:39	Simon	What they stand outside and then walk in?			
	+	Yeah			
1:10:42	Beth	When I have done the chocolate bar task, the students have really struggled with was the idea that three things divided by five people is the same as three fifths, and that actually, the numerator there is the three things you're sharing not the three bits you're getting. If that makes sense.		Very helpful distinction from Beth about the numerator. "Three things divided by 5 people is the same as three fifths"	*Distinctions*
	Alex	One of my students, struggles if there's something she can't get right, again, she will get 95 out of 100. I did that task with the class she is in, my top set year eight, they all came in, they sat down where they wanted to, I didn't let them speak, I let them go back out and try again, and you could tell they were trying to work it out, and the last girl came in and she said, I know what it is, I've just used share in a given ratio. I was like, oh my,			

Time	Name	Transcript	Phase one comments	Phase two comments	Label
		so she's worked out, and outside then told every single kid, the rest of the kids didn't have a clue what they were doing, but she had worked out it was a share in a given ratio so told them all where to sit so they were literally counting and then the last one, all the kids were like, so to that one. Then she came in again and said it was a share in a given ratio problem. I was like, OK.			
	+	[laughter] wow			
1:12:08	Paul	If you were to write down on your piece of paper twelve divided by four, and then show that in a representation, what would that look like? So twelve divided by four and let's start with maybe a rectangle as a representation, how would you do twelve divided by four? [period of time thinking, quietly sharing, comparing]		Getting the group to each try something actively.	*Doing actively*
1:12:41	Joe	[quietly to Alex] I will always go for this one, every time, split it into four, always.			
1:12:48	Simon	Yeah, I would never have even thought that [referring to a rectangle Paul has drawn, split into 3 parts, each labelled with a 4]			
1:12:50 1:13:03 1:13:07 1:13:22	Paul	So twelve divided by four, what we are doing is splitting our twelve into four equal parts, each with 3 in, and secondary teachers in general see this as division. Rather than grouping, where you have twelve and you split it into groups of four, and there are actually three of them. It's a real difference between what most secondary school teachers think of in terms of sharing compared to grouping. The next step is looking at what the whole is and in this case its two divided by five or a fifth of two.		12 divided by 4 and two ways grouping and sharing. A statement about secondary teachers doing one particular way - So given we were a group of 10 secondary school teachers - we could have checked that out.	*Distinctions* *Speaking for others*
	Beth	Yeah and a fifth times two is two fifths.			
	Paul	Yeah.			
1:13:50	Beth	And I wanted to give the teachers a chance to unpick them by themselves.			*Teacher going meta*
1:13:53	Tracy	Yeah, that one is two fifths of one and that is one fifth of two which in the same thing, but what does 0.4 mean in that context?		I think I am showing my sense of the distinction being made – which feels like an important thing for me to do here in terms of my own learning. "Does anybody see that is a different way?" (a move from verbalising my own distinctions/framework to opening it up for difference).	*Distinctions*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
1:14:05	Beth	Yeah. Because I could have taken a number line and shown two split into five parts and then two fifths along that number line.			
	Paul	Or two fifths itself, this is the number two fifths.			
1:14:21 1:14:46	Beth	I had a really interesting conversation with some primary teachers about just that thing. They wanted to teach the bus stop method for division and I asked them what their kids see division as. They said they see it as sharing. Now this is a cautionary tale, because I tried to use Dienes blocks to show long division. In my head, because I'm a secondary teacher, I was sharing, and then I realised the language you use around long division is grouping, it's "how many of this go into that", so you're counting up in groups. So, I was desperately trying to model sharing with Dienes blocks, but realising I wanted groups and it was all a bit of a mess. Luckily, I learned from that experience and in the end the primary teachers planned out a sequence of lessons where they just spent a couple of lessons representing division both ways and talking about sharing and grouping and to try and lead them to using times tables because they wouldn't be able to use the bus stop conceptually if they were sharing. When you say how many of them go into that [pointing at picture of 12 split into 3 group] you're grouping.			*New insight/ awareness*
1:15:20	Tracy	Your grouping in fours, yeah			
1:15:22	Beth	It was a really interesting process planning through that with them because they had to spend quite a lot of time on division by single digits and within the times tables before they could even embark on this new process with the kids and they were really stressed about how they will show progress, when people come to look at my planning, and I explained that they just need to take your time, but because their planning is scrutinised they were really worried that it would look like they weren't moving the kids forward. I had to give them the confidence that this was a necessary process for the kids and not only would they be going forwards in that direction, but in this direction, going deeper.			
1:16:00	Paul	What were the conclusions that the team got to on that? [referring back to the original stimulus – 2 pizzas or one pizza]			
1:16:08	Beth	The conclusion was that one teacher was just very unhappy, no that's two tenths, that's two tenths, and then later on, I'll show you the slides, we did some tasks together, and we looked at… [paused whilst locating a slide]		I can see why did Beth's teacher might see it was 2/10? "can anybody else see that?" "who sees it as 2/10?"	
1:16:18	Tracy	It's the problem with pizzas isn't it	Not sure if meta – an observation	The conversation here, between myself, Paul and Beth, is a little exclusive feeling – especially given we are the ones who have probably grappled with these ideas the most – so my questions in the previous line feels even more important for	*Inclusion*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				the learning of the rest of the group.	
	Paul	It is the problem with pizzas			
1:16:24	Beth	So we looked at this [slide projected on board] we talked about the fact that we've got this issue that fractions are numbers and operators, we talked about that a little bit.			
	Paul	Keep-flip-change			
1:16:39	Beth	We looked at this [slide projected] and it was half-way through this task that the teacher who was unhappy about that being two fifths suddenly went, 'It's the pizza again' when they were looking at this and that was the moment that really helped them to move on to that stage, to make that conceptual leap.		A story that demonstrates teacher learning… teacher seeing a new distinction - they've learnt from one way of seeing a problem and applied it to another	*Teacher going meta*
1:17:05	Paul	What is nice is that you were able to have that open discussion. I've sat in professional development sessions that schools have had and there has been someone sat quietly at the back and they have not been happy with it but they have not been confident enough to say, do you know what I'm just not happy with what you've said there, and been able to break it down, so, having that full out discussion or debate or argument is really important.			
1:17:29	Tracy	Well that's the same as the girl in your class [looking at Lucy]	Linking new comments to old comments, seeing patterns	I see this as trying to return the conversation to being more inclusive, to return to Lucy's issue.	*Patterns/ connections*
	Lucy	Yeah			*Unresolved*
1:17:34 1:17:47	Beth	In the last session we did fractions, operating on fractions, we did dividing and multiplying and we looked at a particular model and it was like the pizza all over again, it became a bit of a jokey reference point, that this was a moment where we became a bit unstuck. I think it was quite interesting that one of the teachers who was particularly unhappy with it, I think, finds it quite challenging to know what to do when a kid doesn't get it in lessons, so I thought it was quite an interesting moment for her to be in the same position, well you're not getting something and how do we unpick that. Which is why I wish we had done a bit more scripting and thinking of questions to ask at that moment, it would have been really nice to give teachers that opportunity.		I feel like I am having to read between the lines a little bit here – I think it is the word 'interesting' that needs a little more work maybe.	*Teacher going meta*
1:18:06	Tracy	So scripting is a mechanism for getting people to think through…	Highlighting, rephrasing	Attempting to highlight strategies – for working with teachers	
1:18:10	Beth	Or just thinking about, if a kid says this, what would you say, even if you're not writing it down, just, almost, modelling it yourself.			
1:18:17	Tracy	Feels like we're back to the start, that you're not just thinking of the questions, you need to think about the consequences of these questions, what are you expecting the students to do and what then are you going to do based on those responses. That was an amazingly rich discussion though, thank you everybody. Time for a cup of tea?		I try to bring it back to the issue at the beginning which was the whole point of the gap task interestingly I'm not sure how much was concluded in relation to the gap	*Unresolved*

Time	Name	Transcript	Phase one comments	Phase two comments	Label
				task… which speaks to me of the importance of having a purpose in mind and to keep returning to it.	

Appendix 3c: Extract used at BSRLM conference with summarised participant comments

Transcript	Comments from BSRLM participants
[laugh] ok so, we were really lucky that Paul had time to come out and bring us up to speed, problem solving is a big issue for us. That whole problem solving, fluency, mastery, all these reasoning things they're all kind of threaded together so I had more clarity, I think, after the session with Paul, that **it was alright that they threaded together, that you didn't have to make them distinct things**. So I decided, I have a top set year ten and we had just done sine rule, cosine rule, and area of a triangle without knowing the perpendicular height. I went for the structure, here are some calculations that a student might have performed based on this stimulus. I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? **I picked that structure because of all the structures that Paul showed me, that was the one that made my head absolutely bend round, and I thought, well if it makes my head bend, let's see how good my year tens really are. It completely split my class massively.** The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure. I don't think I had made a good enough job of making sure they understood the structure. **On reflection I think I should have done something much simpler first. I should have gone for something much simpler** so they understood the idea before throwing in some quite horrific maths on them. I actually I did it and then two days later I taught it and some of those calculations I was thinking why on earth did I do that I can't remember what that's all about. So, but that was quite interesting because you know I think with them I'm kind of almost at the stage where they're testing my maths um and it was quite good for them to see you know I can't remember what that calculation was all about so let's try and pick it apart. I actually started with the same triangle, and asked them to come up with a minimum of three possible examination questions that would need the sine rule, or whatever, to solve. That was fine. That was good, **but there was a massive jump** between that and the structure using the eight calculations. **In terms of the whole department**, we have a lot of non-specialists, I mean I'm a non-specialist, I'm a Physicist, I only did a subject knowledge enhancement course two years ago, but my maths is ok. **I think a lot of my department would really struggle with opening themselves up mathematically to that so I think we probably have to teach the structure with something slightly simpler**, and then encourage them to stay a little bit out of their comfort zone but not to the extent where they are completely freaked out. It took me ages to plan it and the first time I did it, I did it actually with Paul in the room, do you remember? I'd done it wrong, because I then thought I'd come back to it the next day and I thought I'd try it for myself and there were two of the calculations where the actual basic maths didn't make any sense, so it was interesting. **It was quite hairy at times**. **The first task took about five minutes. The calculations task took about ten** so it was quite a meaty starter that could have gone into a whole lesson if I'd felt like it I think. I think **I should have allowed more time and I think I should have actually unpicked each one of the calculations** but as it was the class were forming that kind of horrible, I get it, I really don't get it, and I'm really cheesed off that I don't get it. So **I had to put some emergency repairs in place**. That's me.	-Asking for examples -Challenge assumptions (e.g., ability; getting stuck bad, dichotomies, difficult for me so I'll do with a top set) without making judgements though! -Should/had to… Could = Developing an awareness of choices… Recognising opportunities for this to be made explicit -When the focus is on what did not work… what about what did work and building on that? Noticing changes in people ("I should have done something much simpler first" – the first instance of teacher articulating an awareness of doing something differently)… developing ways of seeing if there is a change in people. The shift in topic – noticing shifts in pattern is possible as an observer – is it useful to draw attention to these? You can only speak for yourself (not globally – "opening themselves up mathematically to that" – speak for yourself only – start that again – what's your evidence) Flagging the negative that forces a change, to do something differently… "emergency repairs" not wrong – but questioning this might lead to "I don't like it when they are stuck" and then we could open up to the group – "what do you do when kids get stuck?" – expanding the space of the possible for *us* and for *them*…

Appendix 3d: All labels by episode

Summary of labels by episode:

Episode one ***Using dissonance***	**Episode two** ***Staying with the detail***	**Episode three** ***Finding conviction***	**Episode four** ***Making it real***	**Chapter nine** ***Going meta***
Dissonance	*Detailed description*	*Should/ought/had to*	*Asking why?*	*Distinctions*
	Slide (from account-of to account-for)	*Speaking for others*	*Unresolved*	*Patterns/connections*
	Doing mathematics	*Inner/outer*	*Commitment*	*Teacher observes change in behaviour*
	It/that/this	*Teaching issue*	*Importance of context*	*Teacher going meta*
	Doing actively	*Teaching strategies*	*Drift*	*Existing frameworks*
	Slide (from mathematics to mathematics teaching)	*Inclusion*	*Research opportunity*	*Change in practice*
				New insight/ awareness

Key to column headings:

Label	for the phenomenon.
Distinction drawn	as distinct from…
Description	of observed behaviour
Potential strategies	* In the moment. ** Anticipatory (setting up).
Functions	of potential strategies.
Notes	on useful additional points.

Summary of working systematically on labels

Episode One: *Using dissonance*

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Dissonance	Consonance.	An experience of disturbance or disruption to an otherwise smooth flow.	* In noticing a moment of dissonance in the self, using this to trigger a prompt or question. * Prompting others in retrospectively re-entering moments of dissonance as a way of identifying issues in relation to practice. * Encouraging others to locate their own moments of dissonance in the future.	- Provoking a more thoughtful response (as opposed to reacting emotionally or judgementally). - Exposing hidden assumptions and biases. - Locating potential issues within practice that can be worked on.	An awareness of dissonance in the self is one aspect. Using the dissonance of others is another aspect.

Episode Two: *Staying with the detail*

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Detailed description	Talking in more general/ abstract terms.	Teacher is describing an account of an event.	* Commenting on when teachers are giving a detailed account, and when they are not, prompting them to describe in detail. * Encouraging a teacher to give a detailed description of what happened in the moment of a lesson, potentially a moment of momentary dissonance.	- Establishing conversation norms in relation to reflecting on past experiences. - Preventing evaluative modes of speaking. - Trapping judgements. - Supporting the development of new insights and awarenesses.	

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
			** Setting up way of talking about experiences at beginning of session.	- Developing new basic-level categories.	
Slide (from 'account-of' to 'account-for')	Staying with the detail of an observable description.	Mode of speaking changes from detailed description to evaluation/judgement/expressed values. Ambiguous term/ phrase/label used (e.g., split). Reason given for making a choice as opposed to describing what happened.	* Prompting a staying in the detail (e.g., by clarifying a subjective term). * Asking a clarifying question. * Reflecting back the mode of speaking.	- Clarifying ambiguous terms/labels (a particular label may mean different things to different people.) - Gaining access to an image of a teacher's classroom. - Delving deeper into the detail.	Sliding is not necessarily problematic, e.g., when a teacher moves from the description of an event to identifying, from that detail, a related issue.
Doing mathematics	Describing the mathematics being done.	Teachers are engaging in doing mathematics together.	* Prompting a mode of doing (as opposed to describing).	- Developing mathematical insights.	
It/that/this	The thing/idea/ phenomenon that *it/that/ this* represents.	Ambiguous use of the word *it, that* or, *this*.	* Asking for clarification.	- Avoiding speaking at cross-purposes.	The object/idea/ phenomenon that *it/that/this* represents.
Doing actively	Describing the teaching scenario.	Teacher running an activity (with the group as opposed to describing activity).	* Prompting a mode of doing/enacting (as opposed to describing how something was/could be enacted).	- Giving direct access to an activity, to experience it more closely.	This label emerged from my recognition of the need to extend the doing mathematics label, *doing actively* does not need to be doing mathematics but it represents the difference between doing and describing doing. Linked to doing mathematics but can apply to teaching mathematics or working with teachers, all of which can be done actively as opposed to talking about the doing.

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Slide (from mathematics to mathematics teaching)	Describing an account of a teaching scenario or a professional development scenario.	The moment doing mathematics becomes talking about teaching mathematics.	* Resisting the slide, by prompting a staying in the mathematics. * Embracing the slide, by relating back to practice (after a period of doing mathematics).	- Resisting common responses to some are of mathematics. - Encouraging new ways of seeing mathematics. - Encouraging new practices.	Sliding is not necessarily problematic, e.g., when a teacher moves from doing mathematics to considering what that might mean for the teaching of mathematics and their practice.

Episode Three: Finding conviction

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Should/ought/had to	Could, choice, multiple possibilities.	Using *should* or *had to* in relation to teaching decisions	* Flagging the use of *should* etc and suggesting shift in mode of speaking. * Opening to the wider group.	- Supporting an awareness of choice, a sense of agency.	Suggesting a lack of choice and limits a sense of agency. Potentially flagging an issue of ownership.
Speaking for others	Speaking for ourselves only.	Teacher making a claim beyond themselves, for example, about a group of individuals.	* Flagging up when a global claim is being made. ** Setting up "we only speak for ourselves" from the beginning.	- Avoiding unhelpful/limiting assumptions.	Limiting the agency of others (as opposed to nurturing).
Inner/outer	*Not* perceived as external.	Referring to a factor (from an external source) that could be interpreted as influencing their practice, or a perceived characteristic of an individual or group of individuals.	* Reporting back on an observation to force a different awareness. * Prompting to avoid unhelpful labels.	- Accumulating alternative positions. - Provoking new awarenesses. - Challenging a preconception/ assumption. - Supporting a process of making it your own.	Can limit expectations of self and others. Moments where a participating mathematics teacher referred to a factor (from an external source) that could be interpreted as influencing a teacher's practice.

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Teaching issue	An observable behaviour. A teaching philosophy.	At basic-level, a meaningful problem encountered in practice. Sometimes being posed as a question, sometimes as a desired outcome.	* Flagging a potential issue. * Working on any potentially ambiguous phrases or terms (e.g., out of comfort zone). * Formulating an issue that others might recognise. * Working with the teacher(s) to formulate an issue that a particular moment or set of moments is speaking to. ** Each person having time to bring to mind a moment of discomfort that might clue an issue.	- Supporting new practices. - Developing a sense of choice. - Supporting the recognition of new categories. - Taking ownership of choices and practice.	Whose issue is it? This category includes teaching issues that do not come from the group members themselves, one aspect of this category is about ownership. Important to agree issues, and that they are consensual (understood in mutual terms).
Teaching strategies	Not at the level of behaviour. At basic-level.	Describing a purpose/action (usually in relation to a particular issue, or to achieve something specific).	*Opening to wider group, to gather multiple strategies around a particular issue or idea. * Prompting for more and different potential strategies, and associated actions. ** Sharing the function of gathering potential teaching strategies and actions.	- Developing a sense of agency (i.e., many effective strategies). - Developing new basic-level categories.	Can come from others but relating to an established teaching issue.

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Inclusion	Absence of participation	Lack of verbal contribution from members of the group	* Inviting comment from those who have not yet spoken. ** Setting up an expectation of having time (limited amount of time, order of speaker established in advance, not waiting for own time to make comments).	- Including all viewpoints - Providing each member with an opportunity to articulate their own issues/perspectives.	Linked to ownership. Being actively involved in forming ideas as opposed to passive.

Episode Four: *Making it real*

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Asking why?	Asking what or how.	Asking for the *cause* of a situation.	* Guiding use of what/how questions as opposed to why. * Focusing on the detail (what). * Seeking similarity across a collection of stories, being explicit about features of specific acts.	- Avoiding the tendency to explain away or justify acts and to classify a situation prematurely. - Informing ways of acting differently in the future (as opposed to hypothesising about the cause of the issue). - Sustaining the energy that might have arisen in the detail.	
Unresolved	Continuing until a resolution is found.	Teacher changes topic of conversation before getting to a resolution (sometimes suddenly).	* Pausing the speaker and bringing the conversation back to the previous unresolved issue. * Checking if the issue has been resolved (as far as is possible)	- Seeking resolution. - Realising possibilities.	Linked to *drift.*

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Commitment	An unformed idea or strategy.	Teacher visually commits to trying something new or doing more of in their future practice.	* After a phase of sharing ideas, prompting a distillation and explicit sharing of commitments. * Visibly collecting commitments from the group as session proceeds.	- Realising possibilities. - Grounding in practice.	Formulating commitments could be done personally, could be shared, could be collected visibly (either at the end or steadily throughout).
Importance of context	Not considering context.	Explicit (or implicit) references made to context.	* Being explicit about the importance of context. * Working with teachers to explore how specific teaching strategies and associated actions could fit with context.	- Realising possibilities. - Grounding in practice.	The importance of school context.
Drift	Staying focussed on a particular point (e.g., collecting strategies, detailed descriptions)	Dialogue (as opposed to monologue) where ideas emerge from one participant to the next.	* Asking group members to summarise the main points that stay with them from the conversation (speak or write) and to translate that into practice.	- Grounding ideas. - Putting ideas into practice as opposed to holding them at a distance.	A lack of focus in the conversation, or the subject of the conversation moves from one point to the next, each point is connected, yet it *drifts* along without resolution (related to the label unresolved). Different to a slide (I use slide in relation to monologue) – drift in relation to multiple people in dialogue.
Research opportunity	No research opportunity.	Teacher articulates a problem that they have not got to the bottom of or asks a question that could form the basis of practitioner research.	* Suggesting a small-scale research project.	- Encouraging practitioner research, critically examining one's practices.	Motivating practitioner research. Looking for opportunities to reframe what a teacher says as a potential research question.

Chapter nine: *Going meta*

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
Distinctions	Not making distinctions	Teacher explicitly commenting on making distinctions (or potentially on not making distinctions). Implicitly, where a distinction is made but not necessarily acknowledged as such.	* Working on making distinctions. * Opening up to the wider group. * Collecting responses on a visible space (like a whiteboard). * Asking for an example that helps us understand the distinction being made/described. ** Facilitating a common experience (e.g., working on some mathematics together) as a reference point/generative exercise. ** Sharing distinctions from external sources and inviting comment (in combination with previous point).	- Supporting the process of making new distinctions. - Support process of seeing more/ differently.	Seeing (noticing), articulating (describing), labelling. Evidence of teacher learning – articulating a (new) distinction. At content level and at meta level. Teachers using distinctions (e.g., open/closed): Important process of sharing how different people are using these terms. Linked to new insights.
Patterns/ connections	Isolated or detached comments/ ideas/issues.	Explicitly making links across different comments over the course of the conversation.	* Drawing attention to the repeated use of a word/phrase/idea. * Asking what is meant by particular word/phrase across different people's use of the word/phrase. ** Visibly gathering themes (e.g., on a whiteboard),	- Checking on understanding of a repeated term (is it being used to mean the same thing?) - Drawing attention to commonalities and persistent ideas/issues. - Checking for resolution (e.g., of a recurring issue).	Patterns can be noticed at content level (e.g., a theme running throughout, or use of a particular word etc). Retrospectively, this might be a process of data analysis, but can happen in the moment of the conversation as well. Patterns/connections also refer to meta-level/process level (e.g., the act of noticing a pattern and explicitly commenting/making links to previous comments). This might be

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
			organising them and making connections explicit. ** Adding to existing collations of ideas/issues/ strategies etc over time, as connections are made.		noticing *types* of comments as opposed to noticing re-occurrence of a word/topic.
Teacher observes change in behaviour	Teacher has observed no change in the behaviour of their students.	Teacher gives an example of where they have observed students behaving in a way they have not observed before.	* Highlighting when a teacher describes a new behaviour (explicitly commenting on this).	- Raising awareness of changes in behaviour (increasing likelihood of noticing more changes in the future). - Increasing capacity to notice and thus triggering new ways of responding.	Observable classroom behaviours/situations/scenarios.
Teacher going meta	Remaining at the level of content. Responding directly.	Teacher makes a comment about the conversation or a comment on the process of the conversation. Teacher comments about a situation/experience at the level above the content of the situation/experience.	* Commenting on when a meta-comment has been made (reflecting back).	- Getting to a new awareness about an experience. - Getting to new distinctions.	Evidence of teacher learning. Sometimes a slide is an example of this, an example of when a slide is useful and demonstrates learning. Teachers also going meta within the conversation itself – taking on established collaborative group norms.
Existing frameworks	Own frameworks.	Using a framework or set of distinctions (e.g., from existing literature or common discourse), in conversation or as a focus of activity.	* Returning to existing framework to add to an emerging set of distinctions from the group. * Publicly collating the categories from the group and then comparing to an existing framework. * Working on making categories visible/ observable	- Enhancing existing distinctions. - Seeing more and differently in relation to practices and being able to comment on them. - Providing a common language within a group.	Whose framework? An existing framework could be seen as an external object, so, it is the process of making sense of the framework/what the individual sees in that framework that is significant in terms of learning.

Label	Distinction drawn	Description	Potential strategies	Functions	Notes
			in teachers' classrooms by getting to detail of examples. ** Introducing an existing framework to add to an emerging set of distinctions from the group.		
Change in practice	No change in practice/Status quo.	Teacher explicitly acknowledging a change in their practice or the practice of another teacher. Over time, a teacher's description of a particular practice changes (e.g., from one feedback session to the next). A change is observed in teachers' practice/my practice (as the teacher educator) in relation to previous practices.	* Commenting when a change in practice has been acknowledged. * Probing for more detail about what might have provoked the change. ** Asking teachers to keep a professional/ research diary.	- Raising an explicit awareness of learning (as change in behaviours), increasing the likelihood of noticing future changes.	This category applies to changes in teachers' practices as well as my own.
New insight/ awareness	No new insight/ awareness.	Teacher explicitly describes having learned from an experience. Following a description, a new summing up occurs in the form of a new insight. Explicitly acknowledging a new awareness.	* Commenting on when a (potentially) new insight is made. * Extending the new insight by prompting for examples.	- Raising awareness of possibility for new insight. -Clarifying the new insight for others. -Creating conditions for further new insights to arise.	Insights/awarenesses could present themselves in many different forms… sometimes following detailed descriptions.

Appendix 3e: Extracts of dialogue gathered by label

Episode One: *Using dissonance*

Dissonance		**Hypothetical utterances**
Session 1 (00:03:10)		
Sam	I have a top set year ten and we had just done sine rule, cosine rule, and area of a triangle without knowing the perpendicular height. I went for the structure, here are some calculations that a student might have performed based on this stimulus [Sam holds up a diagram of a labelled triangle and a separate list of associated calculations, such as $15^2 + 24^2 - 2(15)(24)\cos 32^o$ and, $\sin^{-1}\left[\frac{24 \sin 32^o}{15^2+24^2-2(15)(24)\cos 32^o}\right]$]. I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? I picked that structure because of all of the structures that Paul showed me, that was the one that made my head absolutely bend round. I thought, well if it makes my head bend, let's see how good my year tens really are. It completely split my class massively. The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure. I don't think I had made a good enough job of making sure they understood the structure. On reflection, I think I should have done something much simpler first. I should have gone for something much simpler so they understood the idea before throwing in some quite horrific maths on them […] I actually started with the same triangle and asked them to come up with a minimum of three possible examination questions that would need the sine rule, or whatever, to solve. That was fine. That was good, but there was a massive jump between that and the structure using the eight calculations. In terms of the whole department we have a lot of non-specialists. I mean, I'm a non-specialist, I'm a Physicist, I only did a subject knowledge enhancement course two years ago but my maths is ok. I think a lot of my department would really struggle with opening themselves up mathematically to that so I think we probably have to teach the structure with something slightly simpler and then encourage them to stay a little bit out of their comfort zone but not to the extent where they are completely freaked out. It took me ages to plan it. The first time I did it, I did it actually with Paul in the room, do you remember? I'd done it wrong, because I then thought I'd come back to it the next day and I thought I'd try it for myself and there were two of the calculations where the actual the basic maths didn't make any sense, so it was interesting. It was quite hairy at times. The first task took about five minutes. The calculations task took about ten so it was quite a meaty starter that could have gone into a whole lesson if I'd felt like it I think. I think I should have allowed more time. I think I should have actually unpicked each one of the calculations, but as it was the class were forming that kind of horrible situation where some got it, and others really didn't get it and were really cheesed off that they weren't getting it. So I had to put some emergency repairs in place.	"What do you mean by split?" "Before you move on, shall we just pause and think more carefully about the issue you have raised?" "Start again, how do you know? What is your evidence?" "What do you mean by emergency repairs, what were you repairing?"

Episode Two: *Staying with the detail*

Detailed description		**Hypothetical utterances**
Session 1 (00:01:45)		
Tracy	to get into some of the detail of what happened in the lessons as well the process of working with one another and with your wider departments.	
Session 1 (00:08:19)		
Simon	So, with the structure how did you set it up, in terms of discussion?	
Sam	I asked them to pick one that they could understand and one that they thought was wrong from the list of eight	
Simon	Cool	
Tracy	So there's some wrong ones, some ones that don't….	
Sam	Yeah, it would not give them anything useful, I wrote the answers over here, when I realised I had cocked it up I thought I better write this down so yeah there are numbers from here but they wouldn't actually allow them to access anything, and my IT skills are not up to putting that on a computer so that was a photocopy with my little um visualiser on it.	

Detailed description		**Hypothetical utterances**
Session 1 (00:09:07)		
Simon	We did arcs and sector areas, so I just put a circle up, can you see that there [Simon points to an image including the stimulus and calculations shown]. I gave them these different calculations and said, "right, what do each of these formulas represent on the diagram?" Some of the calculations didn't actually represent anything in relation to the diagram and in those cases, they had to either sketch or at least try and explain what that formula could be showing. So, it was a similar structure to yours [Sam]. Mia and I approached it quite similarly, didn't we?	"What can be said about this, why might this ("pi times 3.6 squared well would that be the area of a smaller circle so I said well would it be half the area of one that's got a radius of 7.2") be a motivating comment for the students?" "Hold on, pause there, can we just work on that for a moment 'pi times 3.6 squared - would that be the area of a smaller circle so I said well would it be half the area of one that's got a radius of 7.2'…"
Mia	Pretty much the same.	
Simon	Yeah, just pairs, talk about it, write something down, discuss with the people around you. That was the most powerful thing, the discussion that came out of it was really, really good. There were a few nice questions like, "if it is three hundred and sixty over two hundred and fifty-two, could that be used to do anything useful?" "What's the point in that?" We just tried to keep the numbers very, very similar. Some of the students were saying, "well maybe it's just a smaller circle". Someone raised the point, "oh there's pi times three point six squared, would that be the area of a smaller circle?" So, I said "well, would it be half the area of one that's got a radius of seven point two?" It just stemmed off from there. But I think it was good putting some in there that were not relevant to the diagram that was quite powerful too.	
Session 1 (00:15:00)		
Tracy	So what was the question that went with that?	"This is a really lovely example of getting into the detail of your experience of the gap task" "Can you say more about the choice of numbers? Is this something you have noticed before or are now noticing since?"
Alex	It was how would you find each of these sections, what would be the easiest starting point?	
Tracy	Nice	
Alex	So then a kid said well I know that 36 over 360 is equivalent of a tenth and then someone else interjected and said well then that means a tenth this will be fifteen ok perfect we'll write this on the board, which will be the next easiest to find and then someone else said well it's got to be seventy two because that's double the amount, and then someone else said well if you've got seventy two we know what one four, four is, and then take it away, then someone said well you can take it away from the rest of it to find what the hundred and eight was and then someone said why would you do that why don't you do thirty six plus seventy two makes one hundred and eight and then someone else said well you could also do one hundred and forty four minus thirty six so again the numbers really.	
Sam	**So the choice of numbers are really important.**	
[Short period of time] **Session 1 (00:16:37)**		
Alex	And then he ran with it so he again saw the 36 over 360 was a tenth	
Tracy	Nice	
Alex	And then was like double, double…	
Tracy	Yeah	
Alex	…take it away, and then again the other one next to him sort of said there's a different way of doing it and again they explained that, so again I felt that this, the numbers **and the resources helped the conversations**	
[Significant period of time] **Session 1 (00:22:09)**		
Mia	That was very thorough	"It is likely to be more powerful to you in terms of what you can learn if you are able to stay with a particular lesson"
Simon	That was pretty thorough	
+	[Laughter]	
Alex	Well I thought we'd rather do one lesson well rather than trying to get in into every lesson.	
Session 1 (00:24:43)		
Joe	I've found with algebra, always, sometimes, never was a wonderful structure, so you give them a statement along the lines of **2a is bigger than a' is this always, sometimes or never true? 'a over b is smaller than a' is this always, sometimes, or never true?** And we got them to prove some of these statements. We also, as part of the do now part of the lesson, all that was around negative numbers to kind of	

Detailed description		**Hypothetical utterances**
	point them in the direction of try different things, try negatives, umm, and what we found was a certain task in particular was **'a over b is smaller than a'** was a really interesting one because I did it with two separate classes and one class was fairly unstructured and I let them plug in what they could I said right you may use integers, negatives, decimals any permutation of it, and we got things like, um, children were dividing by zero and they were getting math error and they were getting to that and we had a whole group who were just at the end were obsessed by the idea of zeros going on forever and infinity. Really engaged and was really interesting. And, um, what happens when you divide a positive by a negative if say a was negative and b was positive what would happen, you know, or double negatives in the situation or even dividing by a decimal […] **if I want them to make a statement, I need, they need to know how to say it, so how do they start it, a over b is bigger than a when and you give them , and they write down the conditions that allow that**, and I felt that was really, really enabled deep learning, on that. […] So it's finding, giving them some constraints and saying what's the biggest solution what's the smallest solution again was incredibly powerful, so I did that, I did one task on just expressions and then I did it, a similar thing on formulae and what I did the second task was get them to think about the strategies, how you, how can you do it in one, how can you make sure you get the right answer first time, which a lot of the more able children really liked as a challenge, but it was still accessible to the less able because they can still pick values play around they can still get to the right solution and unpick it, it's just it enabled that depth of access and understanding. Another one I loved was, with algebra, same different, given two statements, expressions, formulas, ok what's the same, what's different, so 2a, a what's the same, what's different, well they both have a but one is multiplied by two. Um, formulas was different brilliant, was absolutely superb, you gave them I think it was 8t + 5, c = 8t + 5 what's different? Expressions versus formulae, and then we gave them a worded formula and a kind of algebraic formula and got them to pick apart what was the same, what was different.	
Beth	We had the same starting point and our lessons went like this, so Joe kindly lent me two of his classes because I'm not teaching at the minute, and we started with the same structures which was the, here are some expressions, here are some numbers can you make the biggest value for each expression so you know if it was two a they had to substitute a large positive number if it was a takeaway b they had to substitute a large negative number to make it bigger and then we just, the classes kind of went in very different directions. From then on, so**, I was amazed** by how much time we could spend on one tiny concept so we gave them, for my second lesson I gave them this and they had to match up what was the same and the idea was to go onto this which was just looking at the fact that a minus b is the same and minus b plus a what actually happened is they thought that, I'm gonna go back, they thought that these two were the same, **so we spent about twenty minutes exploring why minus b plus a is not the same as minus a plus b, go I get them to substitute, and so they are practising substituting of negatives and that surprised me because that was not what I was planning to do, I had like a whole lesson planned.**	
Session 1 (00:31:19)		
Beth	But they really, they were starting to cotton on to what kind of numbers would give special cases and that was a really nice thing, looking at how their trial and error was working, and the other one was, um, so after this lesson where we were looking at these two and also looking at whether that was the same as minus a plus b getting them to write sentences about why this is always true and why the other one was only sometimes true, we had to say sometimes because they would try zero and zero obviously worked um and it was really nice that it was quite a mixed ability class and I've actually never taught mixed ability before and there were some kids in there who were really struggling with arithmetic and their explanation wasn't quite as mathematically sound but they had this sense that the b was the minus and so although their explanation wasn't quite as nice and articulate and mathematical there was still this sense that the b has the negative and the negative stays with the b so I felt like it was a slightly more superficial understanding but some quite important algebra was going in so that was really nice, but it was just so fascinating how these two classes went in totally different directions.	
Session 1 (00:56:57)		

Detailed description		Hypothetical utterances
Lucy	I did adding fractions with year sevens and what I did was because I liked the proportion questions with different workings out but in a random order So the kids had to choose the correct order and then figure out what the question was. So I did a similar thing with a couple of red herrings in mine. So a few boxes had equivalent fractions in for 1/2 and 2/3 and then the first five times tables for twos and threes and then a wrong answer of 5/6 and 3/5 and then a correct answer and then got the kids to try and put it in the correct order and figure out what the question was.	
Session 2 (00:21:38)		
Simon	Kind of like masking tape, so I gave them each a roll of masking tape and they just put loads of lines, kind of like… that [shows sketch to group], just all over their tables, then I gave them a black board pen and they had to measure the angles and then they had to go round in groups, they each got given a mini white board, and they had to write down anything that looked good on someone else's table, anything that was not so good, and anything else they generally spotted that they thought was interesting, because we hadn't talked about angles in a quadrilateral, and is there more than one way of coming up with the answers that they got, so, like, there was one point where I think someone said oh yeah well they're opposite angles and someone else said well, because I talked about like if you cover your hand over then that's like a straight line.	
Session 2 (00:44:46)		
Sam	we ended up with a smaller version of what might happen, what happens next, where are you going to go, ties in very much to our impact marking where you, at the end of the lesson, basically put a little plenary up with maybe three questions, if the kids get it all right then they get a green line, so that's I am three and they will get some challenge next lesson, if they get two right they get an orange line I am two probably more practice, I mean it's a bit simplistic but, and if they get it, if they really need some help they get a yellow line and that's I am one impact marking one and they become your teacher focus group for the start of the next lesson.	
Session 2 (00:47:24)		
Sam	so if I have eighty skittles and divide it by two is that the same as having eighty skittles, sorry, divide it by four, no two, of god whichever way round it was, I'll get it right in a minute, no sorry, was it eighty divided by four is the same as eighty divided by two plus eighty divided by two, that was it, and if I've got eighty skittles and I divide them by four is that the same as eighty skittles divided by two and then eighty skittles divided by two and then she said I think I'm gonna let you talk about it.	

Slide (from 'account-of' to 'account-for')		Hypothetical utterances
Session 1 (00:03:53)		
Sam	I asked, which of these calculations were possible? What were they trying to find out? Which ones were impossible? Which were pointless? **I picked that structure because of all of the structures that Paul showed me, that was the one that made my head absolutely bend round. I thought, well if it makes my head bend, let's see how good my year tens really are. It completely split my class massively.** The ones who I'm confident will do early entry just went for it and loved it and made up their own and were really having massive arguments about it. Some of the kids just completely failed to understand the structure.	"is challenge something you always value in your teaching?" "What do you mean by split?" "How do you know, what was happening?"
Session 1 (00:05:14)		
Sam	I actually started with the same triangle and asked them to come up with a minimum of three possible examination questions that would need the sine rule, or whatever, to solve. That was fine. That was good, **but there was a massive jump** between that and the structure using the eight calculations	
Session 1 (00:06:04)		
Sam	It took me ages to plan it. The first time I did it, I did it actually with Paul in the room, do you remember? I'd done it wrong, because I then thought I'd come back to it the next day and I thought I'd try it for myself and there were two of the calculations where the actual the basic maths didn't make any sense, so it was interesting**. It was quite hairy at times.**	"Hairy for who?"
Session 1 (00:13:02)		
Alex	They saw this one here [Alex points to the fourth pie chart] where it's got the largest sector and then looked for the largest number from the tables. When we were going	

Slide (from 'account-of' to 'account-for')		**Hypothetical utterances**
	around asking them why they were choosing, the discussions were really rich. It helped that the resource was well designed. It only took me a little while to find it, it was a Median resource. So we used this [pointing to the resource again] and I thought it was really rich. They had that and they had to match each of the letters with a number. I think we sort of explained the next task briefly [showing a different resource] and then they went away and really went for it. Because they hadn't done pie charts, […] we went through the workings and they had the discussion; "do we need to find the last one or do we just take it away from the full amount?" That was a really rich discussion.	
Session 2 (00:22:44)		
Simon	In terms of the reasoning I just thought it was good because I just put a few like kind of prompty questions up on the board just things I wanted them to talk about and then we were talking, we were going 'round each table, talking for like five minutes on each table, yeah, is there more than one way of doing that was kind of the biggest one I think, because there were lots of different ways they could have done it. They were all coming up with all these different ways and why that one was this, then, they'd be like, well, the corner of the table is a right angle so then now we can do this, and quite good, um, but it required no thought, that was the best thing about it [laughing] really.	
Session 2 (00:25:12)		
Lucy	for example, I said what's a quarter of 84 and somebody else said, I know that half of 84 is 42 so a quarter is, so they were breaking it down for somebody else. I think the questions, I wouldn't say that I necessarily planned the questions they kind of came about organically based on the responses they were giving me and based on the mistakes that were coming out of their questions. I was drawing out the misconceptions on the fractions and then making sure that they were answering the questions and when other people were giving them hints I would say, for example, when somebody said a quarter of 84 is 21, I said how do you know and then I encouraged them to use pictures, so draw a circle and then show everything as 84, split it in halves and then split it again, so using different methods	
Session 2 (00:29:44)		
Joe	I taught that normally last term. It didn't go down very well.	
Tracy	What do you mean by normally?	
+	[laughter]	
Joe	"Here's how to do the area of a triangle". "Here's how to do the area of a rectangle, now do some yourself". Pretty much, give them a procedure, give them a formula, you know the eighty percent of the time lesson when you're stuffed for time at the end of term.	"So how would you go about teaching this topic next time?" "Is there another topic you could consider in the same way?"
Session 2 (00:47:56)		
Sam	she spent ages bringing out all these wrong answers, so much so that I was going stop, stop in my head I was going stop, they are convincing themselves that eighty divided by four is the same as eighty divided by two plus eighty divided by two but she just went with it for such a long time, um, and it was a completely different response when we watched that bit and then we watched the next bit of the video which was when she found somebody who had given the opposing answer and she just put, one person, didn't take any other people who'd agreed, and just asked that one person to explain in a lot of detail so she'd taken lots and lots of wrong answers, um, I'd loved to have seen the bit where she'd pulled it around because I was so, there was a poor little boy at the back, and he was so convinced he was right, and my emotional side was, you can tell that boy, and you can tell that boy, and at the end of it he was going yeah alright, but you just knew that in his heart he was gonna go home thinking this is the real world and this is maths	
Session 2 (00:59:37)		
Simon	I did forty percent of fifty pounds equals fifty percent of forty pounds. I put that statement up, and then I walked around. I had a quote board, so anyone that said anything interesting, I just wrote their name on the board and I used that as a discussion point. Out of that, we had a few things that came out, like, "oh it works for that, but what about if it's thirty percent of sixty pounds, or something else?" So, they changed the numbers. They asked, "what about if it's odd?" "Does it matter of it's odd?", "Does it matter if it's even?", "What if I double one value and halve the	"That feels like the type of question you could use in a different topic, would it be worth thinking about that briefly?"

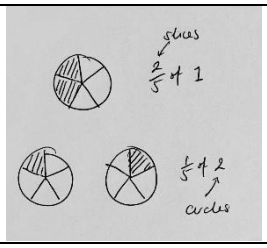
Slide (from 'account-of' to 'account-for')		**Hypothetical utterances**
	other, does it still work?" Then it just led off to lots of other really nice questions. But I hadn't deliberately structured a lesson like that before, and just that open, and we kind of lost our way a little bit. but with a bit more…	
Session 2 (01:03:30)		
Beth	I've been involved in running various bits of professional development recently and one of the topics I've been working with teachers on is fractions. It wasn't meant to be on fractions actually, it was meant to be conceptual understanding and we were going to do lots of topics and then fractions somewhat took over. I had a really interesting response to this [Beth has drawn two images on the whiteboard]. On reflection I wish I had then asked the teachers to have a go at a script, what would happen if a child had come up with this misconception, I really regret not doing it.	"Could you give us a little time to consider this?"

Doing mathematics		**Hypothetical utterances**
Session 1 (00:10:05)		
Simon	"oh there's pi times three point six squared, would that be the area of a smaller circle?" So, I said "well, would it be half the area of one that's got a radius of seven point two?"	
Session 1 (00:12:36)		
Alex	We did a matching exercise. So, there was one pie chart where all the slices had the same area. [Turning to Ellen] what was the second one we all thought they were gonna go for, but they didn't?	"Can we have a few minutes just to have a go for ourselves?"
Ellen	The one when they were all different	
Alex	They were all different. This one [Alex points to the first pie chart on the resource shown that he is displaying on his laptop screen]. Then there is one where the frequency is one, two, three, four, five and they didn't look at that at all. They saw this one here [Alex points to the fourth pie chart] where it's got the largest sector and then looked for the largest number from the tables. When we were going around asking them why they were choosing, the discussions were really rich. It helped that the resource was well designed. It only took me a little while to find it, it was a Median resource. So we used this [pointing to the resource again] and I thought it was really rich. They had that and they had to match each of the letters with a number. I think we sort of explained the next task briefly [showing a different resource] and then they went away and really went for it.	
Session 2 (00:24:31)		
Lucy	divide 126 in the ratio of two to one, if Melissa gives a quarter of her share, how much would she have left?	"Hold on, say that again slowly"
Session 2 (00:39:45)		
Joe	I did an NRICH task, it's the one where they give you two rectangles. One of the side lengths is ten and you have to figure out what the missing length is for the perimeter and area to be equal. […] I was anticipating, they were going to take forever, they would say, "oh what do I do?" But with a little bit of guidance; "how about you try a different side?" "Does it have to be a whole number?" "OK you're very close what should you do now?" Just little question prompts like that.	"Do you want to run this with us, I'm not sure everyone is familiar with the task"
Session 2 (00:40:53)		
Tracy	There's some sort of lower bound is there, two or something	
Joe	Yeah, as long as you have a length above zero, it will fit into a formula, because if you have a b there is a formula that will give you a, so…	
Session 2 (45:53)		
Sam	is eighty divided by two the same as eighty divided by four plus eighty divided by four	

It/that/this		**Hypothetical utterances**
Session 1 (00:34:18)		
Joe	I think there's a really good point you made there about the getting used to it I think the more I did it the better the outcomes were it was quite hard at the start to kind of get my head around it but the more I did it the more they got used to thinking. The easier it became, the better learning we got.	"What is the 'it' you keep referring to?"
Session 2 (00:31:18)		

It/that/this		**Hypothetical utterances**
Miguel	I've started doing this more and more recently	
Session 2 (00:46:50)		
Sam	and it was just such a brilliant way to get the kids to talk	
Session 2 (00:49:33)		
Alex	I was wondering, if you've tried it with the students, if you have, how does it work with SEN students? because I've got..	"What is the 'it' you keep referring to?"
Sam	Sorry, how does?	
Alex	How does it work with SEN students?	
Sam	How does what work?	
Alex	Well, the, those discussions…	
Session 2 (00:50:34)		
Alex	how are we going to differentiate in **this** scenario? For the SEN students who struggle picking up concepts and processes and would **it** be too much for them?	
Session 2 (00:51:18)		
Mia	**That** does definitely happen with high ability kids as well, I was just thinking of a time a couple of weeks ago when I was doing conversions and we were doing area and volume conversions, but part of the starter was just simple conversions and a kid from a top set was convinced that to get from mm to cm, you times by ten and even putting examples up he still was convinced no it was times by ten so even though he knows there are ten mm in one cm he still was convinced you times by ten so I don't really understand how to…	"What is the 'that' here?"

Doing actively		**Hypothetical utterances**
Session 2 (00:30:06)		
Joe	I went back to areas of rectangles and triangles. This time I used a series of diagrams to show the link between the area of a rectangle and the area of a triangle. I went through rectangles first, just put a side value in of say, ten, then I kept changing the bottom length to show that it was multiplying by ten to give the area. Then I changed it around and gave them some to do, did it quickly. Then I did one where I split the rectangle in half. I showed them a few examples, got the idea of halving, changed the way I halved the rectangle, then changed the halving to make a triangle. From there they could make the link that a triangle is always half of a rectangle. I gave them some more examples, so not always halving corner to corner and changing the slant height but keeping the perpendicular height the same, so the area didn't change. I genuinely did not say anything to them apart from at some points to say, "OK, what have I just done?"	"Why don't you show us?"
Session 2 (00:37:42)		
Mia	Can I pick up on another point you said earlier about, because there is something I saw from a teacher when I was training that's really, it's mostly with algebra that I've really used it but he puts like a table up, it's like a, you know a really simple drawing, and he writes things here and the kids have to guess what he's gonna write here	"Would you like me to run this for you all?"
Paul	The teacher game	
Mia	Is that what it is, is it?	
Paul	The teacher game	
Tracy	The function game?	
Paul	The function game	
Simon	You don't say anything	
Mia	But you don't say anything, and if they get it wrong then I just put an unhappy face on the board and if they get it right then happy face, but the kids really enjoy it, um, so it works quite well.	
Session 2 (00:53:17)		
Joe	I did do something very similar, with the same/different so I had three questions, a share in a given ratio question, second question gets more than numbers the same, third question person on the left gets this much and actually on that one the reasoning structure really helped break down that barrier	
Session 2 (01:08:16)		
Tracy	Try and stick with this for a minute.	

Doing actively		**Hypothetical utterances**
+	[laughter]	
Beth	So what does the numerator represent in each case? Does it represent the same thing each time?	
Miguel	Two bits of pizza	
Beth	Both times?	
Miguel	Yep	
Sam	Yep, I would say so, to me the bit that goes on top is the bit we are interested in	
Beth	OK, so what about the denominator? If this is always two bits of pizza, what does the denominator represent?	
Sam	Arrr, and therein lies the conflict	
Miguel	How big the top bits are, they tell you, it's like you've got two bits and the five bits is how big it is out of one, I suppose.	
Beth	OK, so in this case, you think the two are these two? [pointing to the two shaded parts, 1 from each circle – see image below]	
Tracy	I think the two is the two circles. 	
Beth	So here is it, I've got two pizzas, and I split it into one, two, three, four, five parts, so its two things divided into five parts, as opposed to, two out of five parts.	
Session 2 (01:12:08)		
Paul	If you were to write down on your piece of paper twelve divided by four, and then show that in a representation, what would that look like? So twelve divided by four and let's start with maybe a rectangle as a representation, how would you do twelve divided by four?	

Slide (from mathematics to mathematics teaching)		**Hypothetical utterances**
Session 2 (01:03:57)		
Beth	I just asked the teachers to look at these two representations [pointing to a whiteboard]. So, it's a pizza with five parts, two shaded, and then two pizzas cut into fifths essentially, five parts. I asked them, "what is the same and what is different?" and then I asked them "what does the numerator represent in each case, and what does the denominator represent?" I thought that everyone would be quite clear that here [pointing to case 2] it's two pizzas cut into five parts and here [pointing to case 1] it's one pizza cut into five parts with two shaded. It was amazing how controversial it was. One idea was it [case 2] must be two tenths because there are ten parts there [pointing to case 2], rather than it being two fifths of one pizza and what the whole is. It was really, really interesting. I think for me it was interesting because it showed how fixed our ideas of fractions are and how we might unintentionally limit what children think.	
Tracy	Well do you just want to give us a minute to think about this, there might be other people who …	
Beth	Yeah, so the first question was [time spent setting up the board and projector] so the first question was, what's the same, what's different? [more time sorting the projector] What I wished I'd asked was what do you think the kids would say? And then let people think about how they would address that. So, any responses?	
Tracy	Well in both cases you've got two sections highlighted.	
Mia	It looks a bit like a fifth plus a fifth is two tenths, which is a big misconception isn't it.	
Beth	Because there is ten parts in total so your whole is two, yep, any other responses?	
Paul	I was thinking about what you would define the vinculum, the fraction line, as	
Beth	Yeah, so that was the next question, what does the numerator represent in each case and what does the denominator represent, I think this is where we came quite unstuck, which was good, it wasn't a bad thing, it was interesting, so…	

Slide (from mathematics to mathematics teaching)		**Hypothetical utterances**
Tracy	Well do you want to give us a minute to just think about the answers to those questions, what does the numerator represent in each case and what about the denominator?	
Session 2 (01:09:20)		
Miguel	Well, I would say that it's a fractional choice. We've got a whole, we've got one whole, one pizza, and you divide by 2, then you shade it and you've got a half, but when you have two fifths then it gets a bit trickier doesn't it. I think, you've got two, as in the two whole ones, but then when you say split it into five, you need to assume the kids know you mean split each one into five.	
Beth	I could have coloured that one in instead and it may have generated some different responses. Showing they were one part but colouring in the different ones to the ones we were highlighting.	
Paul	You could have two and then split those into groups of five, so you could have had the pizzas then giving you ten quadrants. Or you could share those into two groups of five which would also be a representation of the same thing.	
Beth	Yeah. It made me really reflect on how fixated I have been on this representation when I've been teaching. I don't know if you have ever done the chocolate bar task, when you place one on a table, two on another and three on another and then you ask them to choose where to sit.	
Simon	What they stand outside and then walk in?	
+	Yeah	
Beth	When I have done the chocolate bar task, the students have really struggled with was the idea that three things divided by five people is the same as three fifths, and that actually, the numerator there is the three things you're sharing not the three bits you're getting. If that makes sense.	

Episode Three: *Finding conviction*

Should/ought/had to		**Hypothetical utterances**
Session 1 (00:04:33)		
Sam	I don't think I had made a good enough job of making sure they understood the structure. **On reflection, I think I should have done something much simpler first**. I should have gone for something much simpler so they understood the idea before throwing in some quite horrific maths on them.	"What do you mean by simpler?" "What happened that meant you felt that way?"
Session 1 (00:05:52)		
Sam (R)	So **I think we probably have to teach the structure with something slightly simpler**	"What approach did other people take to using this structure? Did you use it with simpler concepts?"
Session 1 (00:06:33)		
Sam	**I think I should have allowed more time. I think I should have actually unpicked each one of the calculations,** but as it was the class were forming that kind of horrible situation where some got it, and others really didn't get it and were really cheesed off that they weren't getting it. **So I had to put some emergency repairs in place.**	"What kind of emergency repairs?" "What do other people do when their students get stuck?"
Session 1 (00:07:56)		
Sam	I think yeah I think that but **I should have done something to allow them to access the structure because they had to access the structure and access the maths.**	
Session 1 (00:58:30)		
Lucy	**So then you have to ask them what the next steps would be** or to make up a similar question. Or what a harder question would look like, or a wordier a question. […] **You have to ask them leading questions** to get them to respond and to hopefully say the right thing.	

Speaking for others		**Hypothetical utterances**
Session 1 (00:05:34)		
Sam	we have a lot of non-specialists. I mean, I'm a non-specialist, I'm a Physicist, I only did a subject knowledge enhancement course two years ago but my maths is ok**. I think a lot of my department would really struggle with opening themselves up mathematically to that**	"Start that again, how do you know? What's your evidence?" "Try to speak for yourself." "Try not to speak for others, especially entire groups."
[Significant period of time] **Session 1 (00:52:12)**		
Joe	I think maybe **that could be hugely intimidating for non-specialists, that open-endedness, not having the right answer, for somebody who is much less experienced.**	
Session 1 (00:11:03)		
Mia	They really liked the structure.	
Session 1 (00:14:00)		
Alex	So we gave them the workings to put in the correct order to find the number of passengers so we blanked out the passengers, one of the calculations was wrong, the last one, so the actual answer was wrong, um which we changed for the year seven mixed ability.	
Ellen	**Because they wouldn't like that**	"How do you know?"
Alex	Because they wouldn't have liked that it was wrong, um, again they had really good discussions between each other about why they chose them in that order, what the calculations actually meant, I don't think I was as explicit in my explanation to have that discussion so we sort of adapted that so my colleague was more explicit that they needed to think about the, um, what the workings actually meant so I think we sort of learnt from that as a pair, um and then this one was amazing, so um, they worked out the 36 over 360 so again the numbers helped was the easiest one to work out because they could see it was equivalent of a tenth.	
Session 2 (00:10:11)		
Alex	I'm sure people think about questioning but sometimes people don't think that much in advance as part of their planning process…	
Session 1 (00:23:42)		
Simon	If that wasn't there [pointing], would they have chosen that one first, I don't know. Would they possibly have then chosen the largest sector if that wasn't the first one. **Because people like doing things in order don't they.**	"Is that the case for everyone here?"
+	Yeah.	
Session 2 (00:12:50)		
Sam	I think they object to it particularly because it's maths.	
Session 2 (01:12:50)		
Paul	So twelve divided by four, what we are doing is splitting our twelve into four equal parts, each with 3 in, **and secondary teachers in general see this as division**. Rather than grouping, where you have twelve and you split it into groups of four, and there are actually three of them. **It's a real difference between what most secondary school teachers think of in terms of sharing compared to grouping.**	"How do you know?"

Inner/outer		**Hypothetical utterances**
Session 1 (00:17:26)		
Ellen	Even if they could only pick out little bits, they all thought there was something they could access… **but the ones who are being bored to death with the mixed ability** were really happy to have something that they could fly with.	"Maybe we could spend some time as a group thinking about strategies for teaching groups with diverse missed prior attainment"
[Significant period of time] **Session 1 (00:22:45)**		
Tracy	It's interesting though that you got it to work in the two completely different groups	
+	mmmm	

Inner/outer		**Hypothetical utterances**
Ellen	Well the other thing was they both did the breakdown of this task, both groups broke it down exactly the same way they both started with the same, the biggest, then the pairs and left the one that was all different to last, we thought that would be the second one.	
Alex	One student went for the largest first he noticed it was the largest sector and he looked down and he saw there was a seven there.	
Tracy	Right	
Alex	So that was his way, but yeah pretty much most people even because it was the first one that was the first one there perhaps if it was there [pointing] they probably wouldn't have noticed it, they all saw that they were equal, had a discussion about well what do fifths actually mean, well five equal parts.	
[Significant period of time] **Session 1 (00:57:47)**		
Lucy	But some barriers I came across, **because they are mixed ability year sevens** […] **Because it's mixed ability** […] **because it was mixed ability**	"What strategies are there in the group for working with mixed groups?"
[Significant period of time] **Session 1 (01:03:37)**		
Tracy	Because it is interesting Ellen that you taught a mixed year seven group and you didn't have the same barriers that you're [looking at Lucy] describing actually it didn't feel like there was something like adding fractions that stopped them from accessing that task.	
Session 2 (00:11:02)		
Alex	Yeah, so, as a whole school there's been a shift with an onus on, with a focus on literacy and numeracy, so there has been a little bit, um, with regards to oracy in class, so, I think we've sort of linked in that so not accepting twenty…	"That is a lovely example of combining school initiative with your own work on this project"
Ellen	Seventy-two	
Alex	… seventy-two, um, so why is it seventy-two, can you expand on, so trying to get the students to discuss in full sentences, so I think that has actually helped this project, because there's a, there's a wider emphasis on expanding on answers	
Session 2 (00:13:31)		
Ellen	do you find **with the lower ability** that if you say are you sure it, they think that they're wrong then	
Session 2 (00:16:08)		
Sam	our head actually said so after, because it was about different ways of marking, live marking and impact marking which is what we do which I'll talk about later when we do questioning, and he actually said I'm now at the stage where if I don't see marking in books I'm not fussed about it and that is such, you know that's a year on, well even two months ago I would never have thought he would have said that because it was very much, I don't know, back to the walk, what can we show, what can we show, what can we show, but he followed that up with but what I will do is sit down with children and say so, where are you, what do you need to do next, how's your teacher got you to this stage, how is your learning different from the learning of other people in the classroom. So, I think that's, that's really positive that he's not looking just at what is very easy to measure, or that there's no marking in this book, or there's not enough work in this book, to something that's actually more difficult to measure… so holidays for us then	
Session 2 (00:23:47)		
Lucy	because I'm still of that thinking that you need to show, because we are, we have been told that you need to show lots of work in their books	"What is the purpose for you, in showing work in books?"
Session 2 (00:34:58)		
Maria	we haven't got, necessarily, the content in our books but if anybody walks in they can see it is there, this is what we've done.	
Session 2 (00:41:57)		
Joe	It doesn't go onto a form nicely.	

Teaching issue	**Hypothetical utterances**
Session 1 (00:05:52)	

Teaching issue		**Hypothetical utterances**
Sam	and then encourage them to stay a little bit out of their comfort zone but not to the extent where they are completely freaked out.	"What does it look like when a student is in / not in their comfort zone"? "How do others encourage your students to stay out of their comfort zone."
Session 2 (00:40:20)		
Simon	**You know you said about the not revisiting it for a year thing, that's a really big thing for us too**, we've changed our scheme of work completely for year seven straight in with algebra and our last department meeting, we always seem to have like 15 minutes at the start where we are trying to develop an idea or resource or question that is linked to mastery or reasoning around linking back to what we have done before, so for example like we were talking about ratio weren't we and we were trying to link algebra into um ratio um and we are trying to build that throughout so that they don't have to not revisit for a year they're constantly doing fractions, Pythagoras use fractions, whatever, anything and its like that kind of um, what's the word, like cyclical, like just like continuing spiral of like coming back to it.	
Sam	That is something, because we've got so many non-specialists, how do I make this more difficult? Fractions, negative numbers, surds, you know just throw those things in.	
Simon	But then again though, this is what we were saying, we did it for year eight didn't we, and we split off into groups. I think we found trying to plan **a question that links other stuff into that topic for low ability was something we found that particularly difficult**. I don't know whether that was just because we weren't really thinking, or it was just the nature of ratio or whatever, but we've just found that that was harder. **I don't know what other people have found but we're just trying to avoid not revisiting for a year basically and just trying to build in lots of different things. Maths is a connected thing rather than we're gonna do ratio then we're gonna do algebra, then we're gonna do something else. They're all the same, you're teaching everything all the time, that's what we're trying to build in.**	
Session 1 (00:58:07)		
Lucy	My initial thought was **when do I jump in and how long do I leave them to suffer** […] **What I find challenging is knowing when to step in**. Because it's mixed ability. For the weakest kids I probably leave them for a few minutes, and I usually put a timer on the board, so I know how long they've had thinking about it. **I find it really difficult not helping them**. I tried to break it down and give hints to those that were stuck, to try to get them to recollect how they would do it. We were talking earlier about different methods of doing one question, **I want the students to give me more than one method, but typically at the end of the lesson, and only when I know everybody has understood**. Otherwise, I feel that it makes them a lot more confused than they would have been. If you have taught them one way of doing something, and they are still grappling with that method, to have somebody else come and show another way; "I haven't got my head around this way yet, and you are showing me something else". I feel more confident when a student comes to the board to offer a different method once most of them have understood the first method. **They find it hard to explain what they are doing, to put it into words, some of them are not fluent enough to explain**.	"Is them some particular issues in what you have described Lucy, that you would value thinking about a little more with the group?"
Session 2 (00:12:33)		
Joe	I would love to explore how we can get students to express their reasoning in different ways, so maybe pictorially, things like that. It's a real hurdle at the moment, I need to think about training them.	"Does anybody have any stories about …?"
Session 2 (00:13:31)		
Ellen	do you find with the lower ability that if you say are you sure it, they think that they're wrong then	
Session 2 (00:16:58)		
Tracy	one of Paul's prompts was to think about how you measure mathematical reasoning because it's not something you can measure easily	
Session 2 (00:17:47)		

Teaching issue		**Hypothetical utterances**
Sam	Even just, the paper we have just done, paper one question one, higher paper, was, which, it was multiple choice, which of these is the correct formula for the total number of angles in a polygon, and we've taught 180(n-2) and it wasn't it was 360-180n, **and the number of kids who couldn't actually work out that all you do is multiply out the bracket**…	
Session 2 (00:20:39)		
Maria	similar kind of experiences, we're getting **high ability kids that really, really, struggle on a one to one with the teacher, you know, how have you got this? what might happen if? and they just really, really struggle** don't they.	
Session 2 (00:26:11)		
Lucy	There is one child in my group that doesn't like answering questions, and I don't, she has really low self-esteem and I'm not quite sure. I know we propose no opt out, but I don't know how to get her to answer a question without, sort-of,…	"What exactly do you think is the issue there? We can pause and share some strategies"
Tracy	So the issue is that she won't put up her hand to answer a question?	
Lucy	If I ask her to answer a question, and she feels that she is going to get it wrong, she'll say no I'm not answering it. Even though the person next to her is quite confident and would give her a hint. She still doesn't want to.	
Session 2 (00:34:06)		
Paul	Could we learn anything from primaries with regards to reasoning, so, if we're not talking about writing in their books, a lot of primaries still make evidence, they'll keep evidence just as you would in your own mark book, whether that's photographic evidence or a note as to what happened that lesson, I don't think it's something, if your SLT are happy and on board and completely forward with the idea of different lessons look different in books and on work but if they're not then would that maybe support some of those gaps.	
Session2 (00:50:09)		
Alex	**How far do you allow the fixation to go on for?**	
Session 2 (00:59:37)		
Simon	I put up a prompt the other day, like a stimulus, and I guess what I'm basically asking is **how open do you make things for them to be able to facilitate being able to reason and what's too open?**	
Session 2 (01:00:30)		
Simon	Then it just led off to lots of other really nice questions. **But I hadn't deliberately structured a lesson like that before, and just that open, and we kind of lost our way a little bit. but with a bit more…**	

Teaching strategies		**Hypothetical utterances**
Session 1 (01:01:57)		
Lucy	If I had more time, I would have made the questions a bit wordier and possibly put a few more hints on paper for the least able kids, because it was mixed ability. To stop me from actually going over, and instead just encouraging them to read the hints.	"What other stories or strategies are around in the group?"
Session 2 (00:05:58)		
Alex	because previously that's how they've honed the conversations down. So as a result, teachers said, that, I don't think that they were previously pinging back like the old table tennis to students but, I think they were then getting other students more involved in conversations, getting them to be involved in the conversations just as a facilitator. So I think, myself included probably would accept, like yes, amazing, um, without getting them… the number one discussion point or thing people were going to take away and try was convince me so it was quite powerful, convince me, or, how do you know it's right, or, can you convince the person next to you why it's right or can you convince the person they're wrong, or, how do you know they're wrong so it's having the, them having those meaningful conversations, getting them to reason with each other and reason with you about why they are correct	"convince me… does anybody else have similar phrases they use?"
Session 2 (00:12:06)		
Ellen	You know, why is it the right answer, who else, that's the right answer, how did you get that right answer, did anybody get that answer in a different way	
Alex	yeah	
Ellen	Did anyone get a different answer, using this, you know that sort of thing	
Session 2 (00: 13:00)		

Teaching strategies		**Hypothetical utterances**
Joe	But there's also a case to think why do you have to write a sentence are there other ways you can express it?	"We could pause a few minutes here, does anybody else have experiences of expressing reasoning without using words?"
Ellen	Yeah because the sentence doesn't have to be in the English language it could be a maths sentence	
Joe	yeah	
Sam	Yeah, or a diagram… or a graphic representation	
Maria	It's really interesting that you said that because I work with a non-secondary ready class, and um, they are very much, they give me the answers, and I just, you know, I say to them are you sure, can you show me how, and you know they have just started now to do pictorially	
Joe	Oh	
Maria	You know, drawing baskets, drawing eggs, drawing, which is fantastic	
Session 2 (00:26:43)		
Ellen	Could you avoid only asking one person to answer. If they've got mini whiteboards and they all write an answer down, then she's answering it along with everyone else?	"Perhaps we just need to establish exactly what the issue is Lucy, do you think you could get to the issue?"
Lucy	Yeah, I guess so yeah, but I think in her head she's just convinced that she's going to get it wrong even though she's got the ability.	
Alex	What's wrong with getting things wrong? Just celebrate the fact that people are getting things wrong. That's an opportunity to learn.	
Lucy	Yeah, yeah, that's what I say	
Ellen	That's going back to my old, oh what's the program called? You know the one, I used to use it.	
Alex	Was it QI?	
Ellen	QI that's it! Where the wrong answers come up. I was hoping someone would say that. I used to use that a lot. I've forgotten about that.	
Lucy	Oh yeah, absolutely. Like with the fraction misconception, when someone says something like that, I say, "oh thank you, because this is the mistake that people often make", and then I celebrate it. I even make mistakes in front of them, so I try to celebrate mistakes.	
Alex	I use Joe's Nan, and they don't even know who Joe is. It's not Joe it's his Nan that gets things wrong21 [laughter].	
+	[laughter]	
Tracy	So not Alex's Nan, Joe's Nan [laughing]	
+	[laughter]	
Alex	It's not Joe it's his Nan that gets things wrong	
Lucy	But yeah	
Paul	Maybe if it's about self-confidence and it is a serious issue for that student, then if you have a particular question, or you've planned a particular question that you want to ask that student, then you give her lots of time. Say to her, at this time in the lesson, I'm going to ask you this, so have a go on your whiteboard, and we can have a look at it beforehand. If she's worried about being wrong, then that conversation is maybe between you and her, but maybe that would actually help build up her confidence of you being her teacher, and, you know, enable her to speak.	
Lucy	Yeah, yeah, yeah	
Paul	… so something like that with that particular student, you might find that the time, the time you give her gets shorter and shorter to the point where it's only a few seconds maybe, you know that and celebrating the wrongness of stuff	

Inclusion		**Hypothetical utterances**
Session 1 (00:49:34)		
Lucy	The classic question would be if they if there's a statement in an exam paper, and then the student would have to say why that statement is wrong, and they would have to use what they know about that subject, and show some examples, hopefully.	"Would you like to ask a question or make a comment Lucy?"
Session 2 (00:18:13)		
Miguel	You do wonder quite how much, um, I know because it's early days we haven't had that many examples of the papers, um, when we get enough they start repeating	"Simon, Mia, Lucy, I realise we haven't

[21] In the first feedback session, Joe told the group that he often refers to his fictitious nan in reference to making mistakes.

Inclusion		**Hypothetical utterances**
	themselves, you can always revise for a test, I mean like the eleven plus, if you haven't seen it before you are gonna be stuffed, but you can revise for it and they're gonna have, they are eventually going to start repeating those type of questions, there's only so many times you can do that, I'm not saying you don't need to teach mastery…	heard from any of you yet, would you like to offer anything at this point?"
Session 2 (01:16:18)		
Tracy	It's the problem with pizzas isn't it	"I notice this conversation has been between three of us, would anybody else like to make a comment or ask a question?"

Episode Four: *Making it real*

Asking why?		**Hypothetical utterances**
Session 1 (00:07:02)		
Beth	I was interested in what you were saying about the split of the class, **why do you think there were pupils who were really backing off from it**? What was the barrier there?	
Sam	I think about a year ago, I was in a Saturday session with one of my teach first people who had just done some reading by some guy about mathematicians, really good mathematicians, being either inchworms or grasshoppers. My grasshoppers who just fly from one concept to another had absolutely no problem with it at all. The inchworms who are more procedural were saying, I don't know what you want me to do. I understand that if I've got a triangle then I can take the sine, but where do I start with this thing that is completely different? So I think it might have been that. I don't really know.	
Session 1 (01:03:10)		
Simon	Do you think that could be about school culture, or ability? We did our lesson with quite high ability classes so maybe they were more willing to try, or maybe they didn't have a fear of being wrong, I always try to have a no fear culture.	"Perhaps just take us back to the moment again, what do you think is the issue you are grappling with?"
Ellen	Maybe because they are more used to variety	
Beth	Or how close to your normal teaching practice that is.	
+	yeah	
Alex	I think they liked it because it was different didn't they	
Ellen	I didn't think it was that different.	
Beth	Maybe that says it all, that is didn't seem different. It didn't seem different to the kids either.	
Tracy	It is interesting Ellen that you taught a mixed year seven group and you didn't have the same barriers that you have described Lucy. It didn't feel like there was something like adding fractions that got in the way of the students accessing that task.	
Ellen	Maybe it was because the task was slightly easier, or maybe it's because I'm always saying because, or why? Maybe they are used to, how do you know? that's my usual one, how do you know?	
Alex	The kids know that if I can't hear them, it is because they haven't responded with a because.	
	[Laughter]	
Sam	That's nice	
Alex	So after a while the kids will say, oh that's what I need to say, the reasoning behind it. So again, maybe it is because we already have that established.	
Beth	It's interesting to consider if certain structures might match up to certain topics. Maybe the structure was just slightly more aligned with the topic in that instance. So there were just less barriers for the kids to get through.	
Session 2 (00:11:09)		
Tracy	So, I'm interested in your sense that it's thinking through not just the questions but how you expect the students to respond. Is that the shift that you are trying to get…	

Asking why?		**Hypothetical utterances**
Session 2 (00:19:37)		
Sam	The fact that their reasoning is getting better, is it moving them towards the top?	

Unresolved		**Hypothetical utterances**
Session 1 (00:05:30)		
Sam	**In terms of the whole department.**	"Before you move on, would you like to pause and open up to the rest of the group, whether they have questions, comments or ideas?"
Session 1 (00:09:04)		
Simon	Mia and I followed a very similar structure.	
Session 1 (00:59:30)		
Lucy	But what I find challenging is knowing when to step in. Because it's mixed ability. For the weakest kids I probably leave them for a few minutes and I usually put timer on the board so I know how long they've had thinking about it, but I find it really difficult not helping them. But I tried to break it down and give hints to those that were stuck to try to get them to recollect how they would do it. // **We were talking about different methods of doing one question**, I want them to give me another method but typically at the end when I know everybody has understood, otherwise I feel that it makes them a lot more confused than they would have been. If you have taught them one way of doing it, and they are still grappling with that method, to have somebody else come and show another way, well I haven't got my head around this way yet and you are showing me something else. I feel more confident when a student comes to the board to offer a different method once most of them have understood. // **So, they find it hard to explain what they are doing, to put it into words**, some of them are not fluent enough, as you said, to explain, why is it 7/6 and not 5/6 and where did I go wrong, so for some of them, when they came across what the question was, I would have said it's 3/5 because I am just adding across, so some of them struggle to say why is it correct that way or why isn't it a different way.	"What strategies do we had for supporting students verbal explanations?"
Session 2 (00:13:31)		
Ellen	do you find with the lower ability that if you say are you sure it, they think that they're wrong then	
Session 2 (00:13:57)		
Miguel	Do you think that with taking notes then, I know you said there are other ways of doing, I say to my lot you do need to be able to take notes, all I say is it doesn't matter if it is dreadful to begin with, it will only get better, but I also think that is you, if you explain some of the importance of taking notes outside maths or outside even academia, then they're more receptive to it, and um, I just say they've got to make a start, and also if you ask them, if you ask for complete silence usually boredom will drive children to work.	
Session 2 (00:22:22)		
Sam	So you didn't give them any rules at all, like did you insist that there was a set of parallel lines in there or?	
Session 2 (00:31:18)		
Miguel	I've started doing this more and more recently […] so what I'm tending to do more and more is give them, give them questions to do without any help at all and then usually what happens, if I go round the class marking the work, particularly if I've got like an answer, like my own copy of the work myself, I have a work sheet, I'll circle things their getting wrong with a few notes on it then when I go through it on the board I draw out things they've got wrong […] when I'm going through the answers rather than going through the method and giving out the answer, I'll start by giving the answer at the bottom, like I'm not really interested in it, and then, and then talk about the method, because otherwise I think the danger is I think the pupils are thinking what's the number so I can tick it	"So Lucy, what did you take away from that conversation in relation to the issue you raised? Are there any actions you will be putting in place? Anything you will try to do differently?"
Session 2 (00:54:28)		
Tracy	But I'm not convinced your student was doing something algorithmically because I think it's unintuitive that if you're going from mm to cm that you divide, you want to make it…	

Unresolved		**Hypothetical utterances**
Sam	…Make it bigger	
Tracy	Yes bigger, it does feel like that should be right so it something conceptually reversed in what you actually have to do	
Session 2 (00:54:28)		
Tracy	But I'm not convinced your student was doing something algorithmically because I think it's unintuitive that if you're going from mm to cm that you divide, you want to make it…	
Sam	…Make it bigger	
Tracy	Yes bigger, so timsing by ten would make sense, there is something conceptually reversed in what you actually have to do in these kinds of questions	
Session 2 (01:00:39)		
Simon	but with a bit more…	
Sam	Yeah, I like the idea of quote board, it's really nice	
Session 2 (01:01:40)		
Paul	Can I refer back to what you [looking at Sam] were saying about the video and your learning and how that related to the reasoning and the responses in the department as a whole.	
Session 2 (01:17:29)		
Tracy	Well that's the same as the girl in your class [looking at Lucy]	"Well that's the same as the girl in your class, what do you think you will do given that issue next time you teach that class Lucy?"
Lucy	Yeah	
Session 2 (01:18:17)		
Tracy	Feels like we're back to the start, that you're not just thinking of the questions, you need to think about the consequences of these questions, what are you expecting the students to do and what then are you going to do based on those responses.	

Commitment		**Hypothetical utterances**
Session 1 (00:39:08)		
Miguel	I'll have, let's say, five quick questions on adding fractions starting on easy ones and gradually building up and I'll give them maybe, I don't know, between ten and twenty seconds to answer each question, and I'll go through them at the end and say right, take a note of the hardest thing you understand and I'll often use those partly if there's part of my lesson where I've got ten minutes left and I don't want to start something new, um, but I want them to use their time productively and also realise that the main problem with people doing well at maths is recall so if you teach something and don't revisit that for a year the chances are, almost certainly that they will have forgot it. When if you go over it, even briefly they've got a chance to remember it. And I was thinking actually it would be quite a nice structure to do with those quick questions so if I kept them, you know, once I had taught a subject and you'd do, I don't know, let's say these are the answers what are the questions you could keep that slide and then six months later you could say to them right these are exactly the same questions I gave you six months ago, you might change the numbers in them, um, and then see how, it would be a good way of getting them to recall their understanding of it.	"So if you were going to do that Miguel, how would you go about it, would you use exactly the same questions for example?"
Session (00:08:44)		
Alex	I don't know, yeah, somewhere in the middle, but that was it really. So we are breaking that down, so when we now do our meetings we are gonna have people just sort of thinking about what potential questions there are, so, breaking it down, we've got fourteen teachers, so we'll probably have six or seven in a meeting planning a lesson, but we're then gonna have two people responsible for thinking about what potential questions could come up	
Ellen	Um, yeah, and what we've done is we've put people, split people into key stage three and key stage four, and planned the actual lessons going much deeper than we would have gone if it was one person just doing it their way	
Session 2 (00:22:04)		"Great, I will add that to a list of commitments, does anybody else want
Sam	Oh I love that. I'm doing that.	
Session 2 (00:33:41)		

Commitment		**Hypothetical utterances**
		to share their own at this point?"
Sam	So I don't give them the right answer but I think actually in the future I think I will	
Session 2 (00:40:33)		
Joe	my next task is going to be show them the formula for how do you get the perimeter and area equal and then see if they can postulate you can't have	
Session 2 (00:41:07)		
Sam	Teaching in silence is a really nice idea, I'm going to give it a go.	

Importance of Context		**Hypothetical utterances**
Session 2 (00:05:10)		
Alex	The context is, we have a couple of new members of staff in the department, and a PGCE student as well. As a result of this, we said, when you come to doing your planning, you don't have to draw it out on an A3 sheet like we did in the last workshop. But do have a think about what paths the lesson could take. What sort of questioning might come up and how you could approach that?	"How does context make a difference do you think?"
Session 2 (00:11:20)		
Alex	As a whole school there has been a shift in onus on developing literacy and numeracy, so, we've linked the work on this project to that whole school focus. So we won't just accept an answer of, say, seventy-two. Why is it seventy-two? Can you expand on that? Trying to get the students to use full sentences. I think that initiative has helped the work we have done during project, because there is a wider school emphasis on expanding answers and developing oracy in the classroom.	
Session 2 (00:14:38)		
Paul	the previous cohort were really worried that there wasn't enough work being done	
Session 2 (00:20:33)		
Maria	We are trying live marking in our school at the moment and we're trying to incorporate the reasoning within that.	

Drift		**Hypothetical utterances**
Session 2 (00:52:57)		
Alex	It's like those share in a given ratios that, when it came to the harder questions, like, Aaron's got thirty-six things, its shared in a given ratio, still try and share it in a given ratio, even though two parts are thirty-six.	"Why don't we pause for a moment and share how we each approach share in a given ration questions for ourselves"
Joe	I did do something very similar, with the same/different so I had three questions, a share in a given ratio question, second question gets more than numbers the same, third question person on the left gets this much and actually on that one the reasoning structure really helped break down that barrier	
Paul	I think Ellen and I saw a lesson, an interview lesson, the other week that was about sharing into ratios ratio wasn't it, but it was…	
Ellen	Yes	
Paul	… just done, it was just beautiful, I don't get emotional about watching teachers but I was just, yeah, we need to hire that person now, because it was explained in terms of I want to give this group three times more than this group so it was instantly not using that idea and then looking at representations in order to work with the structures and um you know and definitely not going down the let's see how many parts there are, lets add them together, let's, you know there was none of that in the lesson at all but there were a bunch of year	
Ellen	It was a story, it was about giving money to penguins and	
Paul	orangutans	
Ellen	orangutans	
Session 2 (00:55:48)		
Ellen	I always think that when people, **and it is changing the subject slightly**, but when kids have to have scribes in exams, that would disadvantage me terribly, because something happens between here and here that doesn't involve the rest of me sometimes, and if I try to verbalise it, it's sometimes when you're doing a question in class and you haven't come across the question before and kids ask you to explain it I have to do it myself first, so that it goes straight from my head to my pen, without	

Drift		**Hypothetical utterances**
	saying anything, and then once I've got the answer I can work backwards verbalising	
Tracy	The saying gets in the way?	
Ellen	yeah	
Beth	For me that's where this idea of planning questioning comes from, in that actually we need to do something…	
Ellen	But I could not have done the exam using a scribe where the only chance I've got to get marks has to be spoken	
Simon	It's harder to speak than it is to write isn't it	
Ellen	yeah	
Tracy	Yes, it's like something else is going on	
Ellen	And when, when, you know, people have said to me can you tell me, like even my own children they ask me to help them with something, not anymore because they're both cleverer than me, but, um, when they, when they, when they used to ask me to write, and I'd say, oh you have to pass it to me because I can't tell you what I want to write, I have to write it	
+	yeah	
Tracy	And for me that is linked to Joe's idea of showing that set of images, it's as if the words can sometimes get in the way. Why do we need words when we have direct access to something? Students don't necessarily need to verbalise things for themselves, because the reasoning is there, within the structure of what you offer them. For me, that is connected to Ellen needing to write for the words to come.	
Ellen	Yeah, I just want to write it, yeah	
Alex	We've got one teacher in our department who's Charlie Chaplin, he doesn't, he hardly ever speaks, he just does all his stuff on the board, because, same thing, he says that if you're having to listen to the conversation you're not actually thinking, you're just listening, so he'll do lots and lots of examples in silence and then when he gets, sets them off on tasks, again, the kids that don't get it keep looking at the board, the kids that do continue working and he'll just go through example, example, example, on the board, and then he knows when the kids goes, like that, they've got it	
Tracy	They've got it	
Alex	Yeah, and you don't have to disrupt anyone, everyone's working at their own pace and they can get as many examples as they want	
Simon	It's like if you're happy, ignore me, if not, keep listening until…	
Alex	But they don't have to listen because, if you listen, the kids get distracted whereas he just doesn't talk, and just…	
Sam	But ignore me doesn't necessarily mean don't listen does it, it can mean ignore what I'm writing	
Alex	yeah	
Beth	The only thing that I'm kind of thinking there is, is it very procedural, are they following a method and what happens when you want to ask a question, does it follow the same pattern, and is that where your questioning needs to come in because you haven't checked for understanding of the concept, you've checked for replication of the process	"What do you mean by procedural here?"
Tracy	But this [pointing to Joe's images] doesn't feel algorithmic	
Beth	No that doesn't, so it depends on what you're doing	
Tracy	So, it's what is it you're doing	
Beth	Yeah, what is it, what's happening	
Alex	Yeah, yeah	
Session 2 (01:01:50)		
Sam	Yeah, so having been able to have some individual conversations with, how many are we? I think eight, so there are still two people left that I haven't yet had an individual conversation with yet, but you might remember we have maths club on a Friday after school, so they have kind of passively picked up but there is a lot of conversation at the moment about talking, questioning, and planning for the activities that are going to follow on from the questioning, and thinking about the types of responses you might get. Because we've got a lot of non-specialists.	
Maria	Sorry can I jump in. We had a twilight session, so we literally started off with open and closed questions and whether they fully understand that and it was interesting because I don't think the clarity was there.	

Drift		**Hypothetical utterances**
Sam	Particularly with some maths examples. If there is only one answer, is it an open question or a closed question?	
Maria	Yeah. Then obviously, we gave everybody the questioning template which is now on display in their classrooms and just asking them to experiment as much as possible, and seeing where they got with that so we're still trialling.	
Sam	Kind of working this way [gesturing] in this way I would say [gesturing]. We are aiming for there, but it's definitely, we keep embedding it.	
Maria	Yeah, but as I said, the minute we go on our learning walk and take these in now, you know you can hear the questioning from the teacher, you can tick as you go which is lovely.	
Session 2 (00:32:43)		
Beth	Yeah, I did something quite similar, it was year eleven, not key stage three but, we were talking about the multiple-choice questions at the start of the paper. I circled the right answer and said, "why is this correct?", so they didn't need to think about that, they knew which one was correct. Then I gave them the question again three more times with the other three answers circled and said, "now you need to tell me what the question could have been if this was the right answer". It was really interesting because they did things, I didn't expect them to do. It was a ratio one, it was what fraction of the drink is orange? I thought they would just change the type of drink each time, but they also started reordering the ratios as well, which for me, showed a better understanding. I was a bit nervous, is it the right thing to do? Just give them the right answer and hope? But actually, it worked really well because it really did help me to understand what they understood.	"A number of you have contributed to this discussion. I wonder if we can get back to the original issue? Or if you could each articulate the issue you have in mind in the examples you have given."
Sam	We've got a thing in year eleven at the moment where all of our starters is at least one multiple choice question, one, three or five multiple choice questions	
Beth	Are you AQA by any chance [laughing]	
Sam	[laughing] so we're going through that, and sometimes I just put one and say right I want you to right questions that would give me the other three	
Maria	yeah	

Research opportunity		**Hypothetical utterances**
Session 2 (00:18:47)		
Joe	In terms of judging that, I think there is a huge case for professional judgement in relation to kids reasoning. The reason is, if I look at the classes that I have been focussing on their reasoning, then the children who are consistently the best at reasoning, the ones who are able to make the connections, are those who typically fall in around quartile three in terms of their attainment.	
Session 2 (00:42:38)		
Tracy	I reckon you could do a research study on the reasons kids put their hands up.	

Chapter nine: *Going meta*

Distinctions		**Hypothetical utterances**
Session 1 (00:03:11)		
Sam	we were really lucky that Paul had time to come out and bring us up to speed, problem solving is a big issue for us. That whole problem solving, fluency, mastery, all these reasoning things they're all kind of threaded together so I had more clarity, I think, after the session with Paul, that **it was alright that they threaded together, that you didn't have to make them distinct things.**	"Does anybody here see these terms as distinct?" "How are you seeing these as distinct?" "What is the different about each of these terms?" "What does *reasoning* look like? Any examples from your classrooms?"
Session 1 (00:08:06)		
Joe	I was gonna ask, **did you teach this on its own first or did you do the structure as the way of explaining it?**	"Nice question"
Session 1 (00:30:20)		

Distinctions		**Hypothetical utterances**
Beth	But then we went into this one, and it's always, sometimes, never true and what was interesting is that you [Joe] kind of did that as a different lesson where you'd gone on to division and we'd discussed whether we should phrase it differently, so you'd [Joe] phrased it as a minus b is bigger than I think or you phrased it. We looked at whether phrasing it positively made for a different discussion and we kind of expected that the kids would be like yeah, course it is but actually they still weren't like that, they still took a lot of substitution and a lot of trial and error before they were convinced, um, so, there was a whole lesson planned but we actually just didn't get onto that we literally did those three slides and it took a whole hour. Um, what was really nice was at the end, there were two moments that really stood out for me, one was a kid going I think I was wrong Miss and talking about why that is really important to identify that your assumptions aren't always true and the other one was, well two other things, one was they got obsessed with using zero.	
Session 1 (00:45:27)		
Beth	Something I noticed even in my two lessons that I taught them was their use of language it was something I tried to be really careful with. It was minus three add seven and the child wrote minus four as the answer so they said minus three add seven is four but you've got a negative and a positive so it must be minus four. Such a classic. So I was really hammering, it's adding a negative that's the same as subtracting rather than it becoming negative and it was really fascinating to think realistically they've had a term and they've already mis-learned that topic and it was not that they were taught it incorrectly it's that the use of language had perhaps allowed them to slip into some poor assumptions. But my lesson was ok. **And substitution itself was a structure because it was a structure for exploring negatives.**	"Can you say a bit more about how you see substitution as a structure?"
Session 1 (00:47:23)		
Beth	Can I ask something, this is a very general question but I did a little survey with some primary staff to ask what people thought reasoning meant and it was amazing how many of them had the word explaining in their answer, they were all like, reasoning is explaining, its saying what you did, I was wondering how people felt about that because to me there is more to reasoning than just explaining, I was wondering how everyone thought about it.	"Well, we could go around, and each describe a moment where we think we have observed reasoning in action, so what does it look like? Have a few minutes to have a think and to make some notes before we share"
Simon	It's about asking questions, in my head if a student is reasoning they are asking themselves questions they're trying to justify their thinking but also be accepting of why I don't know taking into account different perspectives and ideas, it's not just explaining	
Sam	No explaining is more descriptive isn't it, an explanation is more of a description of what I thought rather than why I thought it maybe?	
Beth	I just thought of a question where for example if it's something add 10 good reasoning would be just do that in your head and add 10 but if they explained the column method for that they've maybe not reasoned very well because they've chosen a really inefficient method so you can explain bad reasoning.	
Paul	I think that maybe a bit of a faux amis. So the idea of, it's like when we hear the term differentiation, as teachers, we think are we talking about attainment, or are we talking about mathematical differentiation. And you know there are two differences with one word, I think, in terms of reasoning, if you are looking at literacy, literacy based subject, or for example, to get a grade A in RE, you would have to show one side of an argument, the second side of an argument and explain your reasoning throughout those. Whereas we're talking about mathematical reasoning, that's justifying and proving, and so, is it may be that case that it is the same word being used in very different ways.	
Session 2 (00:04:45)		
Alex	so we set up into three groups and thought about the script and what could go on throughout the lesson, and explained to people about what makes a rich question trying to make it as open as possible and when closed questions are appropriate and when to use more open questions,	
Session 2 (00:09:32)		
Alex	That's not collaborative planning, that's just someone planning for you	
Session 2 (00:19:50)		
Ellen	There's a lot of memory involved	
Sam	Is it because they can't reason or they can't articulate how they can reason?	
Session 2 (00:20:04)		

Distinctions		**Hypothetical utterances**
Alex	But he's so, so high order level of thinking, he can't, he can't explain to me how he gets answers, and that's the one thing…	
Session 2 (00:35:07)		
Maria	We've got a working wall, so they're display boards, and we call them working walls, they're all on the same colour throughout the maths department and that is standard procedure, SLT know when they're bringing in visitors, you know, they will come in they will find our working walls and they will.	
Sam	And its things like pictures of kids' books, photographs of what they've done on whiteboards, when they've used flipcharts stick that up there, they look a complete mess, well no that's not true they don't look a mess.	
Session 2 (00:37:14)		
Sam	It's a working wall	
Session 2 (00:39:04)		
Paul	Sounds like teaching for mastery that does.	
Session 2 (00:46:54)		
Tracy	Ok, so stop there a sec, what was the response from the children? It feels like she's waiting for a difference, a difference in opinion…	
Sam	yeah	
Tracy	that's when she's going to say it is exciting?	
Session 2 (00:50:56)		
Maria	mine would have been more visual, you know, diagrams and not so much of the, this answer, is this answer the same as this answer because yeah there's the level of confusion.	
Session 2 (00:55:44)		
Tracy	To explain what's going on and is that important or is that just fluency?	
Session 2 (00:58:39)		
Beth	The only thing that I'm kind of thinking there is, is it very procedural, are they following a method and what happens when you want to ask a question, does it follow the same pattern, and is that where your questioning needs to come in **because you haven't checked for understanding of the concept, you've checked for replication of the process.**	"What do you mean by procedural in this instance?"
Session 2 (01:00:45)		
Alex	I did **the same thing**, trick the kids.	"What do the terms open and closed questions mean to you?" "What about others in the group?
Session 2 (01:02:25)		
Maria	we literally started off **with open and closed questions** and whether they fully understand that	
Session 2 (01:10:42)		
Beth	three things divided by five people is the same as three fifths, and that actually, the numerator there is the three things you're sharing not the three bits you're getting, if that makes sense.	
Session 2 (01:12:50)		
Paul	So twelve divided by four, what we are doing is splitting our twelve into four equal parts, each with 3 in, and secondary teachers in general see this as division. Rather than grouping, where you have twelve and you split it into groups of four, and there are actually three of them. It's a real difference between what most secondary school teachers think of in terms of **sharing compared to grouping**.	
Session 2 (01:13:53)		
Tracy	that one is two fifths of one and that is one fifth of two which in the same thing, but what does 0.4 mean in that context?	

Patterns/connections		**Hypothetical utterances**
Session 1 (00:12:32)		"What was significant about the numbers do you think?" "How might you use what you have learned moving forward?"
Alex	Yeah so the numbers	
Ellen	Everything else was the same	
Session 1 (01:02:45)		

Patterns/connections		**Hypothetical utterances**
Tracy	Some of you have spoken about the fact that the structure felt difficult for them because it was the first time they had seen that structure and that might be part of what you are describing maybe?	
Session 2 (00:38:15)		
Tracy	**That is linked to your [looking at Joe] not talking**, because I was interested in that too, that you didn't sound like you were just about the teacher not talking, so what does reasoning look like if there is no talking from anybody?	
Session 2 (00:40:20)		
Simon	You know you said about the not revisiting it for a year thing, that's a really big thing for us too.	
Session 2 (00:55:38)		
Tracy	**That's similar to your idea** that the person who gets the best mark in the test doesn't necessarily know how to …	
Session 2 (00:57:02)		
Tracy	**And for me that is linked to Joe's idea of showing that set of images, it's as if the words can sometimes get in the way**. Why do we need words when we have direct access to something? Students don't necessarily need to verbalise things for themselves, because the reasoning is there, within the structure of what you offer them. **For me, that is connected to Ellen needing to write for the words to come.**	
Session 2 (01:17:29)		
Tracy	**Well that's the same as the** girl in your class [looking at Lucy]	

Teacher observes change in behaviour		**Hypothetical utterances**
Session 1 (00:10:52)		
Mia	There was hardly anybody off task in either of the classes.	
Session 1 (00:33:18)		
Beth	But for me they were a blank canvas and so from the first lesson, I was really surprised that the lower attaining kids that maybe find maths hard actually doing much better in the second lesson, I think I'd slightly adjusted how I approached it, maybe made the modelling a bit clearer, so they knew exactly to write down but they were still able to access it, but you were right there was a different level of reasoning for the kids who'd spotted that it was a rule, that came out in their explanation. It was also, because I didn't know the other adults in the room, obviously I knew you two, but I didn't know the TAs that I was working with, that was really interesting, two different ones, one each lesson and watching, like one was literally holding the pen and writing for a child and the second lesson, the same child was so much more successful and they did not have someone sat with them all of the time, and actually just, they had a kid who was really engaged and really interested and they worked really well together it just really revealed to me how the adults can really influence how the kids are working in the lesson, our assumptions about them. Not that we shouldn't, obviously you know your kids and you know when to put things in place they won't access but maybe sometimes we assume they'll need something that they don't.	"Why were you surprised do you think?" "How do others work productively with TAs?"
Session 1 (00:34:38)		
Tracy	What gave you the sense that the learning was better, what was it they were doing that meant that the learning was better?	
Joe	Well, for example, when I was giving the students the biggest, smallest task, they were reasoning more about which values to use in the lesson when I had given them a selection of values to choose from, so they could select in order to yield the result they wanted. The first time I did it, it was very much trial and error. Whereas the second time was very much more about strategy.	
Tracy	Oh OK.	
Joe	And their different approach was really noticeable.	
Tracy	So same structure both lessons, but in the first you let them choose any numbers and in the second, you gave them a selection to choose from?	
Joe	Yeah, so how they approached it was different and the speed at which they got to the correct answer and the accuracy of calculation was much stronger in the end. The students were not writing four examples per expression and stumbling on the	

Teacher observes change in behaviour		**Hypothetical utterances**
	answer, they were saying "right I think I'm going to get the right answer by following this strategy" and getting there much more quickly.	
Session 2 (00:41:11)		
Joe	Yeah, from what I thought would be kind of do and review of something at quite a low level and I'd have to really go over here's how you do area, here's how you do perimeter, actually it then turned into they did it all themselves, and you know in the class you get hands up all the time, it wasn't sir help me, it was sir look at this, look at this, look at this I did it	
Tracy	Oh, so that's nice, so the difference is in hands.	
Joe	yeah	
Tracy	The reason for the hands going up is very different.	
Joe	I found it!	
Tracy	And hands are something maybe that are measurable, it's back to this, you know what can you measure, so what are they putting their hands up to do what are they putting their hands up to say.	
Joe	Sadly, I'm not able to sight body language as an AFL technique [laughing]	

Teacher going meta		**Hypothetical utterances**
Session 1 (00:22:03)		
Alex	Questions?	
Session 1 (00:35:30)		
Beth	For me it wasn't the fact that there was lots going on like in the first task where there was literally 2a, a-b, 2a+b and a set of about six numbers they had to substitute a mixture of small, large, negative, and positive and there was just a lot of ideas happening there and then the second lesson we were literally focusing on one thing and that just meant they could really delve in and they were just less distracted by some of the other rules and that's why the learning was better. We did less maths but more maths.	"Can you say a bit more about what you mean by less maths is more maths?"
Session 2 (00:09:42)		
Alex	Any questions?	
Session 2 (00:29:22)		
Joe	a lot of what we are discussing now here is very talk based	
Session 2 (00:36:04)		
Sam	and it really feels that displays work to support the learning	
Session 2 (00:48:56)		
Sam	you shouldn't be judgemental should you because it was a fifty-minute lesson and we saw two five-minute clips and there are some rules about IRIS film club that you are not supposed to be judgemental. But it was really interesting watching somebody else, so it wasn't me and it wasn't them, it wasn't my IRIS it wasn't their IRIS, it was a neutral person	
Session 2 (00:58:52)		
Joe	**Going back to the talking thing,** about that Charlie Chaplin, and like Mia's point, just like putting something on the board and they're just thinking without saying anything, I've got a PGCE student taking my group at the minute who will just talk and talk and talk and they lose the clarity, and it's like they actually confuse the situation	
Session 2 (00:59:33)		
Simon	**So going back to your video** of that eighty divided by four or two or whatever, can't remember what it was exactly [laughing]	
Session 2 (01:01:40)		
Paul	Can I refer back to what you [looking at Sam] were saying about the video and your learning and how that related to the reasoning and the responses in the department as a whole.	
Session 2 (01:13:50)		
Beth	And I wanted to give the teachers a chance to unpick them by themselves.	
Session 2 (01:16:39)		
Beth	We looked at this [slide projected] and it was half-way through this task that the teacher who was unhappy about that being two fifths suddenly went, 'It's the pizza	

Teacher going meta		**Hypothetical utterances**
	again' when they were looking at this and that was the moment that really helped them to move on to that stage, to make that conceptual leap.	
Session 2 (01:17:47)		
Beth	I think it was quite interesting that one of the teachers who was particularly unhappy with it, I think, finds it quite challenging to know what to do when a kid doesn't get it in lessons, so I thought it was quite an interesting moment for her to be in the same position	

Existing frameworks		**Hypothetical utterances**
Session 1 (00:32:33)		
Paul	Can I ask a question? You talk about different levels and you know what you said before when we talk about reasoning a lot of the time we think about proof where we go into the algebra and doing that, if you were looking at your mixed attainment group could you refer that to those five steps and start seeing they were also reasoning but on a different level or…	
Session 1 (00:50:14)		
Tracy	So you can reason mathematically on paper, you can reason mathematically verbally, it can be in words, so it's not an easy question, what is reasoning. It's your thing, there's problem solving, there's fluency, they are not obviously distinct are they.	
Sam	Which I find very difficult.	"OK, why don't we collate these different reasoning behaviours".
Tracy	You want to categorise them.	
Lucy	What the guys said on Saturday, the AQA guy, he said, problem solving is more open ended, whereas reasoning, there's usually a right or wrong answer, but then you have to say why, say why you're right	
Ellen	It's making sense of	
Tracy	Making sense of, that's nice	
Joe	Why is it correct, justify your answer	
Simon	Is it realistic? yeah	
Beth	Because I was thinking about when we did substitution, a child that substituted a negative number into a minus b to make it bigger, their number choice was reasoning, they explained it beautifully as well but they had already reasoned by choosing a negative number not just by explaining, it would have felt wrong to say they weren't reasoning until they explained it, because they made the right number choice	
Paul	Because they're explaining their reasoning, the reasoning is not the explanation.	
Joe	The calculation justifies their answer I suppose	
Simon	If you set up a task that is to do with reasoning then my thinking is that it should provoke curiosity so students should be asking questions around it, it shouldn't be just like a… they can explain, I don't want to use the word explain, they can say why it could be that or why it couldn't be going back to the RS, both sides thing, but it should be more like, oh here's a problem, what about that, what if I did this, why did you do that, and it's the questions that are coming from it, that's what…	"This might be a good time to return to the framework you have met previously… how do you see similarities and differences between our list and this one?"
Tracy	That's lovely	
Session 2 (00:56:45)		
Ellen	And when, when, you know, people have said to me can you tell me, like even my own children they ask me to help them with something, not anymore because they're both cleverer than me, but, um, when they, when they, when they used to ask me to write, and I'd say, oh you have to pass it to me because I can't tell you what I want to write, I have to write it.	"That reminds me of a distinction from the literature between different types of knowing, such as tacit or embodied, you might be interested in reading about this Ellen."

Change in practice		**Hypothetical utterances**
Session 1 (00:28:55)		
Joe	The last thing we do, the end of every lesson we get them to reflect on their learning, and I was looking at a lot of their sentences and they're "oh I learnt today how to calculate" and things like that which I thought was not very useful **so I've got them really unpicking what they've been doing in lessons at the end and giving some examples**, the most important thing is to remember that two times a is a 2a for example if a is four, two times four is eight and getting them to really.	"Can you tell us what that unpicking looks like Joe?"
Session 1 (00:37:27)		
Miguel	I think is a bridge too far and to begin with I sort of, because I'd be worried that they'd just sit there and, because they basically tend to be spoon fed. The way that I naturally taught, is that I would explain something and then give them the work, and I've tried to move away from that. My attitude used to be, how can I explain this in the clearest possible way so that the students have to think about it as little as possible to understand it and my attitude now has become what is the least I can possibly do to explain it to them, without deliberately making it confusing, so they can make the rest of the leaps themselves.	"What do you think has led to that change in your practice Miguel?"
Tracy	That's lovely	
Miguel	So they can make the rest of the leaps themselves	
Sam	That's nice	
Tracy	That's really nice isn't it	
Session 2 (00:00:47)		
Tracy	Then I suppose the role of the listeners, everybody else, is to support that person in their reflection, so not to find solutions necessarily but maybe just supporting the person to get into greater depth or you can ask a question if something is not clear, so listening and asking questions. Just spend a few minutes now to gather some thoughts. If you want us to do some mathematics because something has come up then feel free to share some mathematics with us that we can work on together. It is going to go quiet now while you are thinking, and then whoever is wanting to start, in a few minutes or so, they please just start.	
Session 2 (00:05:58)		
Alex	because previously that's how they've honed the conversations down. So as a result, teachers said, that, I don't think that they were previously pinging back like the old table tennis to students but, I think they were then getting other students more involved in conversations, getting them to be involved in the conversations just as a facilitator. So I think, myself included probably would accept, like yes, amazing, um, without getting them… the number one discussion point or thing people were going to take away and try was convince me so it was quite powerful, convince me, or, how do you know it's right, or, can you convince the person next to you why it's right or can you convince the person they're wrong, or, how do you know they're wrong so it's having the, them having those meaningful conversations, getting them to reason with each other and reason with you about why they are correct	"So, these are potential phrases to follow up when a students gives an answer… lets collect them on the board… does anybody have anymore?"
Session 2 (00:10:43)		
Ellen	I can remember there being times in the past where if you still at the board after ten minutes in an observation that is poor teaching whereas this is taking much longer but more kids are getting involved and, and the way that you are at the board is changing as opposed to like, so, so, in fact if anything the time at the board is longer but the usefulness of that time…	
Alex	Is richer isn't it	
Ellen	… is richer yep	
Session 2 (00:29:37)		
Joe	so I did this, copied somebody at the conference completely, linking area of rectangle to area of triangle, I taught that normally last term, it didn't go down very well.	
Session 2 (00:31:18)		
Miguel	I've started doing this more and more recently […] so what I'm tending to do more and more is give them, give them questions to do without any help at all and then usually what happens, if I go round the class marking the work, particularly if I've got like an answer, like my own copy of the work myself, I have a work sheet, I'll circle things their getting wrong with a few notes on it then when I go through it on the board I draw out things they've got wrong […] when I'm going through the answers rather than going through the method and giving out the answer, I'll start	

Change in practice		**Hypothetical utterances**
	by giving the answer at the bottom, like I'm not really interested in it, and then, and then talk about the method, because otherwise I think the danger is I think the pupils are thinking what's the number so I can tick it	
Session 2 (00:35:32)		
Sam	But that, you know, if you look at that, sort of, architype, what five years ago would have been a good display, that you put up for parents evening and stays there all year, you know, at the start of a module they are completely empty, it just says key stage three working wall but then by the end its full of brilliant stuff.	
Session 2 (00: 45:17)		
Sam	But, we've now got to the stage, because we are so good at it, that it now happens in the middle of lessons	

New insight/awareness		**Hypothetical utterances**
Session 1 (00:15:53)		
Sam	So, the choice of numbers is really important in terms of the reasoning.	
Session 1 (00:20:34)		
Tracy	Does it feel like a structure you can apply in lots of different contexts. In fact you've got multiple structures haven't you going on there	"Can you tell us a bit more about how you might go about selecting the small number of examples?" "Who knows what they are teaching tomorrow, perhaps we can try it out?"
Alex	The day after our meeting I did which one oh this structure [putting things in the right order] on completing the square and solving by completing the square	
Tracy	And that was the putting it in the right order one	
Alex	Putting it in the right order and seeing if the, if the workings are correct	
Tracy	yeah	
Alex	So I think that's something that you can run with quite easily. Now obviously something, this matching task, you have to, again **it's all about finding the small examples, small number of examples that actually pick things out**	
Session 2 (00:30:57)		
Joe	I'm just wondering if there's a case with some of this reasoning, less talk is more maybe.	
Session 2 (00:42:59)		
Sam	you know sometimes when you somebody just says the right thing at the right time and you just think yeah that's just it, I was struggling with my PGCE student, getting her to engage and then I was on the teach first mentor day last week, and somebody put a slide up, and it was about learning can't happen without emotional engagement	
Session 2 (00:55:10)		
Mia	I think it's what you said, because I always think of it as you are going from a smaller unit to a bigger unit, so you need less of them, so that's how we sort of talk about it, but you're right, millimetres to centimetres is getting ten times bigger.	
Session 2 (01:14:46)		
Beth	I asked them what their kids see division as. They said they see it as sharing. Now this is a cautionary tale, because I tried to use Dienes blocks to show long division. In my head, because I'm a secondary teacher, I was sharing, and then I realised the language you use around long division is grouping, it's "how many of this go into that", so you're counting up in groups. So, I was desperately trying to model sharing with Dienes blocks, but realising I wanted groups and it was all a bit of a mess. Luckily, I learned from that experience and in the end the primary teachers planned out a sequence of lessons where they just spent a couple of lessons representing division both ways and talking about sharing and grouping.	"Can we slow down just a little Beth, so you can show us?"

Appendix 3f: Partial transcript of feedback session 3

The table below contains **my comments only** as transcribed from the third feedback session. Yellow shading indicates a section of dialogue that is then fully transcribed in the subsequent table.

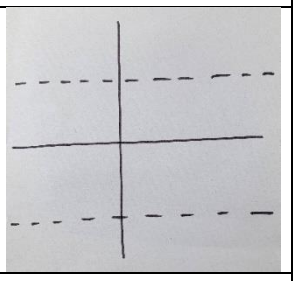
Blue: Labels from process of analysing first two feedback sessions.
Red: New labels arising from analysing feedback session 3.
Green: Labels from mathematics teaching.

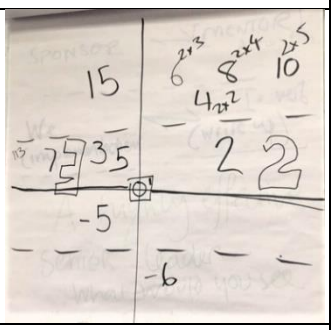
Time	Transcript	Label
0:00	You had a proof task; the gap task was around proof and proving. So, same format as usual then, if the person talking could try and stay as descriptive as possible, and, try to hold back from justifications and I did this because and this felt like this, and try and stick with what you did first of all, to paint us a picture of what happened, and then after that, once we have got a sense of what you did, and what you can remember of what the children did, we can then go into a more open discussion about the any issues you might have encountered and what you might have liked to have done differently. So when you're ready to begin, someone can just start by giving us a picture of what you did.	*Detailed description* *Slide (from account-of to account-for)* *Teaching issues* *Teaching strategies* *Detailed description*
01:55	So what did you do to get them to do that?	*Detailed description*
02:21	Can you remember what they were?	*Detailed description*
03:09	So, the teacher said that? try and convince me	*Detailed description*
03:48	Wow, that's a nice question	
06:22	What's the issue with 2n+1?	
06:38	So as an nth term with one as the first odd number	
06:54	Have you got, is that what you've got there? Your picture	
09:27	So what sorts of questions were they doing that meant that they had to factorise?	
10:50	So does anybody else have other stories about whether they did factorising first or whether factorising came through a task	*Expanding the space of the possible*
12:08	Writing it as two times thirty-five	
12:21	Yeah I guess you are factorising it	
12:40	It feels like it gives a purpose, there is a purpose there of why you would want to factorise	*Creating a need*
14:48	The power of algebra	
14:50	So other stories about the difference between show and prove, did any stories of that emerge from anyone's classrooms? There seemed to be some recognition	*Inclusion* *Ownership*
16:04	Well why don't you do this. Do you want to do this with us?	*Doing mathematics*
16:39	Just pretend we're them.	*Doing actively*
17:17	Can you put pi in there?	
17:27	OK, whole numbers.	
20:19	Can you have a negative prime number?	*Supporting mathematics learning*
20:28	I don't think I've ever encountered negative prime numbers before.	
20:59	But I don't think there are, so, OK, I'll leave it.	*Not imposing*
21:14	Lovely, what came from this then?	*Detailed description*
22:21	What do you mean by, it divided the class?	*Slide (from account-of to account-for)*
24:58	Slow down, say that again and we'll have a go at it	*Doing mathematics*
25:36	Give us a minute, we'll have a go at it	*Slide (mathematics to mathematics teaching)*
25:42	So the task was to write how many different sets?	
26:12	This is the resilience we're talking about	
26:28	I feel like I need to do this systematically	
29:19	This is an amazing task, sorry I'm just…	
33:01	So who did you do this with?	*Detailed description*

Time	Transcript	Label
33:05	And you used that slide?	*Detailed description*
33:41	That might help answer the question 'what does listening to other people's proofs offer?'	*Making it real*
35:20	Oh that's beautiful, so it's the twenty that is the powerful bit isn't it	
37:09	What was the minimally different task?	*Detailed description*
37:29	What's made the difference then there? Because, you seem to have gone from one extreme to the other.	*Slide (account-of to account-for)* *Noticing extremes*
40:32	Where's the proof in here? That's not a leading question, I'm just interested. It feels like there is proof in here.	*Distinctions*
41:10	Proof doesn't equal algebra, but there does maybe need to be some kind of generalisation.	*Distinctions*
41:54	Ah, see I began with three, three, three, three, eight	*Expanding the space of the possible*
43:07	I mean it just seems like with this, even just for the lowest attaining children, then just making sure the numbers add up to twenty is good enough, that's quite a lot going on there.	*Accessible*
43:27	That's your nice picture you showed me in there, of the mean, do you want to, you knew the page off by heart	
43:47	And it shows, it shows the twenty doesn't it	
43:53	But other than that, that lovely image of the rectangle	
44:05	It's that sharing equally isn't it	
44:14	Right school X what have you been up to?	
44:45	So it's a T that's three high and three across	*Detailed description*
45:48	Because everything is just shifted by ten	
47:09	Is it the move from the concrete and pictorial to the abstract that is problematic?	*Distinctions*
48:26	What were you doing before	
48:47	You've got to make those hard enough, I suppose, that you can't just see	*Creating a need*
48:57	Fractions and other horrible things	
49:41	It's got to be in terms of *d*	
50:04	Maybe you could try not allowing them to write the answer, your only allowed to write the calculation that gets you to answer. I'm conscious of time, I think maybe we should hear from you Lucy	*Focus on process* *Inclusion*
51:34	So what did you ask them to do?	*Detailed description*
51:44	Three boxes?	
55:40	So you've got your never true ones that you can disprove with a picture?	*Distinctions*
56:07	That's interesting because you've got number three, which is an always, so were any of them approaching the always ones and coming up with anything?	
56:23	That's not a proof though, is it?	
56:34	So how would anyone here prove that?	*Supporting mathematics learning*
56:49	But what's that based on?	
56:53	But then you have to prove that don't you?	
57:00	Yeah, that's what strikes me about a proof, is actually you're not always directly proving the thing, you've got to come steps back right back to the fundamentals	
57:23	So the 360 bit is arbitrary, but it's the relationship to the triangles, but is that enough, well no, do I believe, do I come right back to the axioms	
57:52	Is that a proof?	
58:07	I don't believe personally that is a proof, just because it looks like it is 360, but can I do something else to prove that they fit perfectly?	
58:32	Yes how many times are you going to have to do this for it to become a proof?	
58:35	It's that generalisation bit isn't it	
59:38	Your 'disproving a conjecture' is just as powerful as proving one	*Thinking mathematically*

Time	Transcript	Label
59:43	I suppose the finding of a counter-example is one mechanism to get them thinking in the general sense, so if I find one that doesn't work it then means it doesn't work for *all* so it is sort of reinforcing that all-ness	
1:01:52	Show that this is always equal to this	
1:02:24	What's quite nice about this particular task is that this is that first structure but within those always ones it is all about proof, why is it always, that's the proof. It's quarter past, we need to stop, thank you very much. I wonder if we can share the things that we have done with one another.	*Making it real*

Section of dialogue fully transcribed (from highlighted section in table above)

Time	Name	Transcript
15:45	Sam	My problem with the years nines was how was I going to make it concrete, how was I going to make proof accessible to them. So for my starter I just put that outline on the board [Sam holds up a sketch] and asked them to shout out numbers, while I put them in the spaces. They had to think about what I was doing. 
16:04	Tracy	Well why don't you do this. Do you want to do this with us?
16:06	Sam	OK then, can I use the board? [a few moments finding space on the flipchart]. So I asked them to give me some numbers.
16:39	Tracy	Just pretend we're them.
16:41	Sam	Yep OK. So I'd like you to give me a number. Maybe your favourite number or your house number and I'm going to put it in one of these boxes and I want you to work out what the boxes are. If you think you have got it, you say, no Miss, I think you have put it in the wrong box.
	Maria	Six
	Sam	Six, OK I am going to put that there.
	Joe	Zero
	Sam	Zero, I'm going to put that there for a minute.
	Simon	Eight
	Sam	Eight
	Maria	Three
	Tracy	Minus five
	Sam	Minus five
	Joe	Pi
	+	[Laughter]
	Sam	It was a bottom set
17:17	Tracy	Can you put pi in there?
	Sam	Can I put pi in there? I did actually say whole numbers to them, where would I put Pi. Ummm no.
17:27	Tracy	OK, whole numbers.
	Alex	Ten
	Lucy	Seven
	Maria	One
	Sam	I'm gonna put it in there for the minute, I'll come back to it.
	Alex	So top left quadrant, are they below the line, seven and three?
	Sam	They are below the dotted line.
	Alex	Oh right, OK.
	Simon	One hundred and thirteen.
17:54	Sam	Thanks! I think it is probably here but I will need to check that out.
	Alex	Minus six

	Sam	Minus six
	Maria	I want to know what goes top left.
	Joe	Fifteen
	Simon	There you go!
18:11	Maria	Yeah OK
	Ellen	So the y-axis is odd and even
	+	Yeah
	Sam	Yep
	Paul	Four
	Joe	Two
	Joe	x-axis is positive and negative?
	Sam	Yep
18:20	Ellen	Two is the only prime even number
	Tracy	What are the dotted lines?
	Ellen	Odd primes and even primes
	Sam	Yeah, so, odd and positive, even and positive, odd and negative, even and negative, and then the primes were in the gap. I just wanted them to think about the numbers and build on something they could do. That brought out, six is two times three, that's 2 times four, that's two times five. 
18:43	Simon	Oh, that is really clever.
	Sam	So that was useful because I knew we were about to go into the double stuff.
18:55	Alex	It is trying to get them to think about the black swan. So, all swans are white, until you find a black swan.
	Paul	Non-standard variation.
	Alex	It is yes. This guy went around doing something similar. For example, six, eight, ten. What is my rule? So, they have to be in order. So, five, six, seven would also be correct. Everyone was saying, ten, twelve, fourteen, you're correct. Twenty, twenty-two, twenty-four, correct. It is going up in twos. But until you try something different. So, it is getting them to think about negative variation. What that black swan is so they can disprove it, rather than just carrying on with the same types.
19:59	Paul	It follows the rule of the elephant doesn't it. What makes an elephant? It is a mammal that is big and grey. You go through elephants until you get to a rhino. Until then everything you described is an elephant. You need the rhino.
20:19	Tracy	Can you have a negative prime number?
	Alex	You should be able to, it's got two factors.
	Joe	Yep. But negative three times negative five would give you fifteen.
20:28	Tracy	I don't think I've ever encountered negative prime numbers before.
	Mia	But minus five has got more than two factors, hasn't it? Hasn't it? Minus one, five, minus five and one.
	Joe	But three also has negative three and negative one.
	Sam	Yes, and we don't talk about those do we.
	Becky	We only talk about positive factors.
	Alex	We've broken maths
	+	[Laughter]
20:59	Tracy	But I don't think there are, so, OK, I'll leave it.
	Sam	That is something to think about.

Appendix 3g: Partial transcript of feedback session 7

The table below contains my comments as transcribed from the seventh feedback session. Yellow shading indicates a section of dialogue that is fully transcribed in the subsequent table.

In the transcript below, I have only included my own comments, and specifically those that I have labelled using existing labels from the process of analysing first two feedback sessions.

Time	Transcript	Label for
11:20	Can you elaborate a little bit more on what you mean by getting from A to B?	*Slide (from account-for to account-for)*
15:57	What is it that tells you they are enjoying it?	*Slide (from account-for to account-for)*
16:22	Can you take us back to the lesson and what happened?	*Detailed description*
18:47	We have got a bit of time after the break to do some maths together so perhaps you could try something out with us?	*Doing mathematics*
23:00	Is there anything else about getting written communication more established?	*Unresolved* *Teaching strategies*
26:35	Don't you want them thinking throughout the lesson?	*Inner/outer* *Importance of context*
27:30	I think that is really important, there is something powerful about doing it as well as talking about it.	*Doing mathematics* *Doing actively*
28:51	Otherwise, it gets stored away, whereas if it is something I can do now, I am more likely to give it a try.	*Making it real*
48:10	Can anybody here suggest a way forward based on that issue?	*Teaching strategy*

Section of dialogue fully transcribed (from highlighted section in table above)

Time	Name	Transcript
20:30	Simon	You know the talking mathematically thing. From a student perspective, in my lessons, I feel like students are a lot more willing to discuss and consider. For example, during an always, sometimes, never activity, producing a counter-example is now a standard thing. They know if something is sometimes true, to give a reason why, and an example of both. So, I've found that they are really good at being able to discuss, but they are not yet able to communicate their reasoning well on a page. So, I think their reasoning, in terms of verbal reasoning, is really good. In terms of their written communication, it is not necessarily coming through yet and I don't know if that is something I am doing.
21:35	Vicky	I've been giving the students sentence starters quite a lot.
21:39	Simon	Yeah, I've never even thought to do that.
21:45	Joe	There is something maybe about committing it to paper that scares some of them. I give the students mini-whiteboards before they commit to paper.
22:00	Vicky	I started giving them sentence starters like "I think… because of…" and "I can show this by…". To start with I would give them different options and they could just cross off the ones they disagree with.
22:30	Harry	What I like is that you have really thought about more than the task, sometimes people think the task is the answer to getting students reasoning, but there is more to it than that.
23:00	Tracy	Is there anything else about getting written communication more established?
23:08	Leo	Yeah, I've got a standard form I use for all of the reasoning activities that I give the classes. So, its essentially an experiment area, followed by a conclusion area. So, by using it in a standard way, you're encouraging the students to experiment first of all, so even if they don't know really what's going on. So, it's really getting them into that habit; if you're stuck with a question, the first thing you do is experiment, you try something. You can get it wrong. Then that conclusion area is for that sentence, that key thing that I want them to take away. In terms of getting them writing what they are thinking, that has been my way into it, and making it routine. I've got about ten or fifteen of these now, and a list of those I want to make. I have seen a difference there in the classes, in that they know what do to, it's the same

		as they have been told before. Even if they don't know what is going on, they know that they need to have a go.
24:35	Vicky	When have you been doing these? At the start of the lesson? At the end?
24:40	Leo	The start of each lesson would be some mixed practice, then we might do some examples and then this would be the finish. Typically, it is about five minutes of them trying things out, maybe writing a conclusion if they feel confident to, and then about five minutes of us talking about it as a class, and getting to the conclusion that I want them to get to.
25:02	Vicky	So, like a class conclusion.
25:09	Leo	Exactly, and then what I will do is wipe off the board, and then the form is double sided, so they can write their own version on the back and then the check is that I ask three different people to read out their own versions, word for word. I am making sure they have clear sentences and that it is coming from them, rather than writing it down from the board.
25:35	+	That's really nice.
25:40	Simon	That is really nice, I'm going to give that a try.

Appendix 3h: Structured activities/structures presented during workshop 1

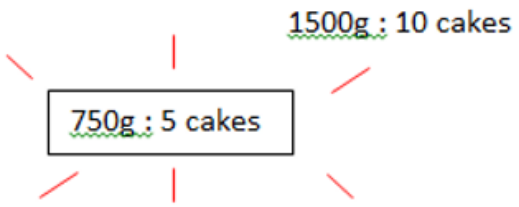
Available from: https://carmelarchimedesmathshub.org.uk/resources-before-import/ks3-reasoning-project/

Generating opportunities *to develop learning and deepen understanding*

Strategies:	***The stimulus question/focus:***
The strategies/activities outlined below offer opportunities for varying degrees reasoning. They invite pupils to reason and justify including; making sense, representing and explaining approaches and calculations	*This would be the focus to which one (or more) of the activities below might be chosen and applied to generate rich reasoning opportunities for pupils in the lesson.* *The stimulus or focus might be a particualr question, context or any mathematical object which might be at the start or at any point in the teaching of the topic.*
The Strategy:	***The stimulus question/focus:*** **A 750g bag of flour costing £1.20 provides enough flour to make 5 sponge cakes**
Hide and reveal *Carefully chosen parts of the presented stimulus are covered up and pupils are asked to consider in turn what each covered bit might be. Thinking time may be given (e.g., in pairs 2 mins –then feedback) with pupils explaining and justifying their suggestion in each case. The fun bit is the revealing (following each suggestion and discussion). This is easily done electronically with a moveable rectangle or physically by removing a posit etc.*	**A [ ]g bag of [ ] costing £[ ] provides enough [ ] to make [ ] sponge cakes**
Sketch a diagram/picture to represent this information *Pupils need to **make sense** of the stimulus and may struggle to do so particularly where the situation or information is unfamiliar, detailed or complicated. A useful strategy for some pupils is to ask them to represent the situation and its information in a drawing/sketch of **their own** - later these might support a link to more mathematical representations – important to emphasise not art lesson, just facts).*	 *Or* 

What do you know?

Using the information/data given to deduce further knowns. Typically expressed in a 'spider diagram' for pairs of data but can be more creative.



What questions could you ask?

Depending on the context and information given in the stimulus, a wide variety of suggestions may offer themselves up to the imagination. Starting fairly open this should then focus on more specific mathematical questions (many of which might anticipate the scope of possible exam questions)

- **What is the cost of one sponge cake? What is the weight of flour needed?**
- **How much flour needed to make _____ sponge cakes?** *(including and fractions)*
- **How much does one gram of flour cost, 1Kg? ..... *etc.***

Here are some calculations - what could be the questions?

Particularly powerful if a selection of data is given in the context from which a variety of questions could be based on. It can offer a range of challenge demanding pupils to talk through their reasoning in each case.

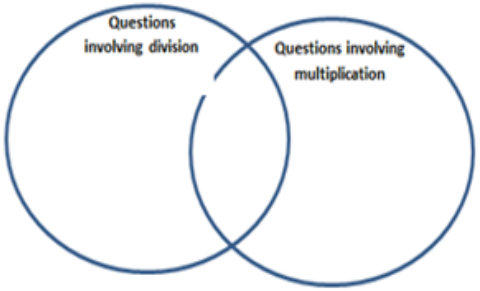
Given the above flour context, what questions could these calculations be? Explain your reasoning.

$\frac{750}{5}$ $\frac{750}{1.20}$ $\frac{1{\cdot}20}{5} \times 50$ $\frac{120}{750}$ $\frac{750}{120}$ $\frac{1.20}{750} \times 1000$ $\frac{1{\cdot}20}{5}$

Classification

Covers a wide range of rich activities which might involve Venn diagram, two way tables, flow chart… grouping – equivalent representations, calculations card sort etc

Or



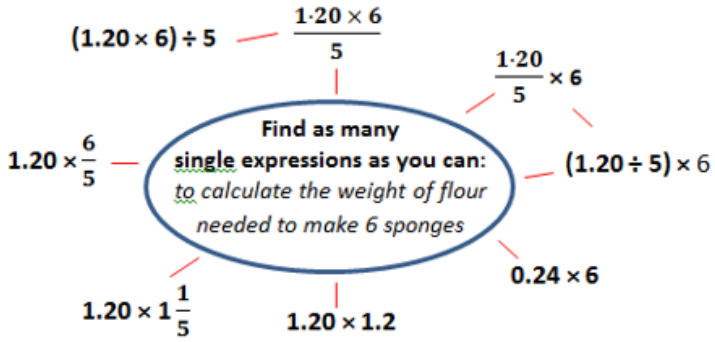
	Solutions one step	Solutions two step
Solutions involving unitary method		
Solutions involving scale factor		

As many ways as you can

Choose a question/expression/ and solve/represent it in as many ways as you can. Rich opportunities for pupils to justify why the different solutions or expressions are equivalent.

Approach A: Find the weight of flour needed to make 8 sponges – *find as many different methods for solution as you can.*

Approach B: Equivalent expressions



Explain/show why the calculations are equivalent

Sequencing solutions *(reading image activity)*

Typically several lines of working out comprising the solution to a problem/s or perhaps a series of images/instructions on how to do something. These might be in a 'cut up' form with the challenge for pupils to put them in order and annotate them to make sense of each line.

750g bag of flour costs £1.20 and has enough flour for 5 sponges. ***A discount is offered for purchases over a certain amount.***

A pupils working out to a particular problem related to this information is shown below. ***Rearrange the working into the order you think the pupil used, deciding what the question was. Next to each step write an explanation of its purpose.***

14.40 − 3.60 = 10.80	*14.40 × 0.25 = 3.60*
0.24 × 60 = 14.40	*1.20 ÷ 5 = 0.24*

What other ways might this problem be solved?

Change the numbers to create your own questions

Keeping the same question structure; change key data/ numbers and solve. How do the calculations change? Is the procedure used still the most efficient?

- ***A ____ g bag of flour costing £____ provides enough flour to make __ sponge cakes. Find the weight of flour needed for ____ cakes.***
- ***Find the cost***

How does the working out change in each case? Is the method used still the best in each case?

Change the context

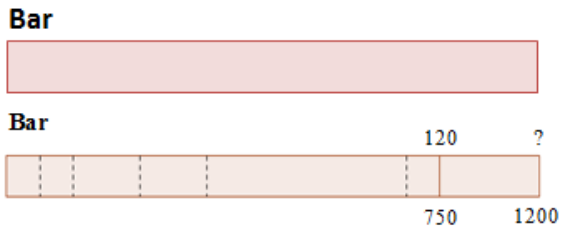
So that the problem changes but the maths is the same.

- ***A 450g jar of drinking chocolate powder costs £___ and is enough to make 30 cups***
- ***5 Litre Tin of paint costs £7.50 and covers 12 square metres***

How do you know you are right?

The challenge to the pupils is consider how they might explain/prove to someone else that their method is right. This is not simply talking though the method or the calculations they have used, but demonstrating to the satisfaction of another pupil why the method gives the right answer. Pupils might work in pairs and be encouraged to use visual images to support explanations.

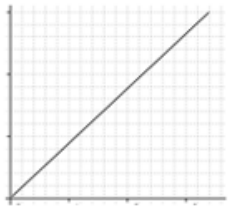
Pupils may choose to construct a visual image –such as a bar model to help them explain why their method works.



Using different models

How might you solve this problem using one of the following: a graph, algebra, enlargement … What are the links? How do key parts of a solution using one model appear or relate to the solution in a different model?

How could you solve this problem using any of the following:



Algebra

$y = kx$

$y = \frac{120}{750} x$

$y = \frac{120}{750} \times 1200$

$y = 0.16 \times 1200$

$y = 192$

Questions to promote reasoning • *Why do you think that …?* • *Can you explain why that is right?* • *How do you know?* • *How did you reach that conclusion?* • *What might explain that …?* • *How is that possible?* • *Can you show me …?* • *Is there another way …?* • *What explanation do you think is best …?* • *Have you tried all the possible cases?* • *Does it always work? Why?* • *What do you notice when …?*	
Sometimes, always, never Present statements, and ask children to decide if they are always true, sometimes true or never true. They need to give reasons for their decisions.	• whole numbers that end in 3 are divisible by 9; • the sum of any three consecutive numbers is a multiple of 3; • numbers that are multiples of both 8 and 5 are multiples of 20; • it is always possible to form a triangle from any three sticks chosen from three 4 cm sticks and three 9 cm sticks; • a quadrilateral with three equal sides must be a trapezium.
Testing conjectures and investigating rich tasks	***Pupils work on rich problems or stating points which give pupils a variety of opportunities to demonstrate their own reasoning or compare with those of others.*** ***Many sources of good problems e.g., NRICH (reasoning with number, algebra, geometry, statistics) E.g.,*** Think of a two-digit number, reverse the digits, and add the numbers together. What happens and why?
Living graphs *Can you create a 'story' to interpret the shape and key points for a given graph in terms of the original problem/context? This will involve explaining and annotating the graph itself and presenting to others.*	***Example: - Washing up graph***

Appendix four: Ethics documents

Appendix 4a: Context one – full ethics application

Name(s): Tracy Helliwell
Proposed research project: PhD
Proposed funder(s): NA
Discussant for the ethics meeting: Alf Coles
Name of supervisor: Laurinda Brown/Alf Coles
Has your supervisor seen this submitted draft of your ethics application? Yes

Please include an outline of the project or append a short (1 page) summary:
Looking at teacher progress – supporting teacher working on an issue within her teaching. Initial meeting to discuss possible project and teaching issues was held on October 19th – meeting notes sent to teacher following this initial meeting to summarise discussion and confirm plan.

Observe lessons every Wednesday (where possible) from October 31st 2016. In the lessons I will make notes – attempting as far as possible to write down what the teacher says/does and what the students say/do – an account of the lesson that will be referred to during discussion after the lesson between myself and the teacher. I might also jot down any questions that arise as I observe. I am happy to participate in the lesson as much or as little as the teacher would like me to – this will be agreed prior to the first lesson observation.

Week 1: 10a period 5 (1:30pm) followed by discussion

Week 2: 11a period 1 (9am) and 10a period 2 (10am) followed by discussion

Discussion can be as long or short as the teacher would like – I am mindful of not taking too much non-contact time. I would like to audio record the discussions after the lessons. I will try to transcribe these quickly after the meeting so this can be sent to the teacher in advance of the following week. We might start each weeks' conversation looking at last weeks' transcript for example and reflecting on it then moving to talk about the lessons. Audio recordings will be between teacher and myself – no recordings of students. Each meeting will include a time (not audio recorded) where we touch base – to discuss any issues – for example – how the project is going, how my participation in the classroom feels, how long we would both like to continue.

Ethical issues discussed and decisions taken:

Researcher access/ exit
I have known the teacher for over two years. She contacted me initially while I was still teaching in schools – we met and talked at length around curriculum. She struck me as somebody who wanted to work on her mathematics teaching – she had trained as a science specialist and had moved to being head of maths in school.

She is also an associate tutor on the ITT programme for which I am a subject tutor which was another reason for communication to take place.

I first contacted her about this research opportunity on October 12th by email – we met on October 19th in her classroom in the school. We had an initial meeting where we agreed some principles and an outline for the project. This was confirmed by follow up email.

Observations and conversations can go on for as long or short a time as the teacher wants them to. We will return to the question of exit as often as is needed – there will be time set aside each meeting to discuss this issue along with any other issues that arise. This part of our discussions will not be audio recorded.

There must be acknowledgement that the possible scenarios could occur: that either wants to continue but the other person does not. This is an issue to be sensitive to.

Information given to participants

Information has been shared with teacher since initial contact. Initial meeting and proposal for project have been communicated with the participating teacher. Ongoing dialogue between myself and the participant with take place.

A letter will also be provided to the head of the school explaining the project.

Participants right of withdrawal

The right to withdraw has been made clear to the participating teacher and this can be done at any point over the time of the research.

Informed consent

Consent has been sought and given.

Complaints procedure

The route through which to make complaints have been detailed for the participating teacher.

Safety and well-being of participants/ researchers

The aim of the research is to support a teacher in working on an issue in their teaching. Time will be given each meeting to ensure that the participant can raise any issues. We have already discussed any issues of workload due to my visits – there is no expectation on the teacher to do anything other than her usual teaching practice – this will be monitored as the presence of any body in a classroom may cause a certain level of stress that needs to be avoided whenever possible, it may be that my visits produce a heightened level of awareness that may be tiring but beneficial to the learning process of the participating teacher. The purpose of the study is to observe and understand progress made towards an issue identified. This should be a positive experience for the participating teacher – the participant has made it very clear that this will be a purposeful experience for her.

It was clear from my initial visit that the priority of the teacher is with her students – this is of upmost importance and will be tracked throughout the project.

Anonymity/ confidentiality

All data collected will be anonymised. Any audio recordings will be stored on the university server and transcribed as soon as possible. Names will not be used in any publications using the data.

Data collection

The primary method of data collection is observation field notes. There will be audio recordings of discussions with the participating teacher after the lesson observations.

Data analysis

Data will be analysed in terms of the progress of the teacher.

Data storage

Audio recorded data will be stored on a password protected laptop until it is transferred onto the university server as soon as is possible the same day.

Data Protection Act

Data will be stored and used in accordance with the Data Protection Act (1998).

Feedback

Transcripts from the discussions will be emailed to the participant in advance of the following meeting – however there is no expectation that this must be read. Field notes will be used as part of the discussion to provide a source of detail from the lessons observed. These observation notes will act as a reference from the lesson – detailing what the participant teacher says during the lessons.

Responsibilities to colleagues/ academic community

My role as researcher in this project mirrors my role as PGCE tutor for the Graduate school of Education – this has been made explicit. If the participant would like me to help support the students in her class in their mathematics, then I am happy to do this and we will continue to discuss and review my role each week.

Reporting of research

Outcomes of the research will be communicated via journal articles and conference presentations and ultimately in my thesis. This will be made clear to participants and permission has been sought for these activities.

If you feel you need to discuss any issue further, or to highlight difficulties, please contact the GSoE's ethics co-ordinators who will suggest possible ways forward.

Signed:	Tracy Helliwell	(Researcher)
Signed:	Alf Coles	(Discussant)
Date:	30/10/2016	

Appendix 4b: Context one – participant information/consent form

Information and Informed Consent Form

The purpose of this study is to track the progress of a teacher working an issue in their teaching. By doing this, some insight may be gained as to how a teacher's practice might change over time by working with a teacher educator. I will also be tracking my own progress as a teacher educator throughout the time of the project as well.

This will involve the researcher on the project (Tracy Helliwell) visiting the participating teacher's school, observing lessons, and taking field notes (photographs of resources used, including whiteboards, may also be taken from the lessons). Reflective discussions after lessons observed with the teacher will be audio-recorded. Audio recordings will be transcribed, and all names kept anonymous, including the name of school, any students, and teachers or any other names given during the recordings. The transcriptions will be used in the research, as may any field notes, photographs of resources used.

All data which is stored will be treated as confidential and handled in accordance with the Data Protection Act (1998) and will be kept on password protected computers and servers. Anonymised data will be used within the research project and within articles and presentations about the research. If you chose not to take part, this will not affect you in any way. Your data will be excluded from any reporting on this research and the audio recorder can be switched off at any point during discussions.

Please tick each box as appropriate:

1.	I understand the information about the project which has been explained to me during the initial meeting.	☐
2.	I have been given the opportunity to ask questions about the project and my participation.	☐
3.	I voluntarily agree to participate in the project.	☐
4.	I understand I can withdraw at any time without giving reasons and that I will not be penalised for withdrawing nor will I be questioned on why I have withdrawn.	☐
5.	I understand that information collected for the research will be kept securely and anonymised.	☐
6.	I give permission for anonymised data to be used in writing about this research (e.g., articles and presentations).	☐
7.	I agree to post-lesson discussions to be audio recorded and transcribed for the purposes of this research, and for field notes, and photographs of resources to be used for the purposes of this research.	☐

_____ _____ _____

Name of Participant Signature Date

If you have further questions about the research, please contact Tracy Helliwell via email [redacted]

If you are unhappy about any of this research, or being asked to participate, then you can contact the Graduate School of Education's ethics coordinator, [redacted]

Appendix 4c: Context one - ethical approval

Subject: Ethics Online Tool: application signed off
1 message

From: Research Governance and Ethics Officer [redacted]
To: [redacted]

Your online ethics application for your research project "Seeing teacher progress" has been granted ethical approval.

Please ensure that any additional required approvals are in place before you undertake data collection, for example NHS R&D Trust approval, Research Governance Registration or Site Approval.

For your reference, details of your online ethics application can be found online here: http://www.bristol.ac.uk/red/ethics-online-tool/applications/44061

Appendix 4d: Context two – full ethics application

Name(s): Tracy Helliwell
Proposed research project: An Enactivist Perspective on Becoming a Mathematics **Teacher Educator:** Using the Discipline of Noticing on Reflective Discussions with Teachers of Mathematics

Proposed funder(s):

Discussant for the ethics meeting: Laurinda Brown

Name of supervisors: Laurinda Brown, Alf Coles

Has your supervisor seen this submitted draft of your ethics application? Yes

Please include an outline of the project or append a short (1 page) summary:

My research problem, within the domain of mathematics teacher educator learning, is becoming a mathematics teacher educator through self-reflection on my own professional trajectory through in-service activity with practising teachers working on what I notice.

My research question is:

> How am I becoming a mathematics teacher educator?

In researching "How am I becoming a mathematics teacher educator?" I will be conceptualising how and what I am learning (in becoming a mathematics teacher educator).

I am the Higher Education Institute lead for one of the projects led by one of the Maths Hubs. There are 35 strategically situated maths hubs across England, each funded by the Department for Education (DfE) and can be described as mathematics leadership networks, which involve schools, colleges and other organisations with mathematics education expertise from across the hub's area. Maths hubs work in partnership with the National Centre for Excellence in the Teaching of Mathematics (NCETM). Each maths hub network includes multiple teacher research groups and projects such as the key stage three (ages 11-14 years) mathematical reasoning project. The reasoning project that I am involved in consists of ten mathematics teachers from five different secondary schools. The project focus is on developing the mathematical reasoning of students at key stage three and the work group meets four times over the year. One role I have been given is to facilitate a 'deep reflection' (a term that has come from the project designers), where each of the ten teachers talk about their recent experiences, from their classrooms and wider departments, which are linked to the project.

I will be attending four meetings of the group in the 2017-18 and facilitating a reflective discussion in three of these meetings; each around an hour long. The meetings will begin with the reflective discussion and I will audio record each one. Additionally, between the first two meetings of the group, I will make a visit to three of the schools of participating teachers when they will have been trying something in a lesson that has stemmed from the work of the reasoning group and audio record the discussions with the teachers after observing the particular lesson.

Ethical issues discussed and decisions taken (see list of prompts overleaf):

Researcher access/exit

I was approached and asked to join the project as a HEI representative by the maths hub lead. Permission will then be sought from the maths hub lead to collect data during the workshops in form of audio recordings. I will then approach each member of the group who will be given an information form and time to consider a response or think of any questions they want to ask. In the initial meeting of the group, no recording will be made but time will be spent in the meeting talking through the proposed research and time for questions to be asked will be given. Consent forms will be signed in the initial meeting.

In terms of access to the individual schools, I will seek to get access through the individual teachers, and I will conform to school policy including the school's ethical policy. If an information form is required, for example, by the head teacher, this will be provided.

Post-lesson conversations can go on for as long or as short a time as the teacher wants them to. We will return to the question of exit as often as is needed – there will be time set aside each meeting to discuss this issue along with any other issues that arise. This part of our discussions will not be audio recorded.

There must be acknowledgement one possible scenario that could occur is that either myself or the participant may want to continue with the visits/feedback but the other does not. This is an issue to be sensitive to.

Information given to participants

Information will be shared with teachers from initial contact. An initial meeting will be set up and the proposal for project will be communicated with the participating teachers. Ongoing dialogue between myself and the participants will continue to take place.

Participants right of withdrawal

The right to withdraw will be made clear to the participating teachers and this can be done at any point over the time of the research. This can be done by either approaching myself or my supervisor: [redacted]

Informed consent

Informed consent will be sought and given. In terms of school permission, this will be sought as required by the school.

Complaints procedure

The route through which to make complaints will be detailed for the participating teachers. If you are unhappy about any of this research, or being asked to participate, then you can contact the Graduate School of Education's ethics coordinators, [redacted] or my supervisor: [redacted]

Safety and well-being of participants/researchers

Time will be given each meeting to ensure that the participants can raise any issues or ask any questions. We will discuss any issues of workload due to my visits - there is no

expectation on the teachers to do anything other than usual teaching practice or practice based on commitment to working on the project which is unrelated to my research.

The presence of anybody in a classroom may cause a certain level of stress that needs to be avoided whenever possible, it may be that my visits produce a heightened level of awareness that may be tiring but beneficial to the learning process of the participating teacher and the students within the classrooms. This should be a positive experience for the participating teacher.

The priority of the teacher is with their students – this is of upmost importance and will be tracked throughout the project.

Anonymity/confidentiality

All data collected will be anonymised. Any audio recordings will be stored on the university server and transcribed as soon as possible. Names will not be used in any publications using the data.

Data collection

The primary method of data collection will be audio-recorded conversations (group and individual).

Data analysis

Data (audio recordings) will be analysed in terms of what I notice so I will be working on the data through transcription – using the raw data (transcriptions) to produce data in the form of noticings (Discipline of Noticing). I will be paying attention to what I do/say in the moment on the audio recording as well as what gets noticed during the transcribing process.

Data storage

Audio recorded data will be stored on a password protected laptop until it is transferred onto the university server as soon as is possible the same day.

Data Protection Act

Data will be stored and used in accordance with the Data Protection Act (1998).

Feedback

Transcripts from the discussions will be emailed to all participants for feedback. However, given possible workload issues, there is no expectation that these will be read.

Observation notes will be used as part of the post-lesson discussion to provide a source of detail from the lesson observed. These observation notes will detail what the participant teacher says during the lessons. They will not form data for the study.

Responsibilities to colleagues/academic community

My role as researcher in this project mirrors my role as PGCE tutor and master's tutor for the Graduate School of Education. If any of the participants that I observe would like me to help support the students in their classrooms, then I am happy to do this, and we will continue to discuss and review my role on an ongoing basis.

Reporting of research

Outcomes of the research will be communicated via journal articles and conference presentations and ultimately in my thesis. This will be made clear to participants and permission has been sought for these activities.

If you feel you need to discuss any issue further, or to highlight difficulties, please contact the GSoE's ethics co-ordinators who will suggest possible ways forward.

Signed:	Tracy Helliwell	(Researcher)
Signed:	Laurinda Brown	(Discussant)
Date:	04/07/2017	

Appendix 4e: Context two – participant information/consent forms

Note: there are two consent forms here. Given at the start of each year of the two-year project with collaborative group of mathematics teachers.

Information and Informed Consent Form

University of BRISTOL

The purpose of this study is to track the progress of teachers working on a common issue (in particular – related to the development of mathematical reasoning of Key Stage 3 students in their classrooms). By doing this, some insight may be gained as to how teachers change practice overtime by working with other teachers and teacher educators. I will also be tracking my own progress throughout the workshops in terms of my role as a teacher educator.

This will involve the researcher on the project (Tracy Helliwell) audio recording reflective discussions with the group of teachers involved on the Key Stage 3 reasoning project – across all four workshops. Audio recordings will then be transcribed but all names kept anonymous, including the names of schools, students, teachers or any other names given during the recordings. The transcriptions will be used in the research. Data may also be collected from written responses to the 'initial thoughts' part of the self-evaluation toolkit and written responses to the school reports which will be filled in after each workshop. Between the workgroup meetings, there is an opportunity for the researcher to visit individual teachers in their schools to observe a lesson and have a post-lesson discussion. If you volunteer for this the researcher will seek consent to audio record the post-lesson discussion and use in the same way to the audio recordings of the reflective discussions.

All data which is stored will be treated as confidential and handled in accordance with the Data Protection Act (1998) and will be kept on password protected computers and servers. Anonymised data will be used within the research project and within articles and presentations about the research. If you chose not to take part, this will not affect you in any way. Your data will be excluded from any reporting on this research and the audio recorder can be switched off when you are talking during discussions.

Please tick each box as appropriate:

1.	I understand the information about the project which has been explained to me during the session on 12/12/2016.	☐
2.	I have been given the opportunity to ask questions about the project and my participation.	☐
3.	I voluntarily agree to participate in the project.	☐
4.	I understand I can withdraw at any time without giving reasons and that I will not be penalised for withdrawing nor will I be questioned on why I have withdrawn.	☐
5.	I understand that information collected for the research will be kept securely and anonymised.	☐
6.	I give permission for anonymised data to be used in writing about this research (e.g. articles and presentations).	☐
7.	I agree to some discussions during workshops to be audio recorded and transcribed for the purposes of this research.	☐

Name of Participant _____ Signature _____ Date _____

If you have further questions about the research, please contact Tracy Helliwell via email [redacted]

If you are unhappy about any of this research, or being asked to participate, then you can contact the Graduate School of Education's ethics coordinator, [redacted]

Information and Consent Form

University of BRISTOL

For this study I am asking the question: How am I becoming a mathematics teacher educator? In researching "How am I becoming a mathematics teacher educator?" I will be considering how and what I am learning through in-service activity with practising teachers working on what I notice.

This will involve the researcher on the project (Tracy Helliwell) audio recording reflective discussions with the group of teachers involved on the Key Stage 3 reasoning project – across three of the four workshops. Audio recordings will then be transcribed but all names kept anonymous, including the names of schools, students, teachers or any other names given during the recordings. The transcriptions will be used to inform the research and will be sent to participants for feedback. Between the workgroup meetings, there is an opportunity for the researcher to visit individual teachers in their schools to observe a lesson and have a post-lesson discussion. If you volunteer for this the researcher will seek consent to audio record the post-lesson discussion and use in the same way to the audio recordings of the reflective discussions.

All data which is stored will be treated as confidential and handled in accordance with the Data Protection Act (1998) and will be kept on password protected computers and servers. Anonymised data will be used within the research project and within articles and presentations about the research. If you chose not to take part, this will not affect you in any way. Your data will be excluded from any reporting on this research and the audio recorder can be switched off when you are talking during discussions. You have the right to withdraw at any point during the study.

Please tick each box as appropriate:

1.	I understand the information about the project which has been explained to me during the first meeting of the workgroup.	☐
2.	I have been given the opportunity to ask questions about the project and my participation.	☐
3.	I voluntarily agree to participate in the project.	☐
4.	I understand I can withdraw at any time without giving reasons and that I will not be penalised for withdrawing nor will I be questioned on why I have withdrawn.	☐
5.	I understand that information collected for the research will be kept securely and anonymised.	☐
6.	I give permission for anonymised data to be used in writing about this research (e.g. articles and presentations).	☐
7.	I agree to some discussions during workshops to be audio recorded and transcribed for the purposes of this research.	☐
8.	If I am visited in school, I understand that the researcher will seek consent to audio-record post-lesson discussions.	☐

Name of Participant _____ Signature _____ Date _____

If you have further questions about the research please contact Tracy Helliwell via email [redacted]

If you are unhappy about any of this research, or being asked to participate, then you can contact the Graduate School of Education's ethics coordinator, [redacted] or my supervisor [redacted]

Appendix 4f: Context two - ethical approval

Subject: Ethics Online Tool: application signed off
1 message

From: Research Governance and Ethics Officer [redacted]
To: [redacted]

Your online ethics application for your research project "An Enactivist Perspective on Becoming a Mathematics Teacher Educator: Using the Discipline of Noticing on Reflective Discussions with Teachers of Mathematics" has been granted ethical approval.

Please ensure that any additional required approvals are in place before you undertake data collection, for example NHS R&D Trust approval, Research Governance Registration or Site Approval.

For your reference, details of your online ethics application can be found online here:
http://www.bristol.ac.uk/red/ethics-online-tool/applications/56185